

Digital Systems- IT131

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Lecture 09: Sequential Circuits Part2

Sequential Circuits

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2. Flip-flops
3. Analysis of clocked sequential circuits
4. Design procedure

Sequential Circuits

Analysis of Clocked Sequential Circuits

- The behavior of a clocked sequential circuit is determined from the inputs, the outputs, and the state of flip-flops
- A clocked sequential circuit can be analyzed by using:
 - (i) The **state equation** obtained by replacing flip-flop input equations in its characteristic equation, or
 - (ii) a **state table**.
- A **state equation** (or **transition equation**) specifies the next state as a function of present state and inputs

Sequential Circuits

- State table - a multiple variable table with the following four sections:
 - Present State - the values of the state variables at time (t) for each allowed state.
 - Input - the input combinations allowed.
 - Next-state - the value of the state at time (t+1) based on the present state and the input
 - Output - the value of the output as a function of the present state and (Mealy) the input.

Analysis (Contd.)

- **Procedure:** Write the Boolean expression for each flip-flop input, which is, in fact, the flip-flop output right after the next active clock edge.
- Consider the circuit shown in next slide
- It consists of two D flip-flops A and B, an input x and an output y
- Since the D input of a flip-flop determines the value of the next state, it is possible to write a set of equations for the circuit

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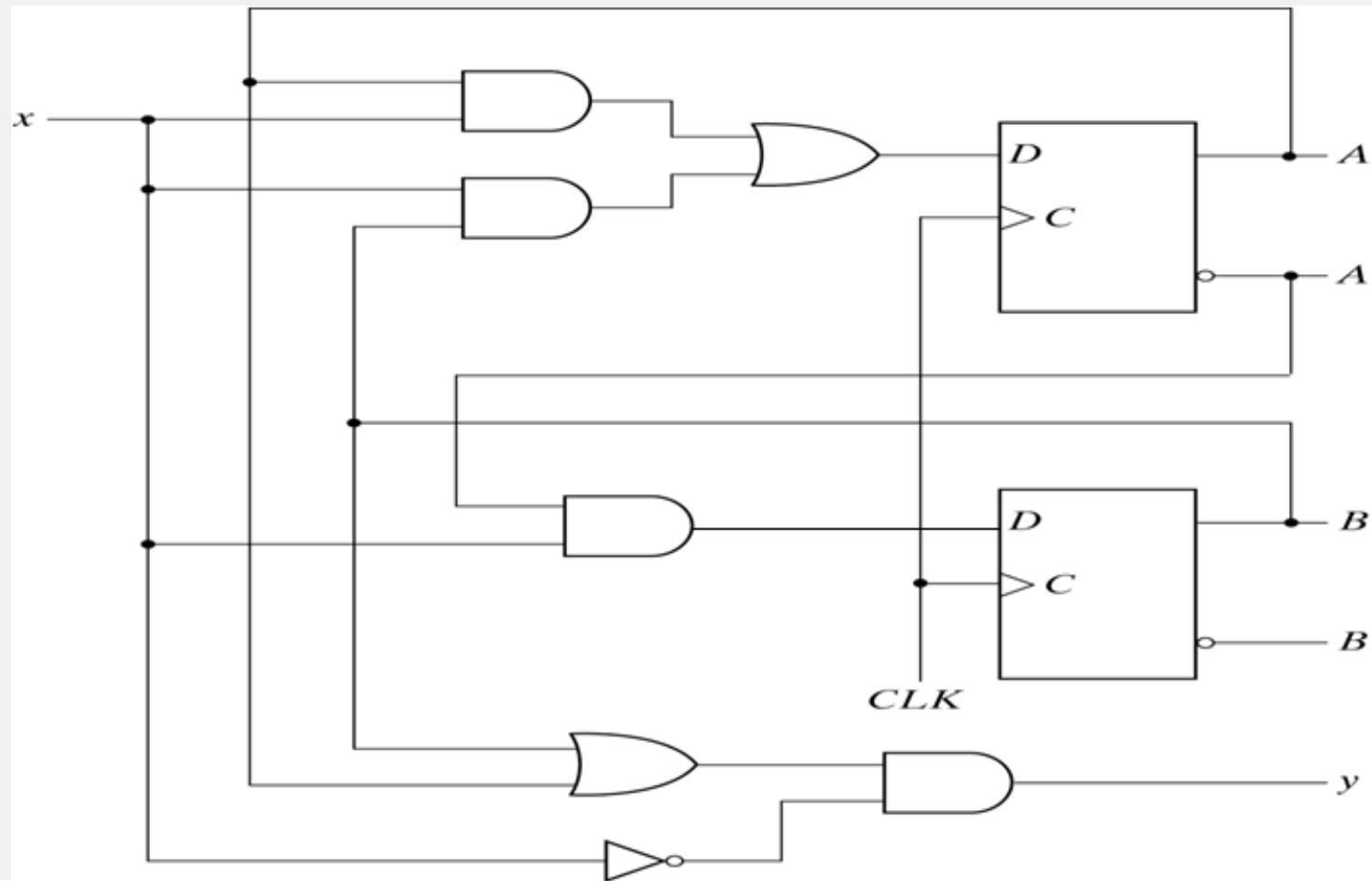


Fig. 5-15 Example of Sequential Circuit

Sequential Circuits

Analysis (Contd.)

