

Probability and Statistics

Prepared by Dr. Alfaisal Hasan

STA 201
Probability and Statistics

Lecture NO 5





مجموعة جامعة
الجميع الذكية
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MIDOCEAN
UNIVERSITY

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STA 201

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Chapter

Topics

- Chapter 1: Introduction To Statistics
- Chapter 2: Descriptive Statistics
- Chapter 3: Probability
- Chapter 4: Discrete Probability Distributions
- **Chapter 5: Normal Probability Distributions**
- Chapter 6: Confidence Intervals
- Chapter 7: Hypothesis Testing with One Sample
- Chapter 8: Hypothesis Testing with Two Samples
- Chapter 9: Correlation And Regression
- Chapter 10: Chi-Square Tests and The F -Distribution

Normal Probability Distribution

5.1 Introduction to Normal Distributions and the Standard Normal Distribution

5.2 Normal Distributions: Finding Probabilities

5.3 Normal Distributions: Finding Values

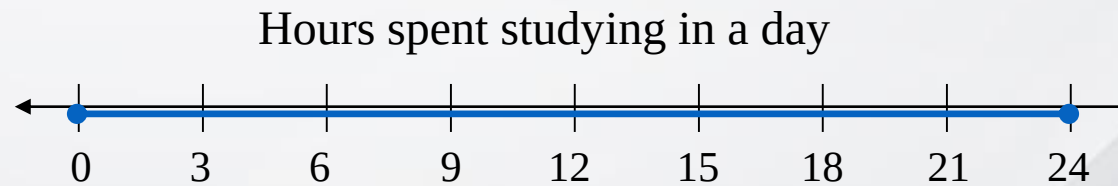
5.4 Sampling Distributions and the Central Limit Theorem

5.5 Normal Approximations to Binomial Distributions

Introduction to Normal Distribution

Continuous random variable

Has an infinite number of possible values that can be represented by an interval on the number line.



The time spent studying can be any number between 0 and 24.

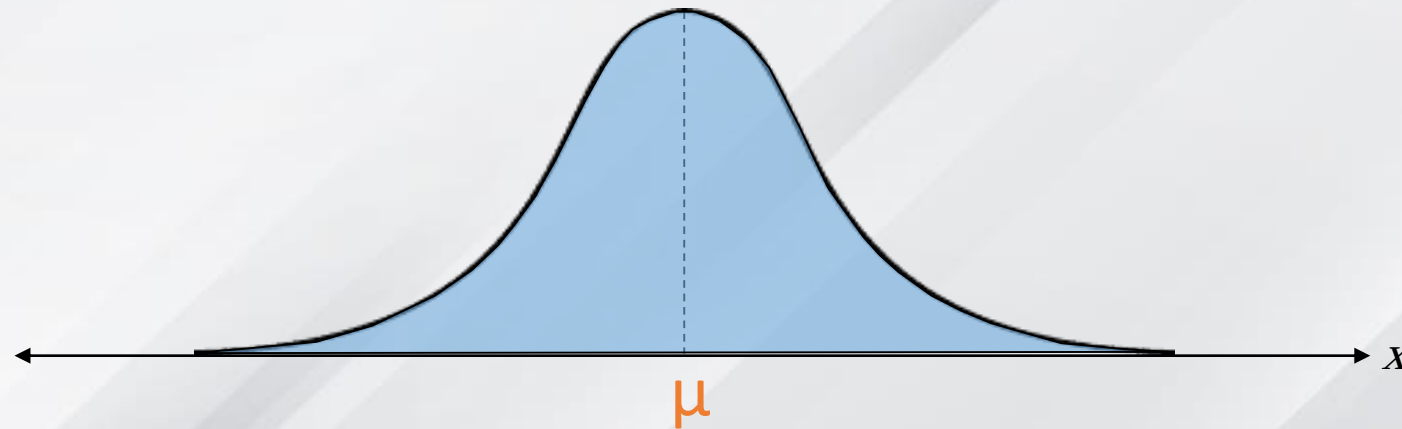
Continuous probability distribution

The probability distribution of a continuous random variable.

Properties of Normal Distribution

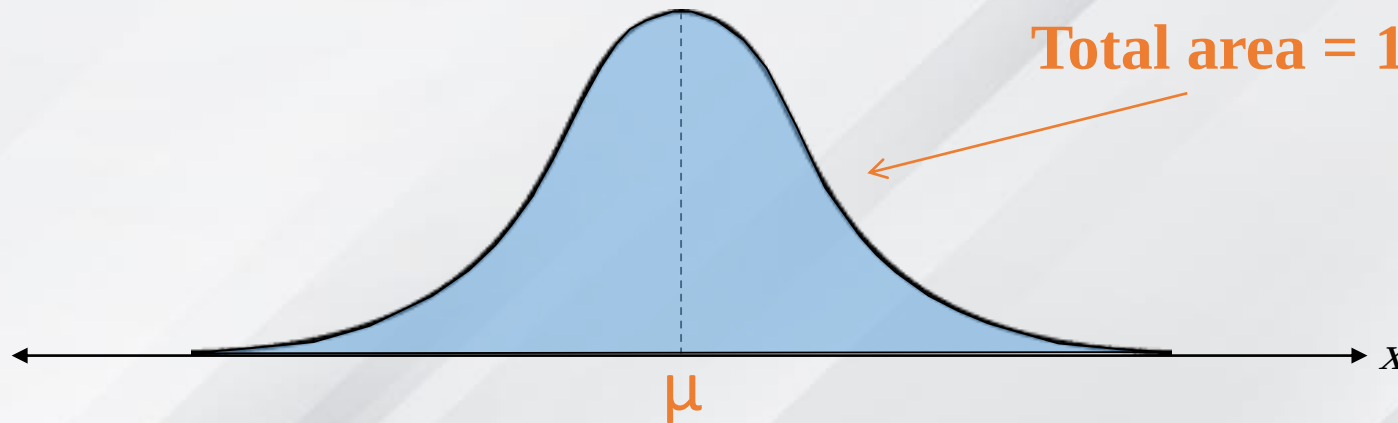
Normal distribution

- A continuous probability distribution for a random variable, x .
- The most important continuous probability distribution in statistics.
- The graph of a normal distribution is called the **normal curve**.



