

Operations Research

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IT 262
Operations Research

Lecture 6



Transportation Applications

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Lecture 6 Topics

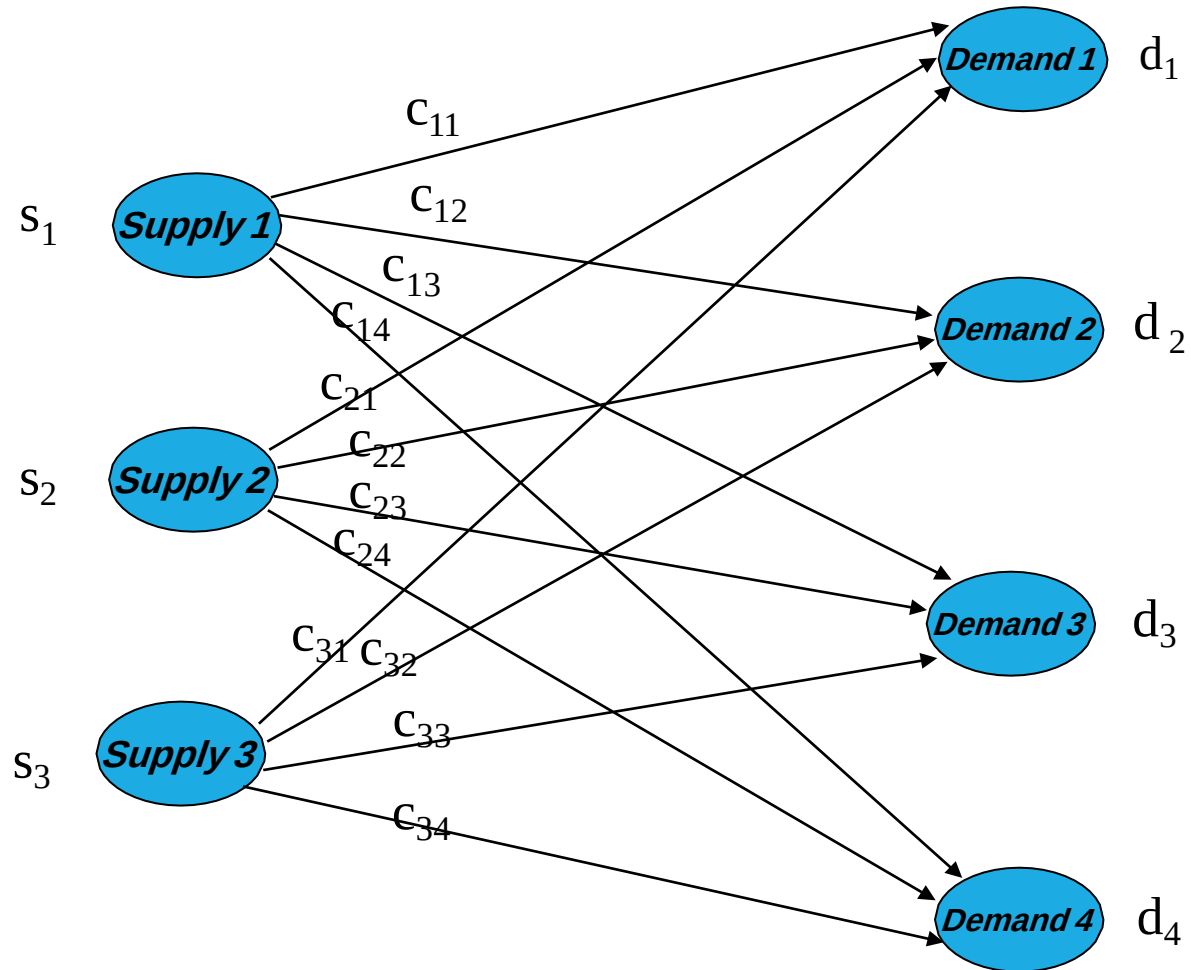
- ❖ **Transportation Model**
- ❖ **Balanced Problem**
- ❖ **Evaluation the total cost of transportation**

Transportation Applications

Basic elements in transport applications

1. A set of supply nodes $\rightarrow n$ nodes
2. A set of demand nodes $\rightarrow m$ nodes
3. Maximum total supply at the supply node $i = s_i$ for all i
4. The total demand at the demand node $j = d_j$ for all j
5. The cost of transport a unit from the supply node i to the demand node $j = c_{ij}$

Transportation Applications



What is the optimal method to transport units from supply nodes to demand nodes?