➤ Flip-flop input equations or excitation equations (for a circuit with D flip-flops, it is the same as the state equation):

$$D_A = Ax + Bx$$

$$D_B = A'x$$

$$y = (A + B)x'$$

➤ As the FF in use is the D FF:

$$Q(t+1) = D$$

Sequential Circuits

Analysis (Contd.)

- The state equation can be written as:

$$A(t+1) = A(t).x(t) + B(t).x(t)$$

$$B(t+1) = A'(t).x(t)$$

- Or simply:

$$A(t+1) = Ax + Bx$$

$$B(t+1) = A'x$$

- Similarly, the present state of output is:

$$y(t) = [A(t) + B(t)]x'(t),$$

or $y = (A + B)x'$

Sequential Circuits

Analysis (Contd.)

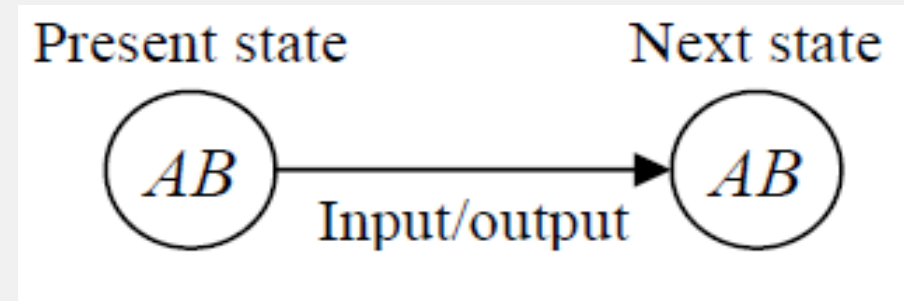
m flip-flops		n inputs	Next state		Output
Present state		Input			
A	B	x	A	B	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

2^{m+n} rows

Sequential Circuits

Analysis (Contd.)

- A **state diagram** is a graphical representation of a state table



- represented by a circle, and the transitions between states are indicated by directed lines connecting the circles
- Number of states = **$2^{\text{number of Flip Flops}}$**
- The binary number inside each circle identifies the state of the flip-flops

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Analysis (Contd.)

- The directed lines are labeled with two binary numbers separated by a slash
- The state diagram follows from the state table
- The state diagram for the given circuit:

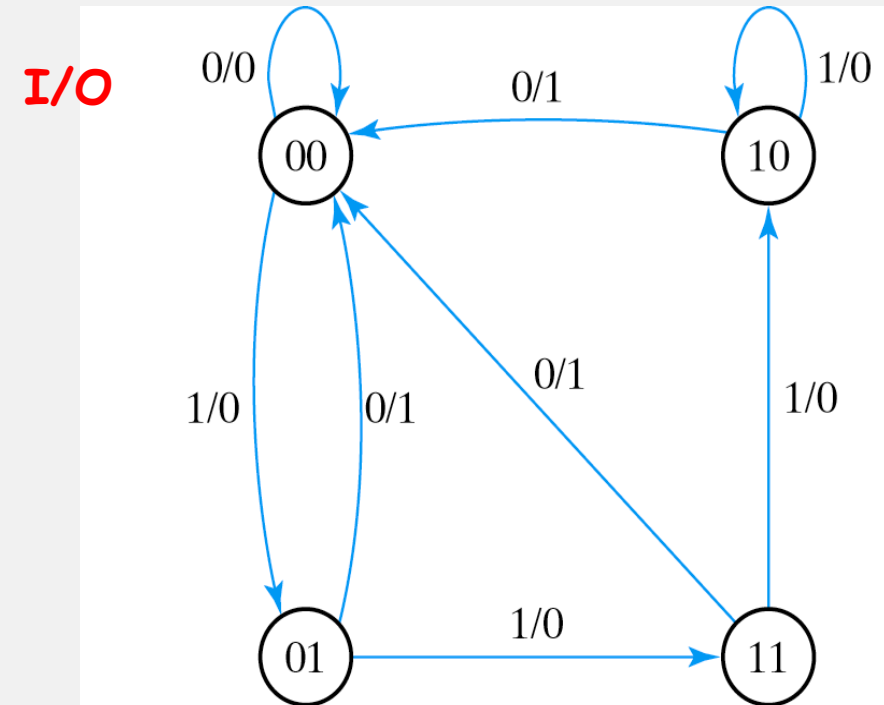


Fig. 5-16 State Diagram of the Circuit of Fig. 5-15

Sequential Circuits

State Diagrams Types

■NOTE:

- **Mealy model** – the output values are determined by both current state and current inputs.
- **Moore model** – the output values are determined **only** by current state.