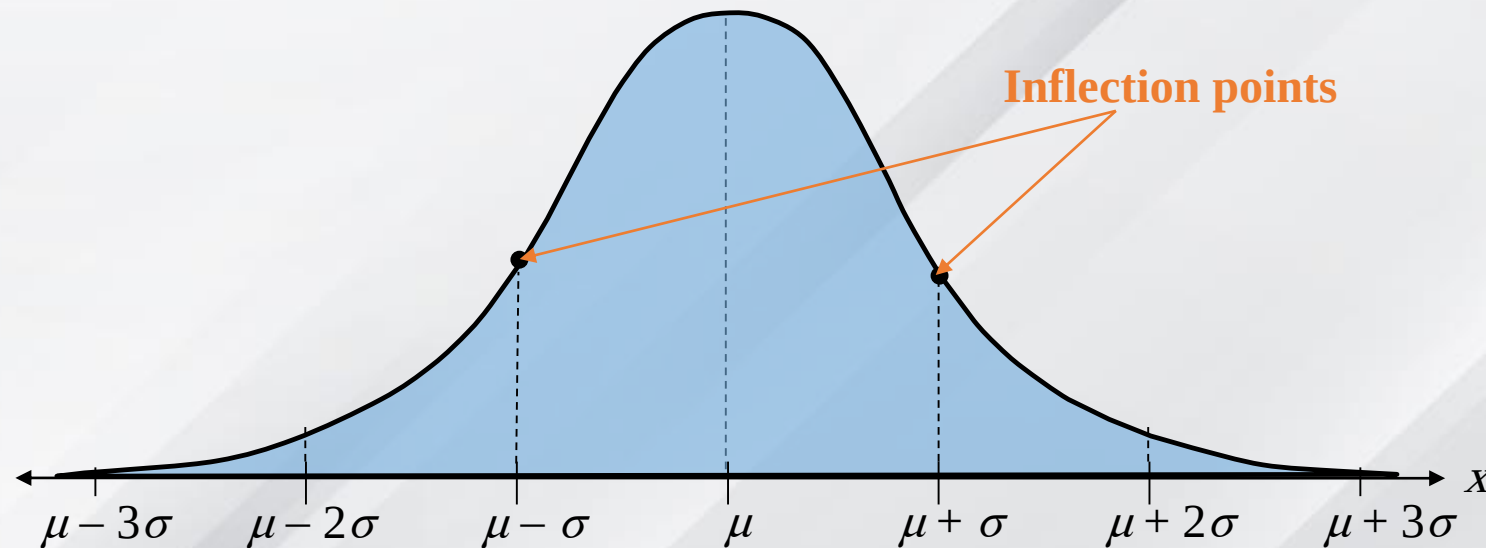
Properties of Normal Distribution

1. The mean, median, and mode are equal.
2. The normal curve is bell-shaped and symmetric about the mean.
3. The total area under the curve is equal to one.
4. The normal curve approaches, but never touches the x -axis as it extends farther and farther away from the mean.



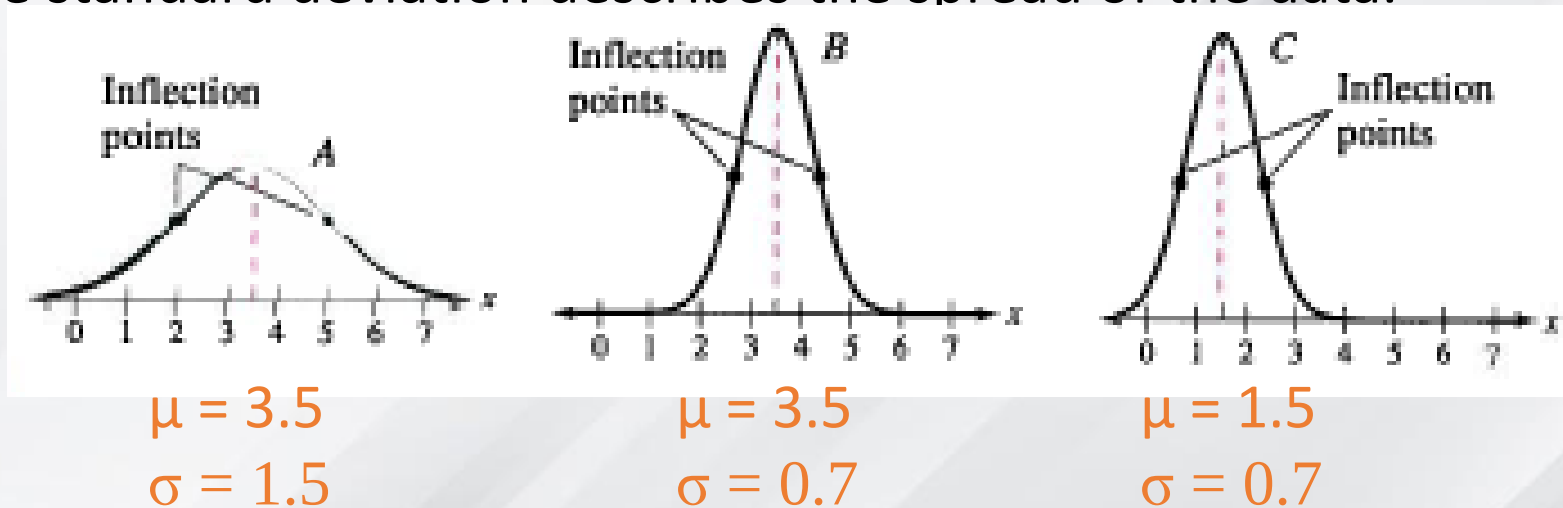
Properties of Normal Distribution

5. Between $\mu - \sigma$ and $\mu + \sigma$ (in the center of the curve), the graph curves downward. The graph curves upward to the left of $\mu - \sigma$ and to the right of $\mu + \sigma$. The points at which the curve changes from curving upward to curving downward are called the **inflection points**.



Means and standard deviations

- A normal distribution can have any mean and any positive standard deviation.
- The mean gives the location of the line of symmetry.
- The standard deviation describes the spread of the data.



Chapter

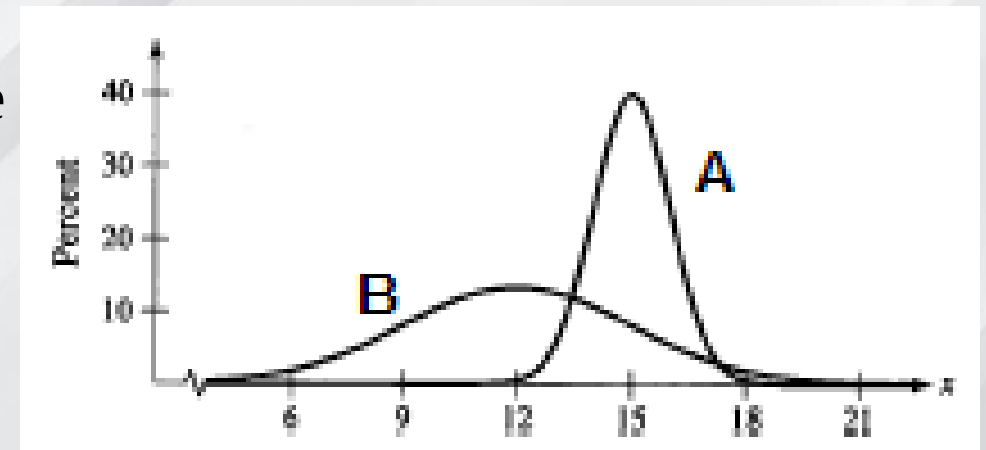
Example

- Which curve has the greater mean?
- Which curve has the greater standard deviation?

Solution:

Curve *A* has the greater mean (The line of symmetry of curve *A* occurs at $x = 15$. The line of symmetry of curve *B* occurs at $x = 12$)

Curve *B* has the greater standard deviation (Curve *B* is more spread out than curve *A*.)



Chapter 10 Convert from normal to standard normal

If x is normal with a mean, μ , and a standard deviation, σ .

Any normal (X) probability can be converted to the standard normal (Z) probability distribution with the following formula

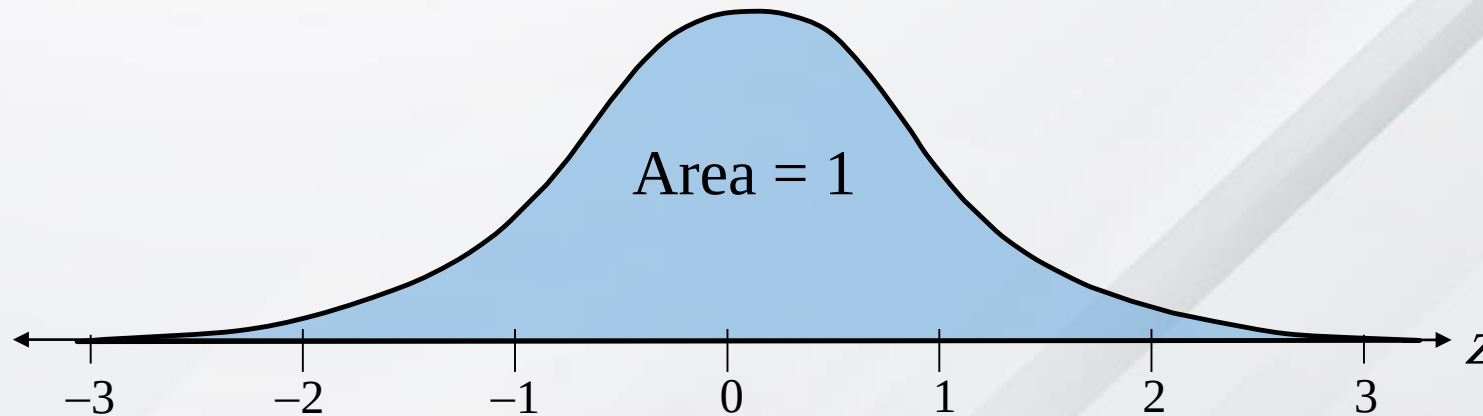
$$Z = \frac{X - \mu}{\sigma}$$

This rule is very important to find the area under the normal curve.