Transportation Applications

The general formula of this mathematical model

x_{ij} = number of units shipped from supply node i to demand node j .

$i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$

Min

- Cost to shipment x_{ij} from supply i to demand j is $= c_{ij} x_{ij}$
- Each demand node must get at least what covers its demand
- Each supply node must send its maximum supply capacity at most

Transportation Applications

The general formula of this mathematical model

x_{ij} = number of units shipped from supply point i to demand point j .

$i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$

$$Z = \sum_{i=1}^n \sum_{j=1}^m c_{ij} x_{ij}$$

Example

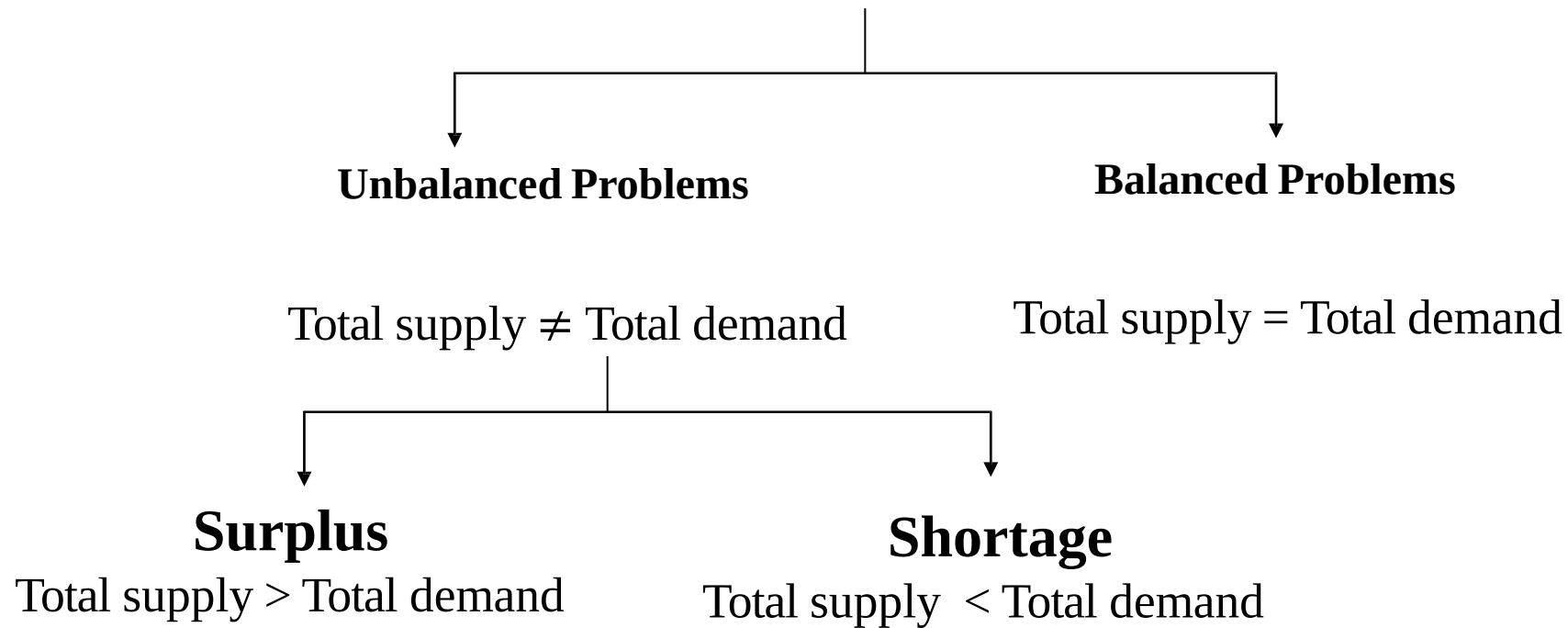
Electricity Company has three power plants in different areas to secure the demand for electricity consumption of four cities. The production capacity of electricity from the station-1, 35 million kilowatts (kwh) per day and the capacity of electricity from the plant Station -2, 50 million kwh per day, while the capacity of electricity from the station -3, 40 million kwh per day. The previous data show that the peak daily consumption of the city-1 is 45 million kwh and the daily peak-day consumption of the city -2 is 20 million kwh and the daily peak-day consumption of the city -3 is 30 million kwh and the peak daily consumption of the city-4 is 30 million kwh. Cause to the distance from the station sites to the cities, there exist a cost to secure ship every million kwh from any station to any city. These costs are shown in the following table:

Transportation Applications

	To			
From	City-1	City-2	City-3	City-4
Station-1	8	6	10	9
Station-2	9	12	13	7
Station-3	14	9	16	5

Transportation Applications

Transportation Models



Transportation Applications

Balanced Problem

Total supply = Total demand

$$\sum_{i=1}^m s_i = \sum_{j=1}^n d_j$$

All constraints are binding:

$$\sum_{j=1}^n x_{ij} = s_i \quad i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} = d_j \quad j = 1, 2, \dots, n$$

$$x_{ij} \geq 0$$

Transportation Applications

Standard Form

$$\begin{aligned}
 & \text{Min} \quad Z = \sum_{i=1}^n \sum_{j=1}^m c_{ij} x_{ij} \\
 & \text{s.t.} \quad \sum_{j=1}^n x_{ij} = s_i \quad i = 1, 2, \dots, m \\
 & \quad \quad \sum_{i=1}^m x_{ij} = d_j \quad j = 1, 2, \dots, n \\
 & \quad \quad x_{ij} \geq 0
 \end{aligned}$$

Transportation Applications

TP Table

	c_{11}	c_{12}	...	c_{1n}	<i>Supply</i>
x_{11}		x_{12}		x_{1n}	s_1
	c_{21}	c_{22}	...	c_{2n}	s_2
x_{21}		x_{22}		x_{2n}	
⋮	⋮	⋮		⋮	⋮
	c_{m1}	c_{m2}	...	c_{mn}	s_m
x_{m1}		x_{m2}		x_{mn}	

Demand

d_1

d_2

...

d_n

Equal

Balanced Problems

Transportation Applications

Initial solution of transportation problems

- The cell (i, j) in transportation table represents the value of the variable x_{ij} .
- The initial table must contain **(m+n-1) cell filled** and the rest of the cells to remain blank.
- Any cell not filled in the initial solution means that the value of the variable associated with this cell is equal to **zero** but is **not written** in the cell.
- Among the filled cells **may exist** that with the **value zero**.
- The sum of the **values** of filled cells in column (j) = the demand on the column (j).
- The sum of the **values** of filled cells in row (i) = the supply on the row (i).

Transportation Applications

Example

<i>Demand</i>		8	6	10	9	<i>Supply</i>
	x_{11}	x_{12}	x_{13}	x_{14}	35	
	x_{21}	x_{22}	x_{23}	x_{24}	50	
	x_{31}	x_{32}	x_{33}	x_{34}	40	
	45	20	30	30		

Transportation Applications

Example

	8		6		10		9
		10		25			
45	9		12		13		7
				5			
	14		9		16		5
		10				30	

Supply

35

50

40

Demand

45

20

30

30

of cells = $6 = 3 + 4 - 1 = m + n - 1$

Solution:

$x_{21} = 45$, $x_{12} = 10$, $x_{32} = 10$, $x_{13} = 25$, $x_{23} = 5$, $x_{34} = 30$

$x_{11} = x_{14} = x_{22} = x_{34} = x_{31} = x_{33} = 0$

	8		6		10		9
x_{11}		x_{12}		x_{13}		x_{14}	
	9		12		13		7
x_{21}		x_{22}		x_{23}		x_{24}	
	14		9		16		5
x_{31}		x_{32}		x_{33}		x_{34}	