Sequential Circuits

Analysis of D FF: summary

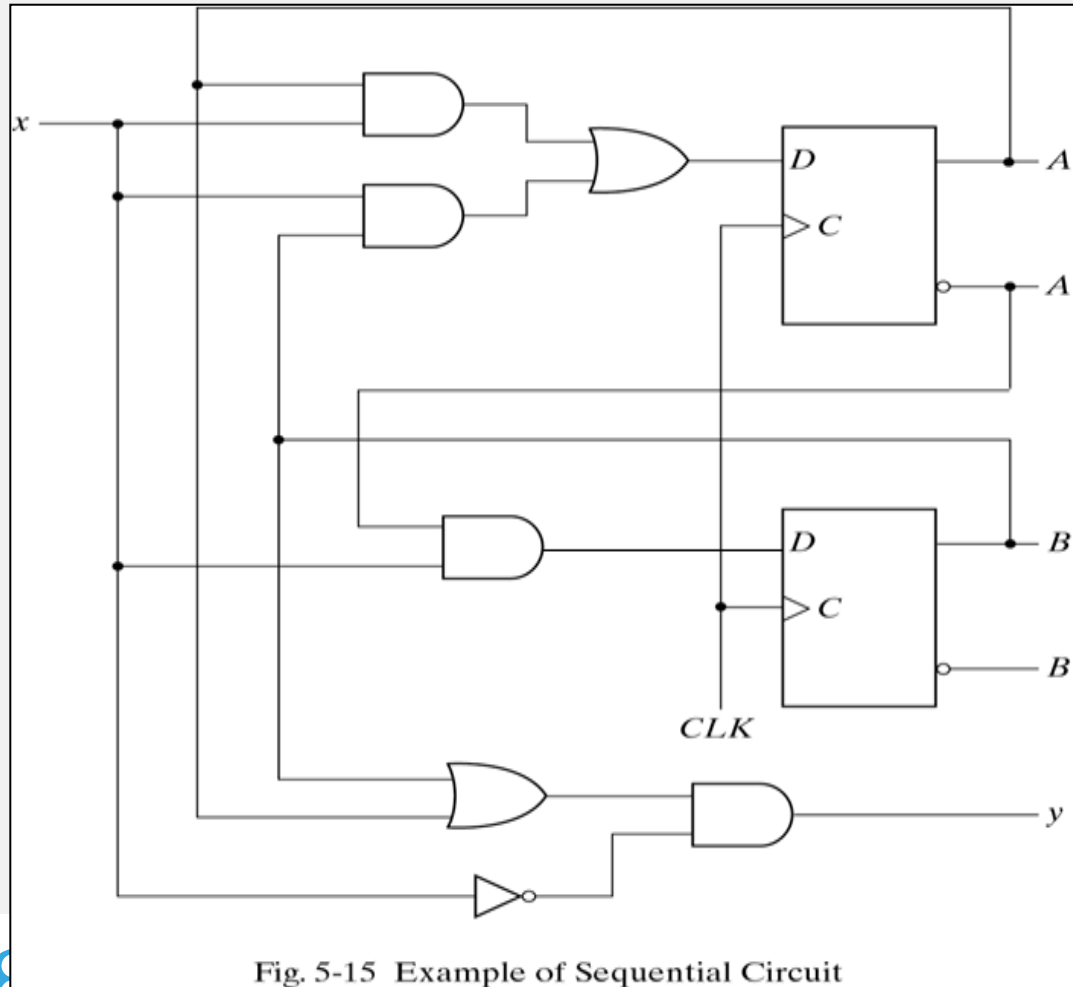


Fig. 5-15 Example of Sequential Circuit

1) FF Input equation:

$$D_A = Ax + Bx$$

$$D_B = A'x$$

2) State equation and output:

Characteristic equation:

$$A(t+1) = D_A, \quad B(t+1) = D_B$$

State equation:

$$A(t+1) = A(t).x(t) + B(t).x(t)$$

$$B(t+1) = A'(t).x(t)$$

Or $A(t+1) = Ax + Bx$

$$B(t+1) = A'x$$

state of output is:

$$y = (A + B)x'$$

3) State table.

4) State diagram (Mealy)

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Analysis with JK flip-flops

- The next state values of a sequential circuit that uses flip-flops such as JK can be derived using following procedure:
 - Determine the flip-flop input equations in terms of the present state and inputs
 - Use corresponding flip-flop characteristic equation to determine the next state values
- We consider the sequential circuit with two JK flip-flops A and B, and one input x

Analysis with JK flip-flops (Contd.)