The Standard Normal Distribution

Standard normal distribution

A normal distribution with a mean of 0 and a standard deviation of 1.



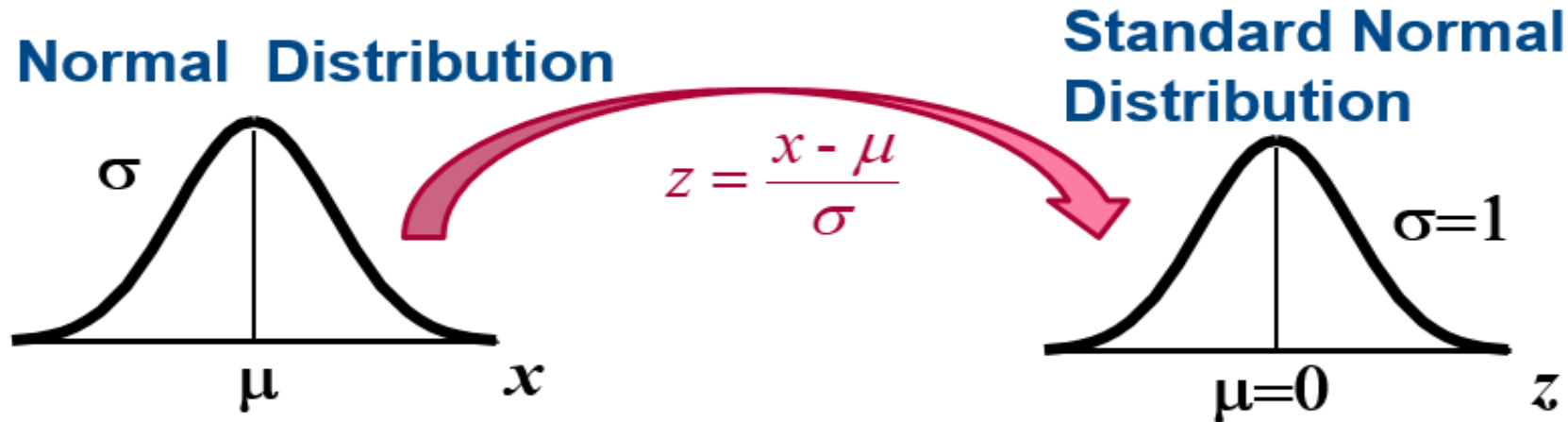
- Any x -value can be transformed into a z -score by using the formula

$$z = \frac{\text{Value} - \text{Mean}}{\text{Standard deviation}} = \frac{x - \mu}{\sigma}$$

The Standard Normal Distribution

Chapter

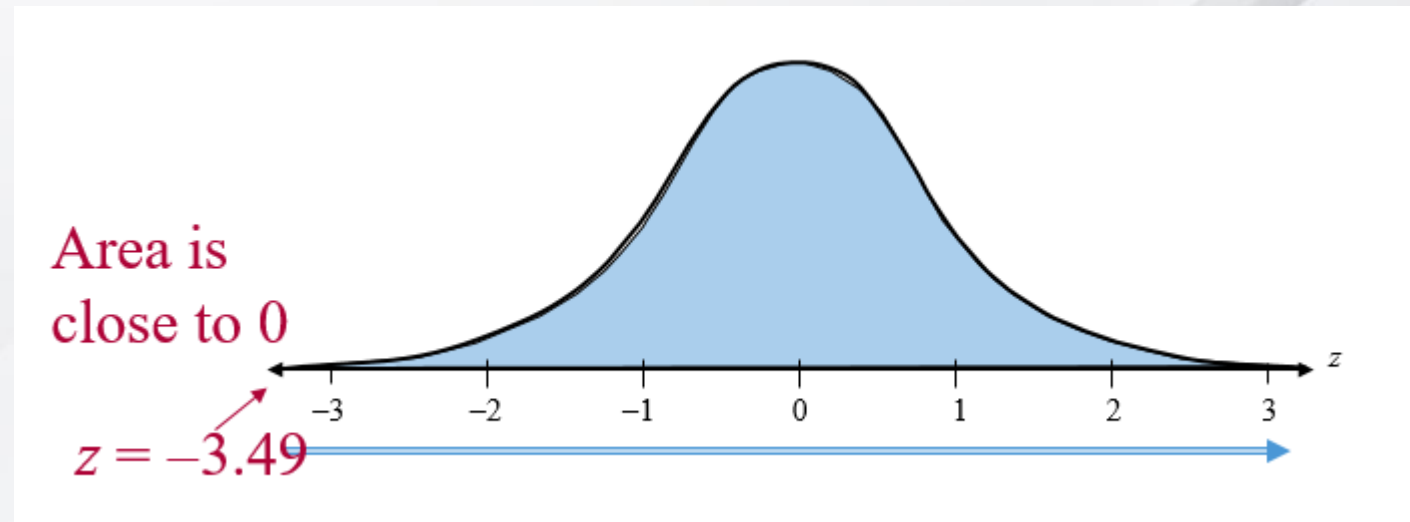
- If each data value of a normally distributed random variable x is transformed into a z -score, the result will be the standard normal distribution.



- Use the Standard Normal Table to find the cumulative area under the standard normal curve.