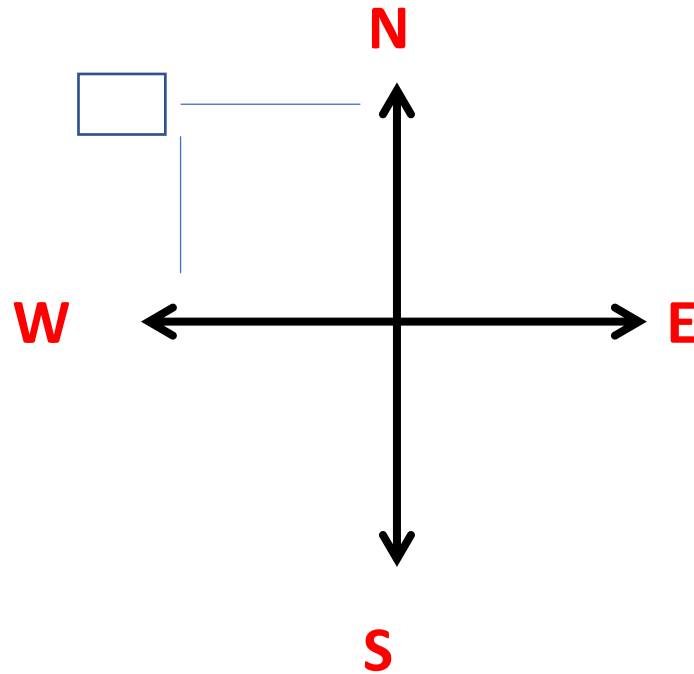
Transportation Applications

Methods to determine the initial solution:

1. North-West Corner Method
2. Least Cost Method
3. Vogel approximation Method

Transportation Applications

1- North-West Corner Method:



Transportation Applications

Initial Basic Feasible Solution (North-West Corner)

	8		6		10		9
	9		12		13		7
	14		9		16		5

Supply

35

50

40

Supply = 125

Demand

45

20

30

30

Demand = 125

Supply = Demand = 125



Balanced Transportation

Transportation Applications

Initial Basic Feasible Solution (North-West Corner)

	8	6	10	9	<i>Supply</i>
35					35
	9	12	13	7	50
	14	9	16	5	40
<i>Demand</i>	45	20	30	30	

Transportation Applications

Initial Basic Feasible Solution (North-West Corner)

	8		6		10		9
35							
	9		12		13		7
	14		9		16		5

Supply

~~35~~ 0

50

40

Demand

~~45~~

20

30

30

10

Transportation Applications

Initial Basic Feasible Solution (North-West Corner)

	8	6	10	9	<i>Supply</i>
35					35 0
	9	12	13	7	50
	14	9	16	5	40
<i>Demand</i>	45	20	30	30	
	10				

Transportation Applications

Initial Basic Feasible Solution (North-West Corner)

	8	6	10	9	<i>Supply</i>
35					35 0
10	9	12	13	7	50 40
	14	9	16	5	40
<i>Demand</i>	45	20	30	30	
	10				
	0				

Transportation Applications

Initial Basic Feasible Solution (North-West Corner)

		8		6		10		9	<i>Supply</i>
	35								35 0
	10	9		12		13		7	50 40
		14		9		16		5	40
<i>Demand</i>	45	20			30		30		
	10								
	0								

Transportation Applications

Initial Basic Feasible Solution (North-West Corner)

		8	6	10	9	<i>Supply</i>
	35					35 0
	9	12	13	7		50 40 20
10		20				
	14	9	16	5		40
<i>Demand</i>	45	20	30	30		
	10	0				
	0					

Transportation Applications

Initial Basic Feasible Solution (North-West Corner)

		8		6		10		9	<i>Supply</i>
	35								35 0
		9		12		13		7	50 40 20
	10		20						
		14		9		16		5	40
<i>Demand</i>	45		20		30		30		
	10		0						
	0								

Transportation Applications

Initial Basic Feasible Solution (North-West Corner)

		8		6		10		9	<i>Supply</i>
	35								35 0
		9		12		13		7	50 40 20 0
	10		20		20				
		14		9		16		5	40
<i>Demand</i>	45		20		30		30		
	10		0		10				
	0								

Transportation Applications

Initial Basic Feasible Solution (North-West Corner)

	8	6	10	9	<i>Supply</i>
35					35 0
9	12	13	7	50 40	20 0
10	20	20			
14	9	16	5	40	
<i>Demand</i>	45 10	20 0	30 10	30	
	0				

Transportation Applications

Initial Basic Feasible Solution (North-West Corner)

	8	6	10	9	<i>Supply</i>
35					35 0
9	12	13	7		50 40 20 0
10	20	20			
14	9	16	5		40 30
		10			
<i>Demand</i>	45	20	30	30	
	10	0	10		
	0		0		

Transportation Applications

Initial Basic Feasible Solution (North-West Corner)

	8	6	10	9	<i>Supply</i>
35					35 0
9	12	13	7		50 40 20 0
10	20	20			
14	9	16	5		40 30
		10			
<i>Demand</i>	45	20	30	30	
	10	0	10		
	0		0		

Transportation Applications

Initial Basic Feasible Solution (North-West Corner)

	8	6	10	9	<i>Supply</i>
35					35 0
9	12	13	7	50	40 20 0
10	20	20			
14	9	16	5	40	30 0
		10	30		
<i>Demand</i>	45	20	30	30	
	10	0	10	0	
	0		0		