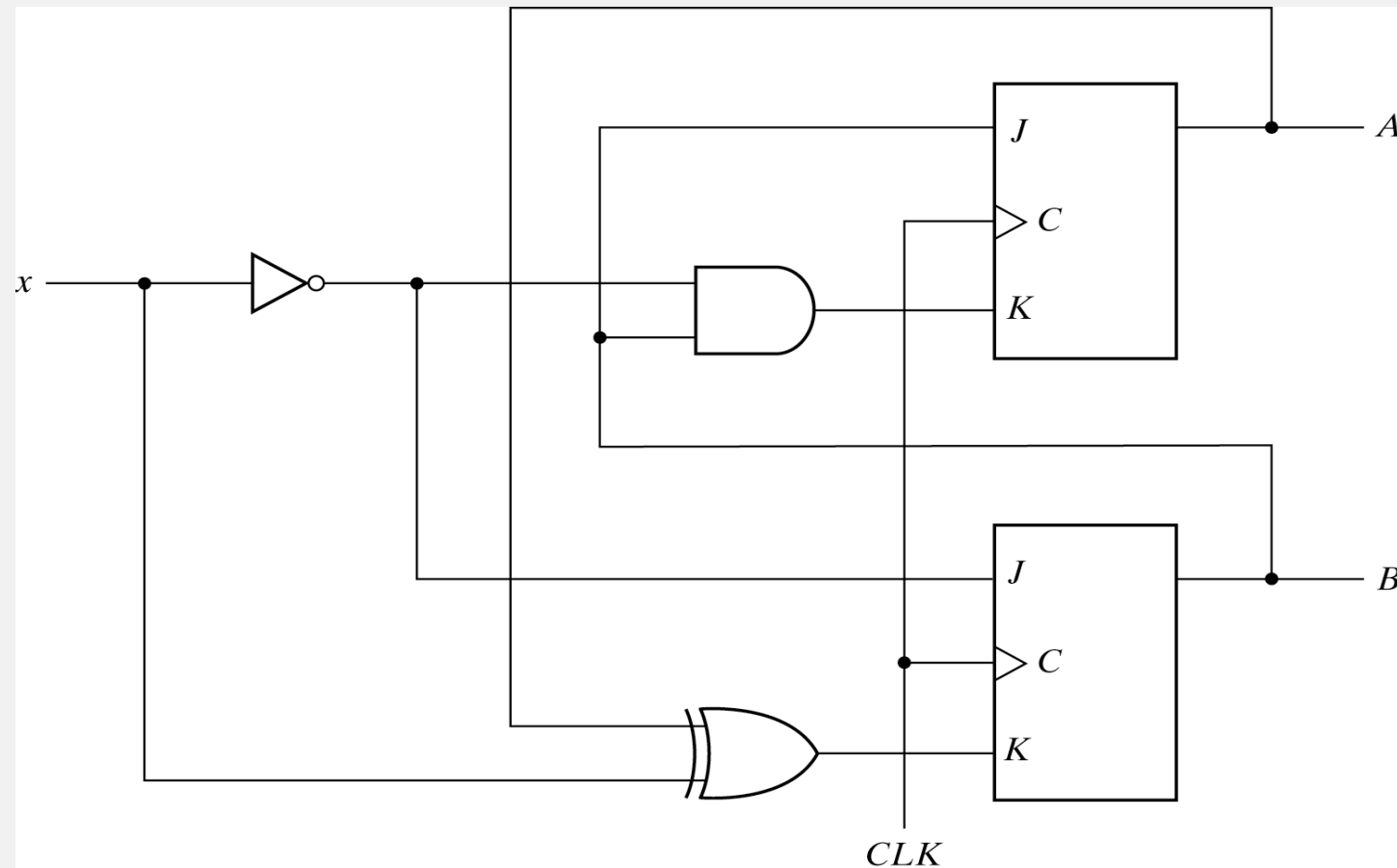


Fig. 5-18 Sequential Circuit with JK Flip-Flop

Sequential Circuits

Analysis with JK flip-flops (Contd.)

- The circuit has no outputs and, so, the state table does not need an output column
- The outputs of the flip-flops may be considered as the outputs in this case
- The circuit can be specified by the flip-flop input equations

$$J_A = B$$

$$K_A = Bx'$$

$$J_B = x'$$

$$K_B = A'x + Ax' = A \oplus x$$

Sequential Circuits

Analysis with JK flip-flops (Contd.)

- find the next state from the state equations by substituting the input equations into the flip-flop characteristic equations.
- Characteristic equation for the JK flip-flop is:

$$A(t+1) = J_A A' + K'_A A$$

$$B(t+1) = J_B B' + K'_B B$$

Analysis with JK flip-flops (Contd.)