Properties of the Standard Normal Distribution

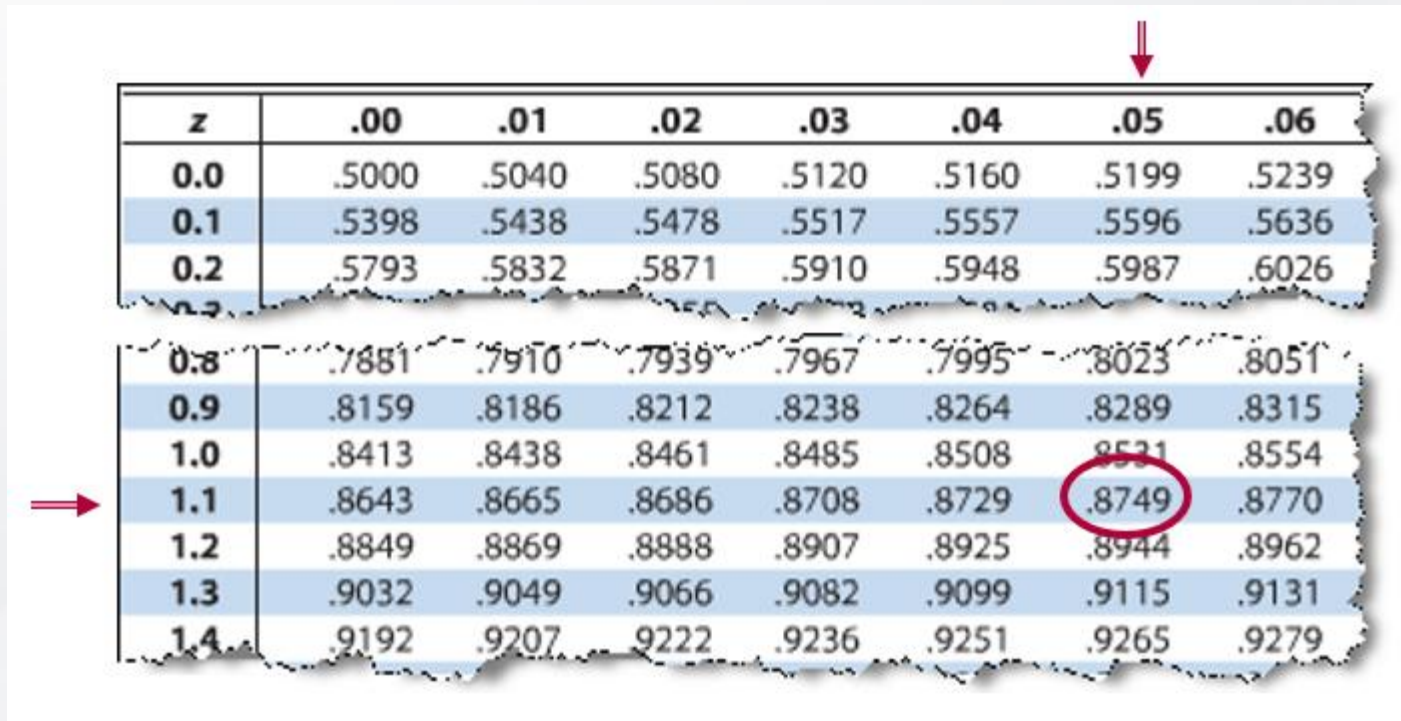
1. The cumulative area is close to 0 for z-scores close to $z = -3.49$.
2. The cumulative area increases as the z-scores increase.



Example: Using The Standard Normal Table

Chapter

- Find the cumulative area that corresponds to a z-score of 1.15.



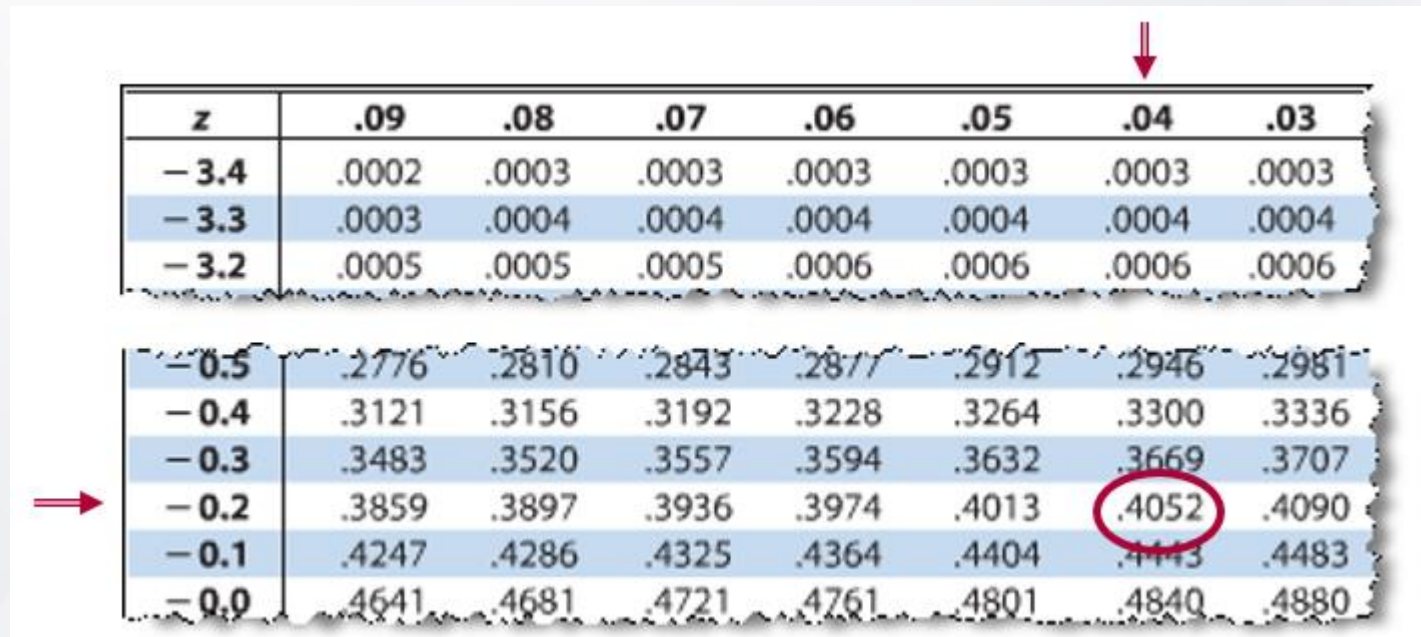
z	.00	.01	.02	.03	.04	.05	.06
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7122
0.6	.7257	.7291	.7324	.7357	.7389	.7421	.7453
0.7	.7580	.7613	.7643	.7673	.7703	.7732	.7761
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279

The area to the left of $z = 1.15$ is 0.8749.

Example: Using The Standard Normal Table

Chapter

- Find the cumulative area that corresponds to a z-score of -0.24.



<i>z</i>	.09	.08	.07	.06	.05	.04	.03
-3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003
-3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004
-3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006
-0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981
-0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336
-0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707
-0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090
-0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483
-0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880

The area to the left of $z = -0.24$ is 0.4052.

The Standard Normal Table

Chapter

<i>z</i>	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
− 3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
− 3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
− 3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
− 3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
− 3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
− 2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019
− 2.8	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026
− 2.7	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035
− 2.6	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047
− 2.5	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062
− 2.4	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082
− 2.3	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107
− 2.2	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139
− 2.1	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179
− 2.0	.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228
− 1.9	.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287
− 1.8	.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359
− 1.7	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446
− 1.6	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548
− 1.5	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668
− 1.4	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808
− 1.3	.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968
− 1.2	.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151
− 1.1	.1170	.1190	.1210	.1230	.1251	.1271	.1292	.1314	.1335	.1357
− 1.0	.1379	.1401	.1423	.1446	.1469	.1492	.1515	.1539	.1562	.1587
− 0.9	.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841
− 0.8	.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119
− 0.7	.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358	.2389	.2420
− 0.6	.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676	.2709	.2743
− 0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015	.3050	.3085
− 0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446
− 0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821
− 0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207
− 0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602
− 0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000

The Standard Normal Table

Chapter

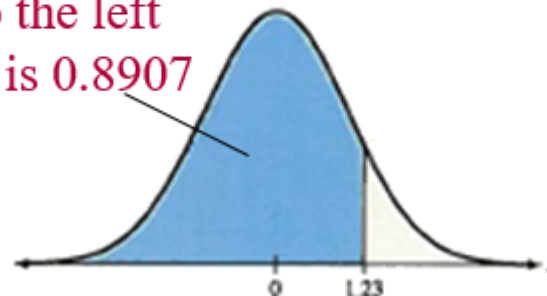
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Finding Areas Under the Standard Normal Curve

Chapter

1. Sketch the standard normal curve and shade the appropriate area under the curve.
2. Find the area by following the directions for each case shown.
 - a. To find the area to the *left* of z , find the area that corresponds to z in the Standard Normal Table.

