

Business Mathematics

Prepared by Dr. Alfaisal Hasan

MTH 112
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Lecture NO 6



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Chapter

Topics

- Chapter 1: Mathematical preliminaries
- Chapter 2: The straight line and applications
- Chapter 3: Functions and Graphs
- Chapter 4: Limits and Continuity
- Chapter 5: Financial Mathematics
- **Chapter 6: Differentiation**
- Chapter 7: Methods of Differentiation
- Chapter 8: Function of several variables
- Chapter 9: Integration and applications
- Chapter 10: Linear algebra and applications

Objectives

To differentiate functions using the power rule, constant rule, constant multiple rule, and sum and difference rules.

Differentiation

- Used to find the “slope” of a function at a point.
- Used to find the “slope of the tangent line” to the graph of a function at a point.
- Used to find the “instantaneous rate of change” of a function at a point.
- Computed by finding the limit of the difference quotient as Δx approaches 0. (Limit Definition)

Differentiation

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Differentiation

$f'(x)$ “ f prime of x ”

y' “ y prime”

$\frac{dy}{dx}$ “the derivative of y with respect to x ”

$\frac{d}{dx}$ is a verb. “Take the derivative with respect to x ...”

Differentiation

- Differentiation is the process of computing the derivative of a function.

You may be asked to:

- Differentiate.
- Derive.
- Find the derivative of...

Differentiation

$$\frac{d}{dx}[c] = 0, \quad c \text{ is a constant}$$

The derivative of a constant function is zero.

Differentiation

- Find the derivative of:

$$f(x) = 7$$

$$f'(x) = 0$$

$$y = -3$$

$$\frac{dy}{dx} = 0 \quad \text{or} \quad y' = 0$$

Differentiation

- Find the derivative of:

$$f(x) = -12$$

$$f'(x) = 0$$

$$y = 5$$

$$\frac{dy}{dx} = 0 \quad \text{or } y' = 0$$

Differentiation

$$\frac{d}{dx}[x^N] = Nx^{N-1}, \quad N \text{ is any real number}$$

$$\frac{d}{dx}[x] = 1$$

Differentiation

- Differentiate:

$$f(x) = x^3$$

$$f'(x) = 3x^2$$

$$g(x) =$$

$$g'(x) =$$

$$y = x^9$$

$$\frac{dy}{dx} = 9x^8$$

Differentiation

$$\frac{d}{dx}[c(f(x))] = c(f'(x)), \quad c \text{ is a constant}$$

- The derivative of a constant times a function is equal to the constant times the derivative of the function.

Differentiation

- Find the derivative of

$$f(x) = 5x^3$$

$$f'(x) = \frac{d}{dx}(5x^3)$$

$$= 5 \frac{d}{dx}(x^3)$$

$$= 5(3x^2)$$

$$= 15x^2$$

Differentiation

- Differentiate:

$$f(x) = 4x^3$$

$$f'(x) = 4(3x^2)$$

$$f'(x) = 12x^2$$

Differentiation

- Find the derivative of:

$$y = x^{\frac{1}{3}}$$

$$\frac{dy}{dx} = \left(\frac{1}{3} x^{-\frac{2}{3}} \right)$$

$$\frac{dy}{dx} = \frac{1}{3x^{\frac{2}{3}}}$$

Differentiation

- Find the derivative of:

$$y = 2x^{\frac{1}{3}}$$

$$\frac{dy}{dx} = 2 \left(\frac{1}{3} x^{-\frac{2}{3}} \right)$$

$$\frac{dy}{dx} = \frac{2}{3x^{\frac{2}{3}}}$$

Differentiation

- Find the derivative of:

$$f(x) = \frac{4x^2}{5} = \frac{4}{5}x^2$$

$$f'(x) = \frac{4}{5}(2x)$$

$$f'(x) = \frac{8}{5}x$$

Differentiation

- Find the derivative of:

$$f(x) = \frac{4x^2}{5} = \frac{4}{5}x^2$$

$$f'(x) = \frac{4}{5}(2x)$$

$$f'(x) = \frac{8}{5}x$$

Differentiation

Function	Rewrite	Differentiate	Simplify
$f(x) = \frac{5}{2x^3}$	$f(x) = \frac{5}{2}x^{-3}$	$f'(x) = \frac{5}{2}(-3x^{-4})$	$f'(x) = -\frac{15}{2x^4}$