Transportation Applications

Initial Basic Feasible Solution (North-West Corner)

		8		6		10		9	<i>Supply</i>	
	35								35	0
		9		12		13		7	50	40 20 0
	10		20		20				40	30 0
		14		9		16		5		
					10		30			
<i>Demand</i>	45	20	30	30						
	10	0	10	0						
	0				0					

Transportation Applications

Initial Basic Feasible Solution (North-West Corner)

	8	6	10	9	<i>Supply</i>
35					35
	9	12	13	7	50
10	20	20			
	14	9	16	5	40
		10	30		
45	20	30	30		

cell filled = $(m+n-1) = 4+3-1 = 6$
The result is True

Total Cost = $(35*8) + (10*9) + (20*12) + (20*13) + (10*16) + (30*5) = 1180 \$$

2- Least cost Method:

Transportation Applications

Initial Basic Feasible Solution (Least-Cost Method)

	2	10	20	11	<i>Supply</i> 15
	12	7	9	20	25
	4	14	16	18	10

Supply = 50

Demand 5 15 15 15

Demand = 50

Supply = Demand = 50



Balanced Transportation

Transportation Applications

Initial Basic Feasible Solution (Least-Cost Method)

	2	10	20	11	<i>Supply</i>
5					15 10
	12	7	9	20	25
	4	14	16	18	10
<i>Demand</i>	5	15	15	15	
	0				

Transportation Applications

Initial Basic Feasible Solution (Least-Cost Method)

		2	10	20	11	<i>Supply</i>
	5					15 10
		12	7	9	20	25
		4	14	16	18	10
<i>Demand</i>	5 0	15	15	15		

Transportation Applications

Initial Basic Feasible Solution (Least-Cost Method)

		2	10	20	11	
	5					<i>Supply</i> 15 10
	12	15	7	9	20	25 10
	4	14	16	18		10
<i>Demand</i>	5 0	15 0	15	15		

Transportation Applications

Initial Basic Feasible Solution (Least-Cost Method)

		2		10		20		11		
	5									
		12		7		9		20		
			15							
		4		14		16		18		
<i>Demand</i>	5		15		15		15			
	0		0							

Supply
~~15~~ 10
~~25~~ 10
 10

Transportation Applications

Initial Basic Feasible Solution (Least-Cost Method)

		2		10		20		11		
	5									
		12		7		9		20		
			15		10					
		4		14		16		18		
<i>Demand</i>	5		15		15		15			
	0		0		5					

Supply
~~15~~ 10
~~25~~ ~~10~~ 0
 10

Transportation Applications

Initial Basic Feasible Solution (Least-Cost Method)

		2		10		20		11		<i>Supply</i>
	5								15	10
		12		7		9		20	25	10 0
			15		10					
		4		14		16		18	10	
<i>Demand</i>	5		15		15		15			
	0		0		5					

Transportation Applications

Initial Basic Feasible Solution (Least-Cost Method)

		2		10		20		11		
	5							10		
		12		7		9			20	
			15		10					
		4		14		16			18	
<i>Demand</i>	5		15		15		15		15	
	0		0		5		5			

Supply
~~15~~ ~~10~~ 0
~~25~~ ~~10~~ 0
 10

Transportation Applications

Initial Basic Feasible Solution (Least-Cost Method)

		2		10		20		11		
	5							10		
		12		7		9			20	
			15		10					
		4		14		16			18	
<i>Demand</i>	5		15		15		15		15	
	0		0		5		5			

Supply

~~15~~

~~25~~

10

~~10~~

~~10~~

0

0

Transportation Applications

Initial Basic Feasible Solution (Least-Cost Method)

		2		10		20		11		
	5							10		
		12		7		10	9		20	
			15							
		4		14		5	16		18	
<i>Demand</i>	5		15		15		15		15	
	0		0		5		5			
					0					

Supply

~~15~~

~~25~~

~~10~~

~~10~~

~~10~~

5

0

0

Transportation Applications

Initial Basic Feasible Solution (Least-Cost Method)

		2		10		20		11		
	5							10		
		12		7		10	9		20	
			15							
		4		14		5	16		18	
<i>Demand</i>	5		15		15		15		15	
	0		0		5		5			
					0					