➤ Substituting the values of J_A and K_A , and J_B and K_B , from the input equations, we obtain state equations for A and B as:

$$A(t+1) = BA' + (Bx')'A = A'B + AB' + Ax$$

$$B(t+1) = x'B' + (A \oplus x)'B = B'x' + ABx + A'Bx'$$

Sequential Circuits

Analysis with JK flip-flops (Contd.)

Present state		Input	Next State	
<i>A</i>	<i>B</i>	<i>x</i>	<i>A</i>	<i>B</i>
0	0	0	0	1
0	0	1	0	0
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	1	1

$$A(t+1) = A'B + AB' + Ax$$

$$B(t+1) = B'x' + ABx + A'Bx'$$

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Analysis with JK flip-flops (Contd.)

➤ The state diagram of the sequential circuit is shown right-side. Since the circuit has no outputs, the directed lines out of circles are marked with one binary number only to designate the value of input x .

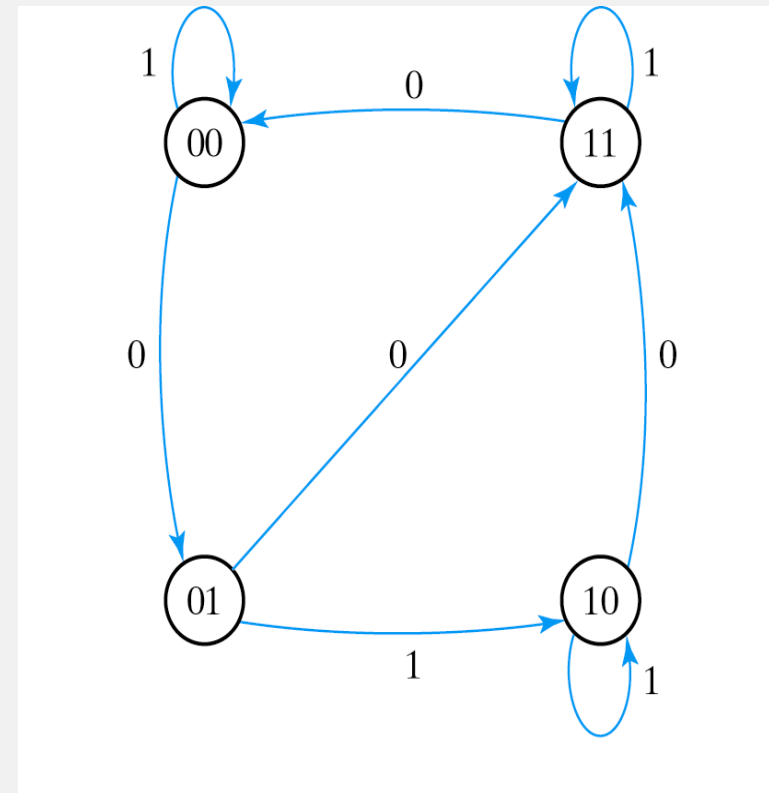
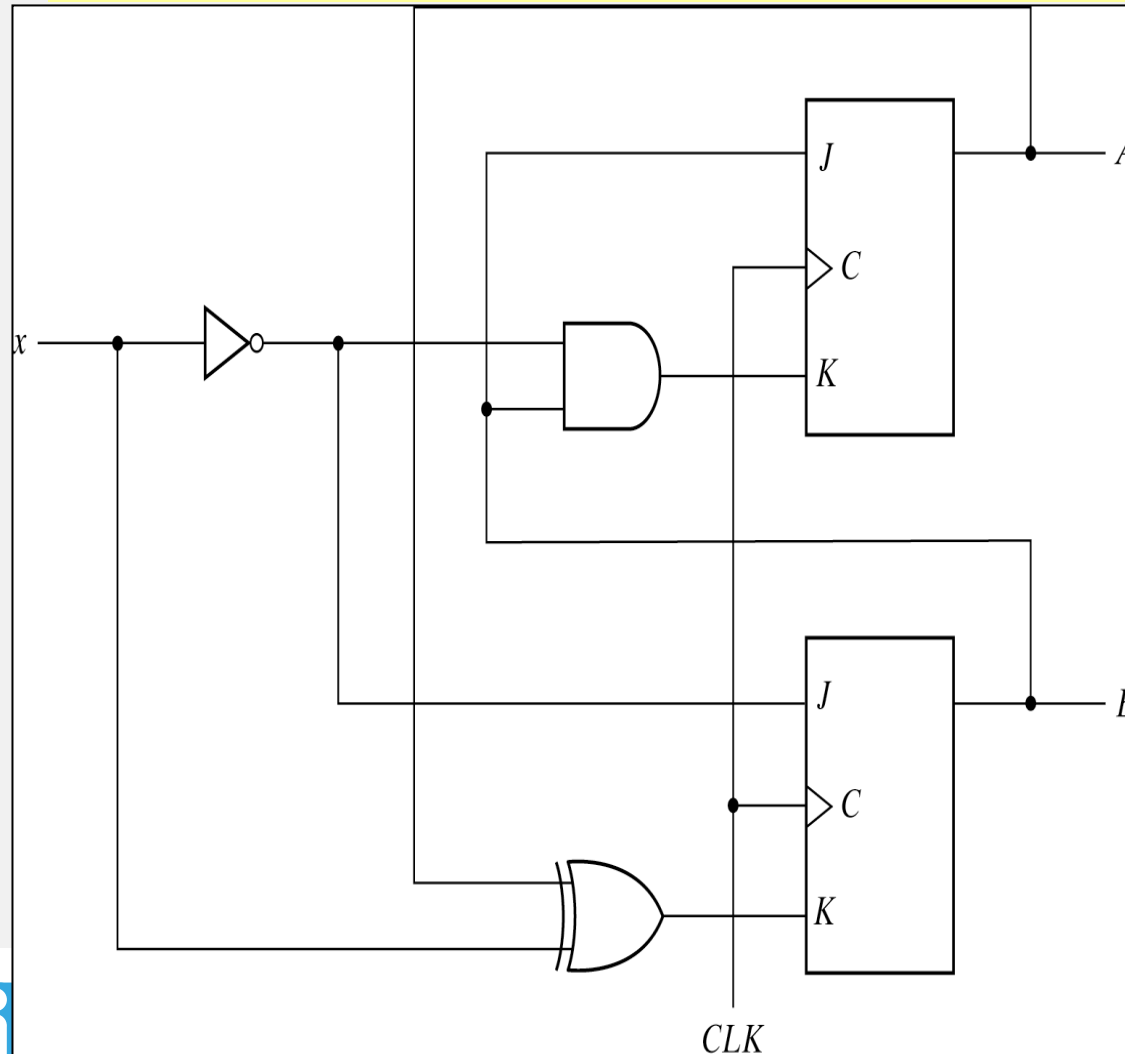