2. The area to the left of $z = 1.23$ is 0.8907



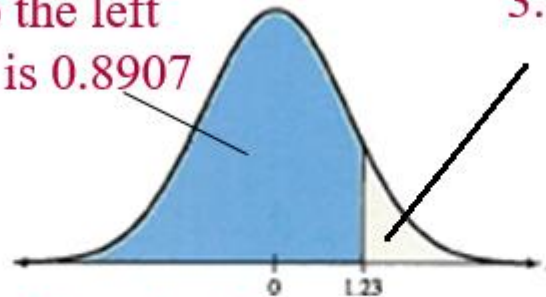
1. Use the table to find the area for the z -score

Finding Areas Under the Standard Normal Curve

To find the area to the *right* of z , use the Standard Normal Table to find the area that corresponds to z . Then subtract the area from 1.

2. The area to the left of $z = 1.23$ is 0.8907

3. Subtract to find the area to the right of $z = 1.23$:
 $1 - 0.8907 = 0.1093$.



1. Use the table to find the area for the z -score

Normal Distributions: Finding Probabilities

Chapter

If X value equal to 600, we have to transform this value to standard value by the rule to be able to use the table:

$$z = \frac{x - \mu}{\sigma} = \frac{600 - 500}{100} = 1$$

Example: Finding Probabilities for Normal Distributions

A survey indicates that people use their computers an average of 2.4 years before upgrading to a new machine. The standard deviation is 0.5 year. A computer owner is selected at random. Find the probability that he or she will use it for fewer than 2 years before upgrading. Assume that the variable x is normally distributed.

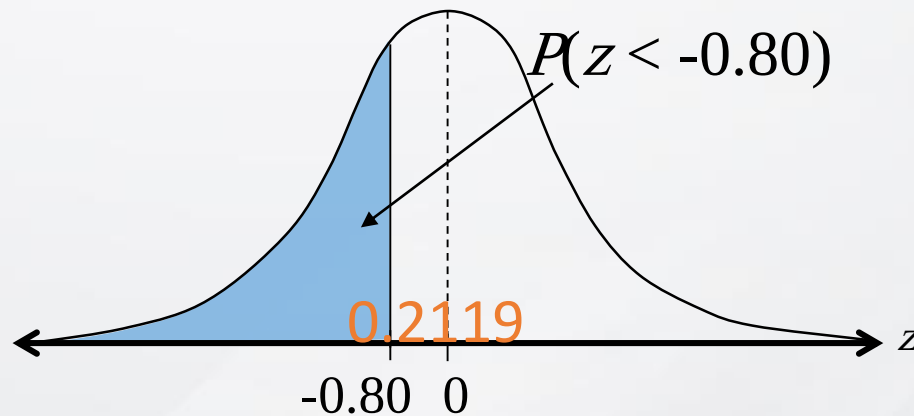
Solution:

We need to find $P(x < 2)$



Example: Finding Probabilities for Normal Distributions

$$z = \frac{x - \mu}{\sigma} = \frac{2 - 2.4}{0.5} = -0.80$$



z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
-3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
-3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
-3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
-3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
-3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
-2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019
-2.8	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026
-2.7	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035
-2.6	.0035	.0036	.0037	.0038	.0039	.0040	.0041	.0042	.0043	.0044
-2.5	.0044	.0045	.0046	.0047	.0048	.0049	.0050	.0051	.0052	.0053
-2.4	.0053	.0054	.0055	.0056	.0057	.0058	.0059	.0060	.0061	.0062
-2.3	.0062	.0063	.0064	.0065	.0066	.0067	.0068	.0069	.0070	.0071
-2.2	.0071	.0072	.0073	.0074	.0075	.0076	.0077	.0078	.0079	.0080
-2.1	.0080	.0081	.0082	.0083	.0084	.0085	.0086	.0087	.0088	.0089
-2.0	.0089	.0090	.0091	.0092	.0093	.0094	.0095	.0096	.0097	.0098
-1.9	.0098	.0099	.0100	.0101	.0102	.0103	.0104	.0105	.0106	.0107
-1.8	.0107	.0108	.0109	.0110	.0111	.0112	.0113	.0114	.0115	.0116
-1.7	.0116	.0117	.0118	.0119	.0120	.0121	.0122	.0123	.0124	.0125
-1.6	.0125	.0126	.0127	.0128	.0129	.0130	.0131	.0132	.0133	.0134
-1.5	.0134	.0135	.0136	.0137	.0138	.0139	.0140	.0141	.0142	.0143
-1.4	.0143	.0144	.0145	.0146	.0147	.0148	.0149	.0150	.0151	.0152
-1.3	.0152	.0153	.0154	.0155	.0156	.0157	.0158	.0159	.0160	.0161
-1.2	.0161	.0162	.0163	.0164	.0165	.0166	.0167	.0168	.0169	.0170
-1.1	.0170	.0171	.0172	.0173	.0174	.0175	.0176	.0177	.0178	.0179
-1.0	.0179	.0180	.0181	.0182	.0183	.0184	.0185	.0186	.0187	.0188
-0.9	.0188	.0189	.0190	.0191	.0192	.0193	.0194	.0195	.0196	.0197
-0.8	.0197	.0198	.0199	.0200	.0201	.0202	.0203	.0204	.0205	.0206
-0.7	.0206	.0207	.0208	.0209	.0210	.0211	.0212	.0213	.0214	.0215
-0.6	.0215	.0216	.0217	.0218	.0219	.0220	.0221	.0222	.0223	.0224
-0.5	.0224	.0225	.0226	.0227	.0228	.0229	.0230	.0231	.0232	.0233
-0.4	.0233	.0234	.0235	.0236	.0237	.0238	.0239	.0240	.0241	.0242
-0.3	.0242	.0243	.0244	.0245	.0246	.0247	.0248	.0249	.0250	.0251
-0.2	.0251	.0252	.0253	.0254	.0255	.0256	.0257	.0258	.0259	.0260
-0.1	.0260	.0261	.0262	.0263	.0264	.0265	.0266	.0267	.0268	.0269
-0.0	.0269	.0270	.0271	.0272	.0273	.0274	.0275	.0276	.0277	.0278

$$P(X < 2) = P(z < -0.80) = 0.2119$$

Example: Finding Probabilities for Normal Distributions

A survey indicates that for each trip to the supermarket, a shopper spends an average of 45 minutes with a standard deviation of 12 minutes in the store. The length of time spent in the store is normally distributed and is represented by the variable x . A shopper enters the store. Find the probability that the shopper will be in the store for between 24 and 54 minutes.

Solution:

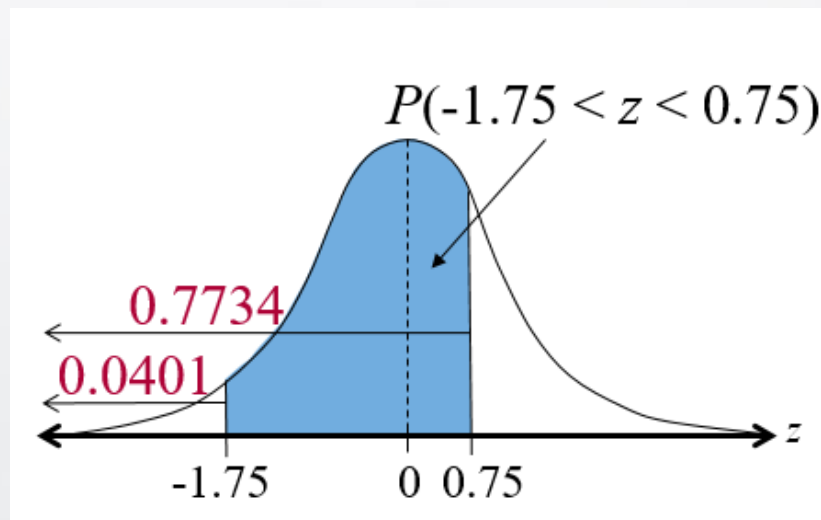
We need to find $P(24 < x < 54)$



Example: Finding Probabilities for Normal Distributions

$$z_1 = \frac{x - \mu}{\sigma} = \frac{24 - 45}{12} = -1.75$$

$$z_2 = \frac{x - \mu}{\sigma} = \frac{54 - 45}{12} = 0.75$$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7421	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133