Supply

~~15~~ ~~10~~ 0

~~25~~ ~~10~~ 0

~~10~~ 5

Transportation Applications

Initial Basic Feasible Solution (Least-Cost Method)

		2		10		20		11		
	5							10		
		12		7		10	9		20	
			15							
		4		14	5	16		5	18	
<i>Demand</i>	5		15		15		15			
	0		0		5		5			
					0		0			

Supply

~~15~~ ~~10~~ 0

~~25~~ ~~10~~ 0

~~10~~ ~~5~~ 0

Transportation Applications

Initial Basic Feasible Solution (Least-Cost Method)

	2	10	20	11	<i>Supply</i>
5				10	15
	12	7	9	20	25
	15	10			
	4	14	5	16	10
			5	18	
<i>Demand</i>	5	15	15	15	

cell filled = $(m+n-1) = 4+3-1 = 6$
The result is True

Total Cost = $(5*2) + (15*7) + (10*9) + (10*11) + (5*16) + (5*18) = 485$ \$

3- Vogel approximation Method(VAM)

VAM is improved version of the least cost method that generally, but not always, produces better starting

- Initial Basic Feasible Solution (VAM)

	6	8	10	<i>Supply</i> 150
	7	11	11	175
	4	5	12	275

Demand 200 100 300

Demand = 600

Supply = 600

Supply = Demand = 50



Balanced Transportation

- Initial Basic Feasible Solution (VAM)

	6		8		10
	7		11		11
	4		5		12

Supply
150

175

275

Demand 200 100 300

penalty 2 3 1

penalty

2

4

1

- Initial Basic Feasible Solution (VAM)

	6		8		10
175	7		11		11
	4		5		12

Supply
150

~~175~~ 0

275

penalty
2
4
1

Demand ~~200~~ 100 300

25

penalty 2 3 1

- Initial Basic Feasible Solution (VAM)

		6		8		10	<i>Supply</i> 150	penalty 2
	175	7		11		11	175 0	
		4		5		12	275	1
<i>Demand</i>	200		100		300			
penalty	25	2	3		2			

- Initial Basic Feasible Solution (VAM)

		6		8		10	<i>Supply</i> 150		
	175	7		11		11	175 0		2
		4	100	5		12	275 175		1
<i>Demand</i>	200		100			300			
	25		0						
penalty	2		3			2			

- Initial Basic Feasible Solution (VAM)

		6		8		10	<i>Supply</i> 150		
	175	7		11		11	175 0		4
		4	100	5		12	275 175		8
<i>Demand</i>	200		100			300			
	25		0						
penalty	2					2			

- Initial Basic Feasible Solution (VAM)

	6		8		10	Supply	
						150	
175	7		11		11	175	0
25	4	100	5		12	275	175
							150
Demand	200	100			300		
	25	0					
	0						
penalty	2				2		

- | | | | | |
|-----|---|-----|----|----|
| | 6 | 8 | 10 | |
| 175 | 7 | 11 | 11 | |
| 25 | 4 | 100 | 5 | 12 |
- Supply
150
- ~~175~~ 0
- ~~275~~ ~~175~~ 150
- 10
- 12
- nd
- ~~200~~
- ~~100~~
- 300
- ~~25~~
- 0
- penalty

2

- | | | | | | | |
|------------|---|------------|----|------------|----|--|
| | 6 | | 8 | | 10 | |
| 175 | 7 | | 11 | | 11 | |
| 25 | 4 | 100 | 5 | 150 | 12 | |
- Demand: 200, 100, 300
 Supply: 150, 175, 150
 Penalty: 10, 0, 12
 Cost: 25, 0, 150
 Total Cost: 2

- Initial Basic Feasible Solution (VAM)

	6	8	10	Supply	
175	7	11	11	175	0
25	4	100	5	150	12
Demand	200	100	300		
	25	0	150		
	0				

penalty

- Initial Basic Feasible Solution (VAM)

	6	8	150	10	Supply 150	0
175	7	11		11	175	0
25	4	100	5	150	275	175
Demand 200	25	100	150			0

- Initial Basic Feasible Solution (VAM)

	6		8	150	10	<i>Supply</i>
						<i>150</i>
175	7		11		11	<i>175</i>
25	4	100	5	150	12	<i>275</i>

Demand 200 100 300

cell filled = $(m+n-1) = 3+3-1 = 5$
The result is True

Total Cost = $(175*7) + (25*4) + (100*5) + (150*10) + (150*12) = 5125$ \$



Thank You