Differentiation

Function	Rewrite	Differentiate	Simplify
$f(x) = \frac{3}{x^5}$	$f(x) = 3x^{-5}$	$f'(x) = 3(-5x^{-6})$	$f'(x) = -\frac{15}{x^6}$

Differentiation

Function	Rewrite	Differentiate	Simplify
$g(x) = \frac{7}{3x^{-2}}$	$g(x) = \frac{7}{3}x^2$	$g'(x) = \frac{7}{3}(2x)$	$g'(x) = \frac{14}{3}x$

Differentiation

Function	Rewrite	Differentiate	Simplify
$h(x) = \sqrt{x}$	$h(x) = x^{1/2}$	$h'(x) = \frac{1}{2}x^{-1/2}$	$h'(x) = \frac{1}{2x^{1/2}}$ $h'(x) = \frac{1}{2\sqrt{x}}$

Differentiation

Function	Rewrite	Differentiate	Simplify
$j(x) = \frac{1}{2\sqrt[3]{x^2}}$	$j(x) = \frac{1}{2x^{2/3}}$ $j(x) = \frac{1}{2}x^{-2/3}$	$j'(x) = \frac{1}{2} \left(-\frac{2}{3}x^{-5/3} \right)$	$j'(x) = -\frac{1}{3x^{5/3}}$

Differentiation

Function	Rewrite	Differentiate	Simplify
$g(x) = \frac{2}{x^{-5}}$	$g(x) = 2x^5$	$g'(x) = 2(5x^4)$	$g'(x) = 10x^4$

Differentiation

Find the derivative of

$$f(x) = \frac{3}{\sqrt{x}}$$

$$f'(x) = \frac{d}{dx}(3x^{-1/2})$$

$$= 3\left(-\frac{1}{2}x^{-3/2}\right)$$

$$= -\frac{3}{2x^{3/2}}$$

Find the derivative of $f(x) = 3\sqrt{x}$

$$f(x) = 3x^{\frac{1}{2}}$$

$$f'(x) = 3\left(\frac{1}{2}\right)x^{-\frac{1}{2}}$$

$$f'(x) = \frac{3x^{-\frac{1}{2}}}{2} = \frac{3}{2x^{\frac{1}{2}}}$$

$$f'(x) = \frac{3}{2\sqrt{x}}$$

Find the derivative of $f(x) = 7\sqrt[4]{x}$

$$f(x) = 7x^{\frac{1}{4}}$$

$$f'(x) = 7\left(\frac{1}{4}\right)x^{-\frac{3}{4}}$$

$$f'(x) = \frac{7x^{-\frac{3}{4}}}{4} = \frac{7}{4x^{\frac{3}{4}}}$$

$$f'(x) = \frac{7}{4\sqrt[4]{x^3}}$$

Differentiation

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

The derivative of a sum is the sum of the derivatives.

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

The derivative of a difference is the difference of the derivatives.

Differentiation

- Differentiate:

$$f(x) = 5x^2 + 7x - 6$$

$$f'(x) = 10x + 7$$

Differentiation

- Differentiate:

$$g(x) = 4x^6 - 3x^5 - 10x^2 + 5x + 16$$

$$g'(x) = 24x^5 - 15x^4 - 20x + 5$$

Differentiation

Find the derivative of

$$f(x) = 4x^5 + 3x^4 - 8x^2 + x + 3$$

$$\begin{aligned} f'(x) &= \frac{d}{dx}(4x^5 + 3x^4 - 8x^2 + x + 3) \\ &= 4 \frac{d}{dx}(x^5) + 3 \frac{d}{dx}(x^4) - 8 \frac{d}{dx}(x^2) + \frac{d}{dx}(x) + \frac{d}{dx}(3) \\ &= 4(5x^4) + 3(4x^3) - 8(2x) + 1 + 0 \\ &= 20x^4 + 12x^3 - 16x + 1 \end{aligned}$$