z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
-3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
-3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
-1.8	.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359
-1.7	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446
-1.6	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548
-1.5	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668
-1.4	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808
-1.3	.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968
-1.2	.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151
-1.1	.1170	.1190	.1210	.1230	.1251	.1271	.1292	.1314	.1335	.1357
-1.0	.1379	.1401	.1423	.1446	.1469	.1492	.1515	.1539	.1562	.1587
-0.9	.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841
-0.8	.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119

$$P(24 < x < 54) = P(-1.75 < z < 0.75) = 0.7734 - 0.0401 = 0.7333$$

Example: Finding Probabilities for Normal Distributions

Find the probability that the shopper will be in the store more than 39 minutes. (Recall $\mu = 45$ minutes and $\sigma = 12$ minutes)

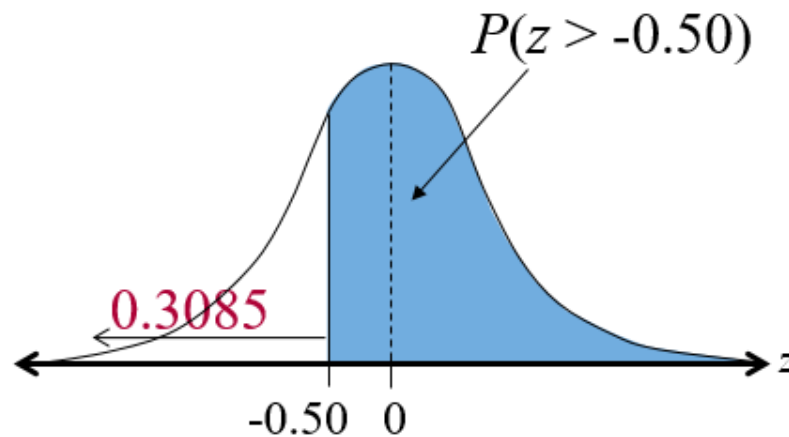
Solution:

We need to get $P(x > 39)$



Example: Finding Probabilities for Normal Distributions

$$z = \frac{x - \mu}{\sigma} = \frac{39 - 45}{12} = -0.50$$



z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
-3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
-3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
-3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
-0.5	.3776	.3810	.3842	.3877	.3912	.3946	.3981	.3915	.3858	.3085
-0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446
-0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821
-0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207
-0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602
-0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000

$$P(x > 39) = P(z > -0.50) = 1 - 0.3085 = 0.6915$$

5.3 Normal Distributions: Finding Values

Chapter

- Find a z -score given the area under the normal curve
- Transform a z -score to an x -value
- Find a specific data value of a normal distribution given the probability

Example: Finding a z-Score Given an Area

Chapter

Find the z-score that corresponds to a cumulative area of 0.3632.

- Locate 0.3632 in the body of the Standard Normal Table.

z	.09	.08	.07	.06	.05	.04	.03
-3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003
-3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004
-3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006
-0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981
-0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336
-0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707
-0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090
-0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483
-0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880

The z-score
is -0.35.

- The values at the beginning of the corresponding row and at the top of the column give the z-score.

Transforming a z-Score to an x-Score

Chapter

To transform a standard z -score to a data value x in a given population, use the formula

$$x = \mu + z\sigma$$

Example: Finding an x-Value

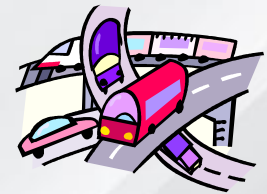
Chapter 1

The speeds of vehicles along a stretch of highway are normally distributed, with a mean of 67 miles per hour and a standard deviation of 4 miles per hour.

Find the speeds x corresponding to z -scores of 1.96, -2.33, and 0

Solution: Use the formula $x = \mu + z\sigma$

- $z = 1.96$: $x = 67 + 1.96(4) = 74.84$ miles per hour
- $z = -2.33$: $x = 67 + (-2.33)(4) = 57.68$ miles per hour
- $z = 0$: $x = 67 + 0(4) = 67$ miles per hour



Sampling Distributions and the Central Limit Theorem

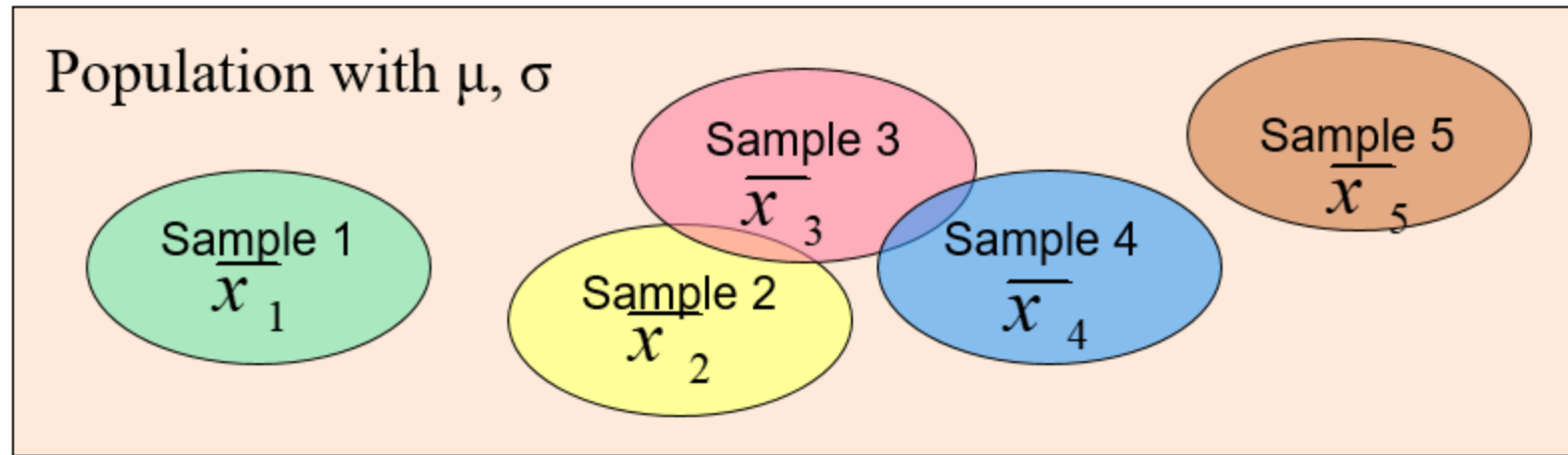
Find sampling distributions and verify their properties

Interpret the Central Limit Theorem

Apply the Central Limit Theorem to find the probability of a sample mean

Sampling distribution of sample mean

Chapter



The sampling distribution consists of the values of the sample means, $\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4, \bar{x}_5, \dots$

Properties of Sampling distribution of sample mean

1. The mean of the sample means, $\mu_{\bar{x}}$, is equal to the population mean μ .

$$\mu_{\bar{x}} = \mu$$

2. The standard deviation of the sample means, $\sigma_{\bar{x}}$, is equal to the population standard deviation, σ divided by the square root of the sample size, n .