Fig. 5-19 State Diagram of the Circuit of Fig. 5-18

Sequential Circuits

Analysis of JK FF: summary



1) FF Input equation:

$$J_A = B \quad K_A = Bx'$$

$$J_B = x' \quad K_B = A \oplus x$$

2) State equation and output:

Characteristic equation:

$$A(t+1) = J_A A' + K'_A A$$

$$B(t+1) = J_B B' + K'_B B$$

State equation:

$$A(t+1) = BA' + (Bx')'A$$

$$= A'B + AB' + Ax$$

$$B(t+1) = x'B' + (A \oplus x)'B$$

$$= B'x' + ABx + A'Bx'$$

3) State table.

4) State diagram.

Sequential Circuits

Analysis with T flip-flops

➤ Similar to the analysis with JK flip-flops, the state table can be obtained by using characteristic table or the characteristic equation

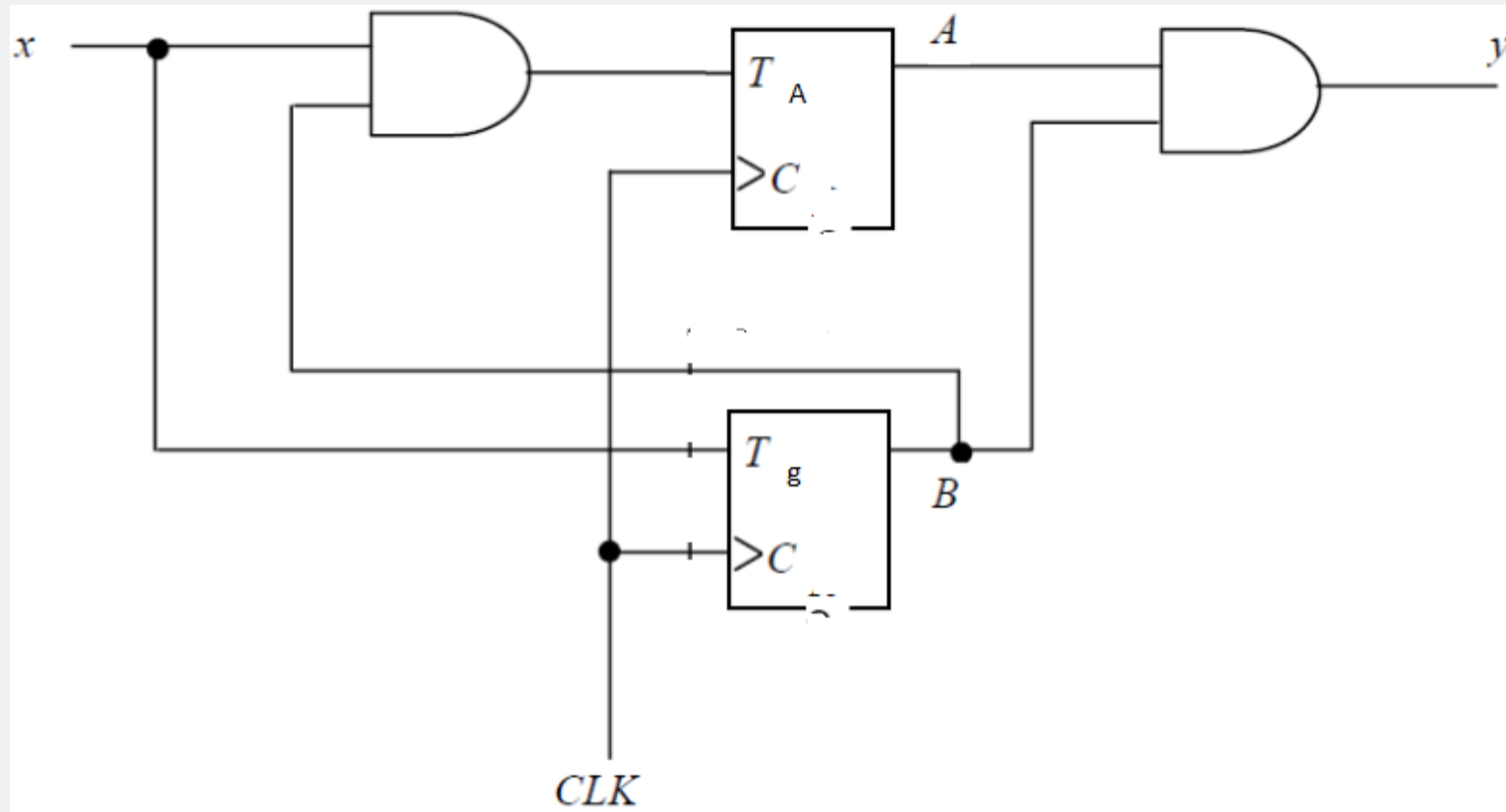
$$Q(t+1) = T \oplus Q = T'Q + TQ'$$

➤ Consider the sequential circuit given in the next slide

➤ It has two flip-flops A and B, one input x, and one output y

Sequential Circuits

Analysis with T flip-flops (Contd.)



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Sequential Circuits

Analysis with T flip-flops (Contd.)

- It can be described algebraically by two input equations and one output equation

$$T_A = Bx, \quad T_B = x, \quad y = AB$$

- The characteristic equations are

$$A(t+1) = T_A \oplus A, \quad B(t+1) = T_B \oplus B$$

- Substituting the input equations with characteristic equations we get

$$A(t+1) = (Bx)'A + (Bx)A' = AB' + Ax' + A'Bx$$

$$B(t+1) = x \oplus B$$

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Analysis with T flip-flops (Contd.)

➤ The state table is as follows:

Present state		Input	Next State		Output
<i>A</i>	<i>B</i>	<i>x</i>	<i>A</i>	<i>B</i>	<i>y</i>
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	1	0
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	1