Differentiation

- Given that $f(x) = x^3$, then find the value of $f'(4)$

$$f'(x) = 3x^2$$

$$f'(4) = 3(4)^2$$

$$f'(4) = 48$$

Differentiation

- Given that $f(x) = 4x^3$, then find the value of $f'(1)$

$$f'(x) = 4(3x^2)$$

$$f'(x) = 12x^2$$

$$f'(1) = 12(1)^2$$

$$f'(1) = 12$$

Differentiation

Given that $y = x^{\frac{1}{3}}$, then find the value of $\frac{dy}{dx}$ at $x = 1$

$$\frac{dy}{dx} = \left(\frac{1}{3} x^{-\frac{2}{3}} \right)$$

$$\frac{dy}{dx} = \frac{1}{3x^{\frac{2}{3}}}$$

$$\frac{dy}{dx} \text{ at } x = 1 \quad \longrightarrow \quad = \frac{1}{3(1)^{\frac{2}{3}}} = \frac{1}{3}$$

Differentiation

Given that $f(x) = \frac{4x^2}{5}$, then find the value of f' at $x = 0$

$$f(x) = \frac{4x^2}{5} = \frac{4}{5}x^2$$

$$f'(x) = \frac{4}{5}(2x)$$

$$f'(x) = \frac{8}{5}x \quad \longrightarrow \quad f'(0) = \frac{8}{5}(0) = 0$$

Differentiation

Find the derivative of: $g(x) = 5x^7$, then find the value of $g'(-1)$

$$g'(x) = 35x^6$$

$$g'(-1) = 35(-1)^6 = 35$$

Differentiation

Differentiate: $f(x) = 5x^2 + 7x - 6$
, then find the value of $f'(0)$

$$f'(x) = 10x + 7$$

$$f'(0) = 10(0) + 7$$

$$f'(0) = 7$$

Differentiation

Differentiate: $g(x) = 4x^6 - 3x^5 - 10x^2 + 5x + 16$
, then find the value of $g'(1)$

$$g(x) = 4x^6 - 3x^5 - 10x^2 + 5x + 16$$

$$g'(x) = 24x^5 - 15x^4 - 20x + 5$$

$$g'(1) = 24(1)^5 - 15(1)^4 - 20(1) + 5$$

$$g'(1) = -6$$

Given the function

$$f(x) = 4x^3 + 6x - 1$$

Find the value of

$$f'(0)$$

$$f'(x) = 12x^2 + 6$$

$$f'(0) = 12(0)^2 + 6$$

$$f'(0) = 6$$

Given the function $f(x) = x^9 + 12x - 2$

Find the value of $f'(1)$

$$f'(x) = 9x^8 + 12$$

$$f'(1) = 9(1)^8 + 12$$

$$f'(1) = 21$$

Differentiation

The derivative of the product of two differentiable functions is given by

$$y = f(x) \cdot g(x)$$

$$\frac{dy}{dx} = y' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\frac{dy}{dx} = y' = f' \cdot g + f \cdot g'$$