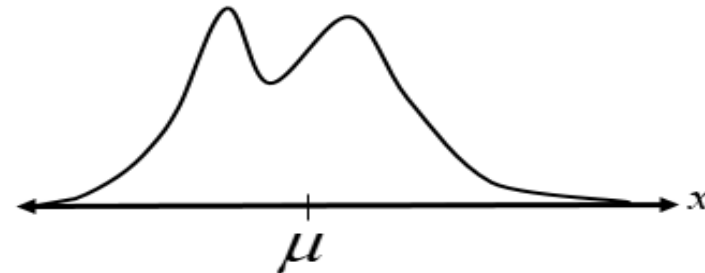
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

- Called the **standard error of the mean**.

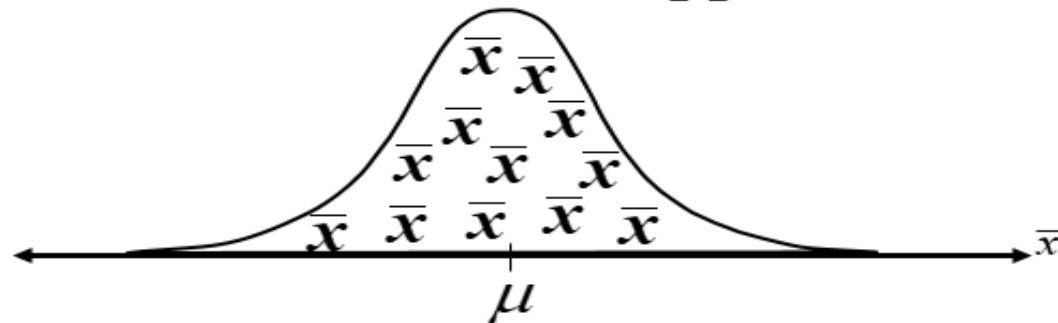
Central Limit Theorem

Chapter

1. If samples of size $n \geq 30$, are drawn from any population with mean $= \mu$ and standard deviation $= \sigma$,



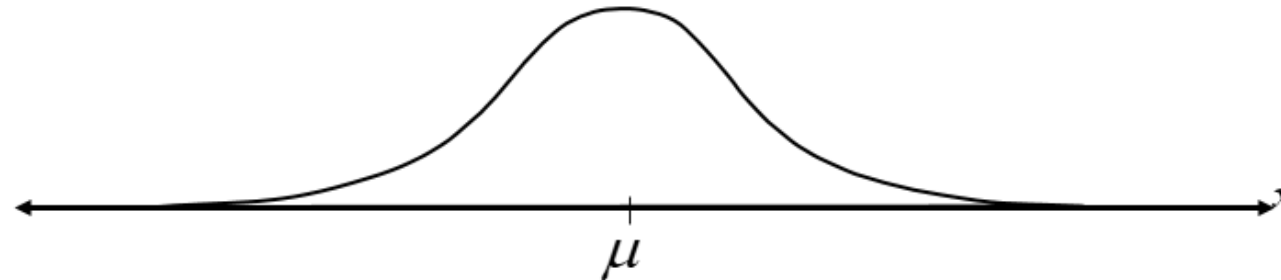
then the sampling distribution of the sample means approximates a normal distribution. The greater the sample size, the better the approximation.



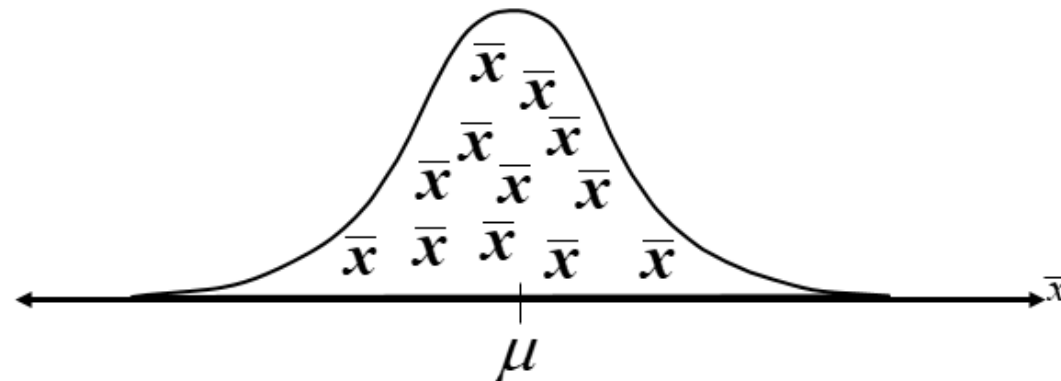
Central Limit Theorem

Chapter

2. If the population itself is normally distributed,



the sampling distribution of the sample means is normally distribution for **any** sample size n .



Central Limit Theorem

Chapter

- In either case, the sampling distribution of sample means has a mean equal to the population mean.

$$\mu_{\bar{x}} = \mu$$

- The sampling distribution of sample means has a variance equal to $1/n$ times the variance of the population and a standard deviation equal to the population standard deviation divided by the square root of n .

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \quad \text{Variance}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \quad \text{Standard deviation (standard error of the mean)}$$

Example: Probabilities for x and $\bar{x}$

Chapter

A bank auditor claims that credit card balances are normally distributed, with a mean of \$2870 and a standard deviation of \$900.

1. What is the probability that a randomly selected credit card holder has a credit card balance less than \$2500?

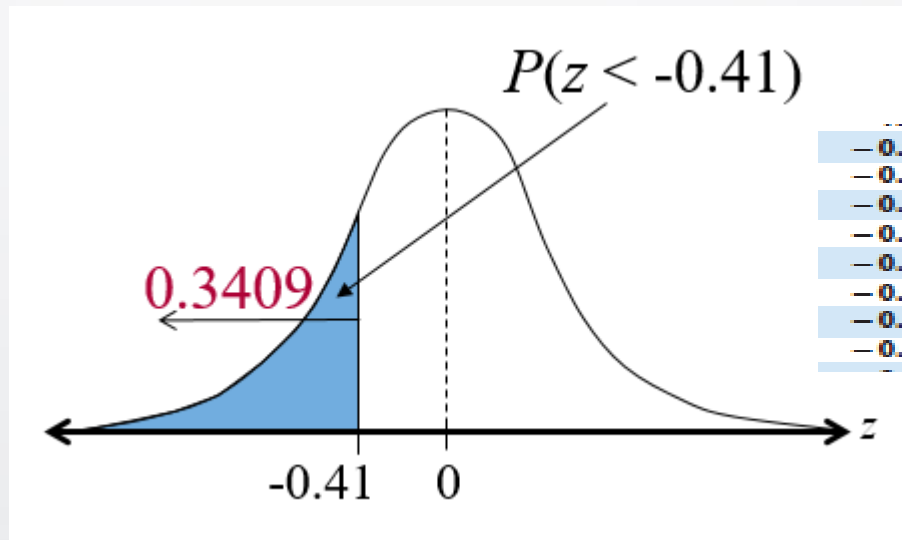


Solution:

You are asked to find the probability associated with a certain value of the random variable x .

Solution: Probabilities for x and $\bar{x}$

$$z = \frac{x - \mu}{\sigma} = \frac{2500 - 2870}{900} \approx -0.41$$



-0.9	.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841
-0.8	.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119
-0.7	.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358	.2389	.2420
-0.6	.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676	.2709	.2743
-0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015	.3050	.3085
-0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446
-0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821
-0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207

- $P(X < 2500) = P(Z < -0.41) = 0.3409$

Example: Probabilities for x and $\bar{x}$

Chapter

2. You randomly select 25 credit card holders. What is the probability that their mean credit card balance is less than \$2500?



Solution:

You are asked to find the probability associated with a sample mean $\bar{x}$.