Differentiation

$$\text{Given that } y = (x^2 + 2).(3x - 5)$$

Then

$$y' = (2x)(3x - 5) + (x^2 + 2)(3)$$

$$y' = 6x^2 - 10x + 3x^2 + 6$$

$$y' = 9x^2 - 10x + 6$$

Differentiation

Find the derivative of

$$f(x) = (2x^2 - 1)(x^3 + 3)$$

$$f'(x) = (2x^2 - 1) \frac{d}{dx}(x^3 + 3) + (x^3 + 3) \frac{d}{dx}(2x^2 - 1)$$

$$= (2x^2 - 1)(3x^2) + (x^3 + 3)(4x)$$

$$= 6x^4 - 3x^2 + 4x^4 + 12x$$

$$= 10x^4 - 3x^2 + 12x$$

Differentiation

Find the derivative of

$$f(x) = x^3(\sqrt{x} + 1)$$

$$f'(x) = x^3 \frac{d}{dx}(x^{1/2} + 1) + (x^{1/2} + 1) \frac{d}{dx}x^3$$

$$= x^3 \left(\frac{1}{2}x^{-1/2} \right) + (x^{1/2} + 1)3x^2$$

$$= \frac{1}{2}x^{5/2} + 3x^{5/2} + 3x^2$$

$$= \frac{7}{2}x^{5/2} + 3x^2$$

Find the derivative of $f(x) = (2x^4 - 1)(x^3 + 15)$

$$f(x) = (2x^4 - 1)(x^3 + 1)$$

$$f'(x) = (8x^3)(x^3 + 1) + (2x^4 - 1)(3x^2)$$

$$f'(x) = 8x^6 + 8x^3 + 6x^6 - 3x^2$$

$$f'(x) = 14x^6 + 8x^3 - 3x^2$$

Find the derivative of $f(x) = (3x^5 - 2)(x^2 + 1)$

$$f(x) = (3x^5 - 2)(x^2 + 1)$$

$$f'(x) = (15x^4)(x^2 + 1) + (3x^5 - 2)(2x)$$

$$f'(x) = 15x^6 + 15x^4 + 6x^6 - 4x$$

$$f'(x) = 21x^6 + 15x^4 - 4x$$

Differentiation

- The derivative of the quotient of two differentiable functions is given by

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \quad (g(x) \neq 0)$$

Differentiation

Find the derivative of

$$f(x) = \frac{x}{2x - 4}$$

$$f'(x) = \frac{(2x - 4) \frac{d}{dx}(x) - x \frac{d}{dx}(2x - 4)}{(2x - 4)^2}$$

$$= \frac{(2x - 4)(1) - x(2)}{(2x - 4)^2}$$

$$= \frac{2x - 4 - 2x}{(2x - 4)^2} = -\frac{4}{(2x - 4)^2}$$

Differentiation

Find the derivative of

$$f(x) = \frac{x^2}{5x - 2}$$

$$f'(x) = \frac{(5x - 2) \frac{d}{dx}(x^2) - x^2 \frac{d}{dx}(5x - 2)}{(5x - 2)^2}$$

$$= \frac{(5x - 2)(2x) - x^2(5)}{(5x - 2)^2}$$

$$= \frac{10x^2 - 4x - 5x^2}{(5x - 2)^2} = \frac{5x^2 - 4x}{(5x - 2)^2}$$

Differentiation

Find the derivative of

$$f(x) = \frac{x^2 + 1}{x^2 - 1}$$

$$f'(x) = \frac{(x^2 - 1) \frac{d}{dx}(x^2 + 1) - (x^2 + 1) \frac{d}{dx}(x^2 - 1)}{(x^2 - 1)^2}$$

$$= \frac{(x^2 - 1)(2x) - (x^2 + 1)(2x)}{(x^2 - 1)^2}$$

$$= \frac{2x^3 - 2x - 2x^3 - 2x}{(x^2 - 1)^2} = -\frac{4x}{(x^2 - 1)^2}$$

Find the derivative of $f(x) = \frac{x^4 + 2}{5x^2 + 7}$

$$f(x) = \frac{x^4 + 2}{5x^2 + 7}$$

$$f'(x) = \frac{(4x^3)(5x^2 + 7) - (x^4 + 2)(10x)}{(5x^2 + 7)^2}$$

$$f'(x) = \frac{20x^5 + 28x^3 - 10x^5 - 20x}{(5x^2 + 7)^2}$$

$$f'(x) = \frac{10x^5 + 28x^3 - 20x}{(5x^2 + 7)^2}$$

Find the derivative of

$$f(x) = \frac{x^3 + 3}{6x^4 + 1}$$

$$f(x) = \frac{x^3 + 3}{6x^4 + 1}$$

$$f'(x) = \frac{(3x^2)(6x^4 + 1) - (x^3 + 3)(24x^3)}{(6x^4 + 1)^2}$$

$$f'(x) = \frac{18x^6 + 3x^2 - 24x^6 - 72x^3}{(6x^4 + 1)^2}$$

$$f'(x) = \frac{-6x^6 - 72x^3 + 3x^2}{(6x^4 + 1)^2}$$

Un Solved Examples

Find the derivative of

$$f(x) = x^4 + 2$$

Un Solved Examples

Find the derivative of

$$f(x) = x^7 - 5$$

Un Solved Examples

Find the derivative of

$$f(x) = \sqrt{x}$$

Un Solved Examples

Given the function

$$f(x) = x^3 + 2$$

Find the value of

$$f'(1)$$

Un Solved Examples

Given the function $f(x) = 5x^2 + 7x - 6$

Find the value of

$$f'(0)$$

Un Solved Examples

Find the derivative of

$$f(x) = (2x^3 - 1)(x^2 + 5)$$

Un Solved Examples

Find the derivative of

$$f(x) = \frac{x^3}{3x + 2}$$

Un Solved Examples

Find the derivative of

$$f(x) = \frac{4}{x^6}$$

Un Solved Examples

Find the derivative of

$$f(x) = \frac{2}{x^{-5}}$$



Thank You