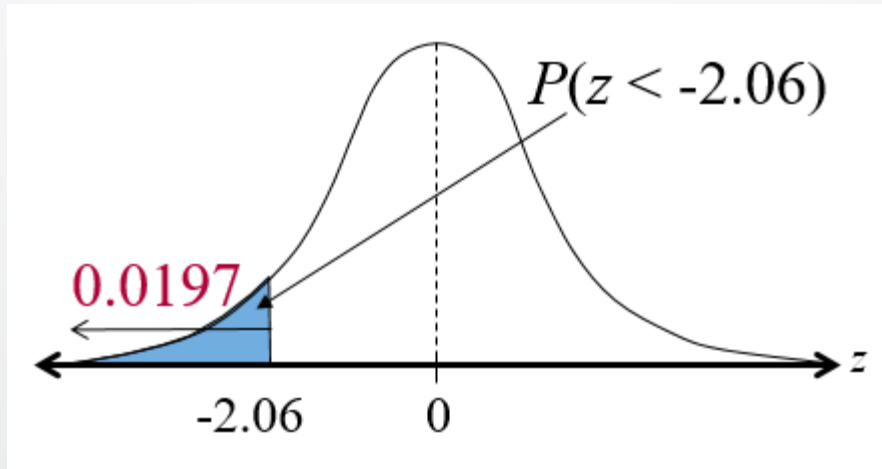
$$\mu = 2870$$

$$\frac{\sigma}{\sqrt{n}} = \frac{900}{\sqrt{25}} = 180$$

Example: Probabilities for x and $\bar{x}$

Chapter

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{2500 - 2870}{\frac{900}{\sqrt{25}}} \approx -2.06$$



z	.09	.08	.07	.06	.05
-3.4	.0002	.0003	.0003	.0003	.0003
-3.3	.0003	.0004	.0004	.0004	.0004
-3.2	.0005	.0005	.0005	.0006	.0006
-3.1	.0007	.0007	.0008	.0008	.0008
-3.0	.0010	.0010	.0011	.0011	.0011
-2.9	.0014	.0014	.0015	.0015	.0016
-2.8	.0019	.0020	.0021	.0021	.0022
-2.7	.0026	.0027	.0028	.0029	.0030
-2.6	.0036	.0037	.0038	.0039	.0040
-2.5	.0048	.0049	.0051	.0052	.0054
-2.4	.0064	.0066	.0068	.0069	.0071
-2.3	.0084	.0087	.0089	.0091	.0094
-2.2	.0110	.0113	.0116	.0119	.0122
-2.1	.0143	.0146	.0150	.0154	.0158
-2.0	.0183	.0188	.0192	.0197	.0202
-1.9	.0233	.0239	.0244	.0250	.0256
-1.8	.0294	.0301	.0307	.0314	.0322

$$P(x < 2500) = P(z < -2.06) = 0.0197$$

5.5 Normal Approximations to Binomial

Chapter
Distributions

Determine when the normal distribution can approximate the binomial distribution

Use the normal distribution to approximate binomial probabilities

Normal Approximation to a Binomial

Chapter

- The normal distribution is used to approximate the binomial distribution when it would be impractical to use the binomial distribution to find a probability.

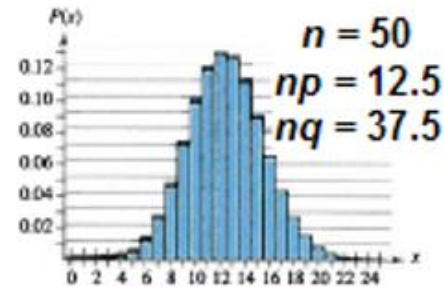
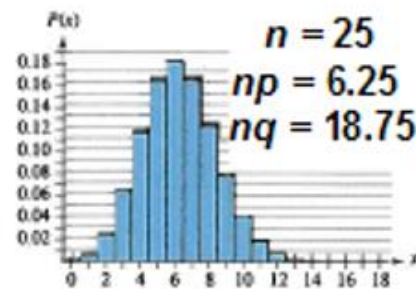
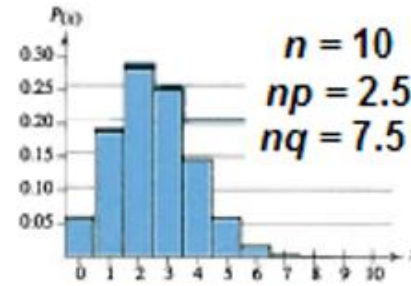
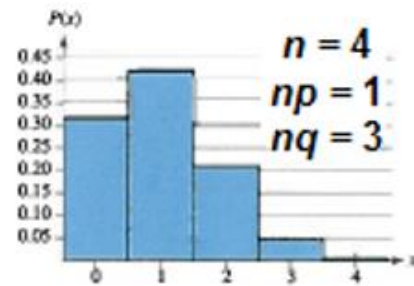
Normal Approximation to a Binomial Distribution

- If $np \geq 5$ and $nq \geq 5$, then the binomial random variable x is approximately normally distributed with
 - mean $\mu = np$
 - standard deviation $\sigma = \sqrt{npq}$

Normal Approximation to a Binomial

Chapter

- Binomial distribution: $p = 0.25$



- As n increases the histogram approaches a normal curve.

Normal Approximation to a Binomial

Chapter

- The normal distribution is used to approximate the binomial distribution when it would be impractical to use the binomial distribution to find a probability.

Normal Approximation to a Binomial Distribution

- If $np \geq 5$ and $nq \geq 5$, then the binomial random variable x is approximately normally distributed with
 - mean $\mu = np$
 - standard deviation $\sigma = \sqrt{npq}$

Approximating the Binomial

Chapter

Decide whether you can use the normal distribution to approximate x , the number of people who reply yes. If you can, find the mean and standard deviation.

1. Fifty-one percent of adults in the U.S. whose New Year's resolution was to exercise more achieved their resolution. You randomly select 65 adults in the U.S. whose resolution was to exercise more and ask each if he or she achieved that resolution.

Solution: Approximating the Binomial

Chapter

- You can use the normal approximation

$$n = 65, \quad p = 0.51, \quad q = 0.49$$

$$np = (65)(0.51) = 33.15 \geq 5$$

$$nq = (65)(0.49) = 31.85 \geq 5$$

- Mean: $\mu = np = 33.15$
- Standard Deviation: $\sigma = \sqrt{npq} = \sqrt{65 \cdot 0.51 \cdot 0.49} \approx 4.03$

Chapter Summary

Chapter

- Found probabilities for normally distributed
- Found a z -score given the area under the normal curve
- Transformed a z -score to an x -value
- Interpreted the Central Limit Theorem
- Used the normal distribution to approximate binomial probabilities

Un Solved Examples

Chapter

In Normal Distribution If $X = 700$, $\mu = 600$ and $\sigma = 100$ then $Z = ?$

في التوزيع الطبيعي إذا كان $X = 700$, $\mu = 600$ and $\sigma = 100$ فإن قيمة $Z = ?$

Un Solved Examples

Chapter

In Normal Distribution If $X = 24$, $\mu = 45$ and $\sigma = 12$ then $Z = -1.75$

في التوزيع الطبيعي إذا كان $X = 24$, $\mu = 45$ and $\sigma = 12$ فإن قيمة $Z = -1.75$

Un Solved Examples

Chapter

In Normal Distribution If $Z = -2.33$, $\mu = 67$ and $\sigma = 4$ then $X = 10$

في التوزيع الطبيعي إذا كان $Z = -2.33$, $\mu = 67$ and $\sigma = 4$ فإن قيمة $X = 10$

Un Solved Examples

Chapter

If in Binomial distribution $n = 4$, $p = 0.25$, $q = 0.75$ then we can approximate it to normally disputation

إذا كان في توزيع ذي الحدين $n = 4$, $p = 0.25$, $q = 0.75$ فإنه يمكن تقريبه إلى توزيع طبيعي



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Thank You