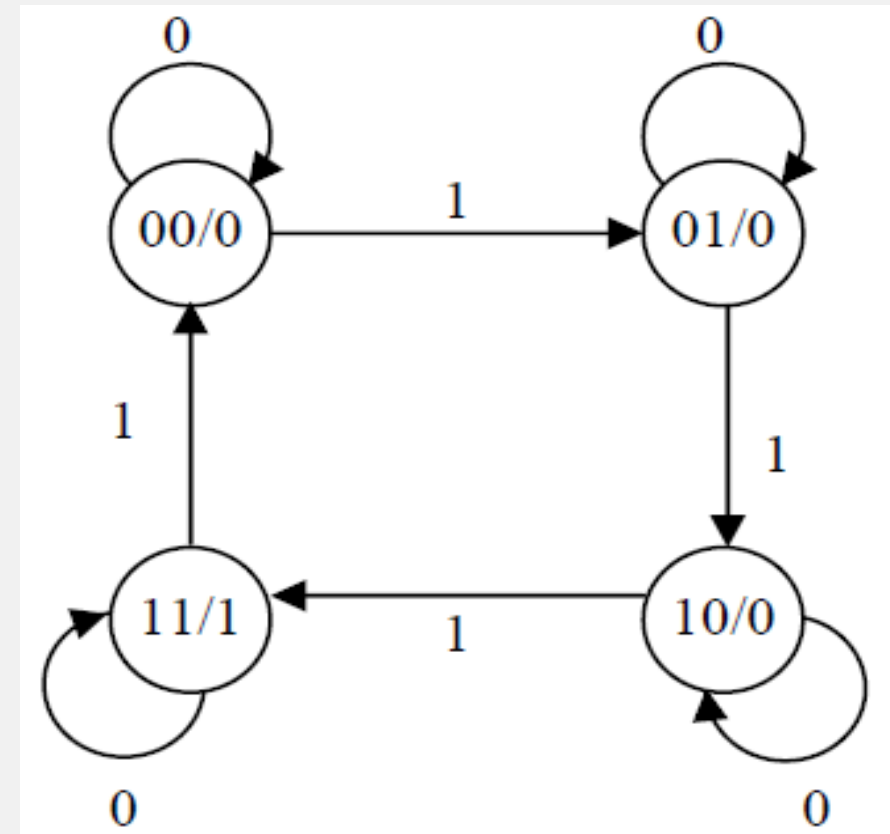
$$A(t+1) = AB' + Ax' + A'Bx$$

$$B(t+1) = x \oplus B$$

Sequential Circuits

Analysis with T flip-flops (Contd.)

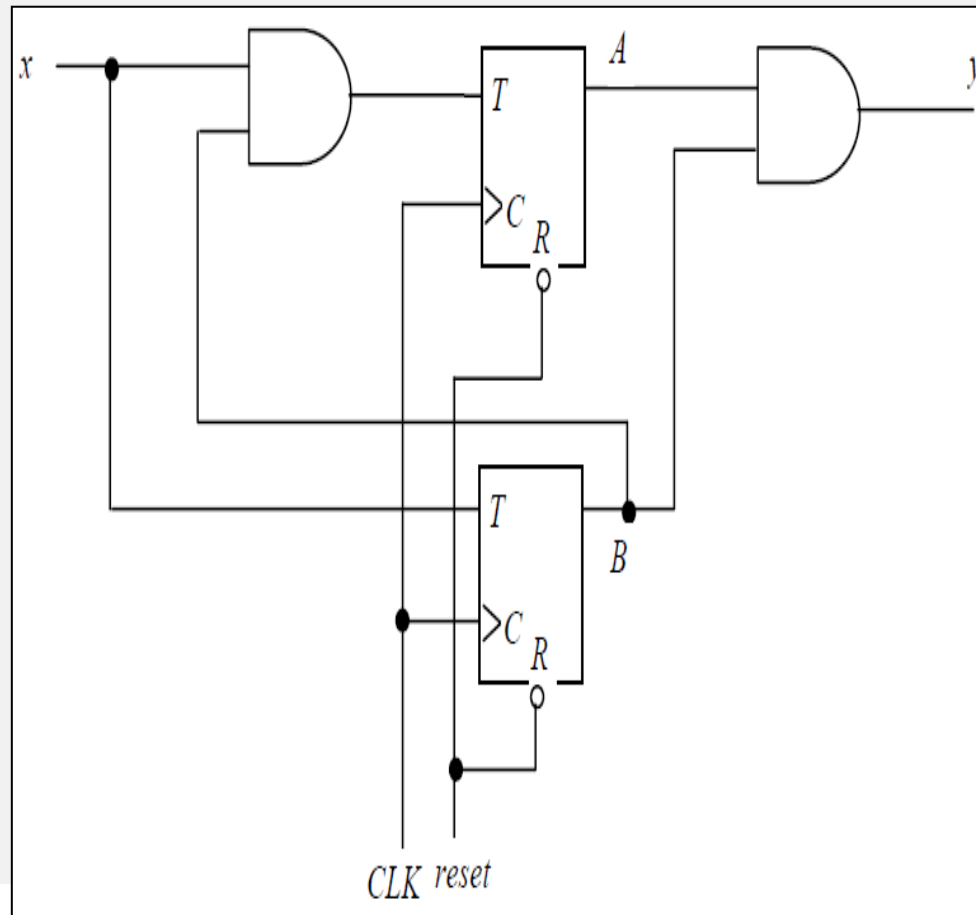
- The state diagram is in right-side
- The output is only a function of current state, not the input, and that is why the output is written inside state circles



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Sequential Circuits

Analysis of T FF: summary



1) FF Input equation:

$$T_A = Bx, \quad T_B = x,$$

2) State equation and output:

Characteristic equation:

$$A(t+1) = T'_A A + T_A A'$$

$$B(t+1) = T'_B B + T_B B'$$

State equation:

$$\begin{aligned} A(t+1) &= (Bx)'A + (Bx)A' \\ &= AB' + Ax' + A'Bx \end{aligned}$$

$$\begin{aligned} B(t+1) &= x'B + xB' \\ &= x \oplus B \end{aligned}$$

State of the output:

$$y = AB$$

3) State table.

4) State diagram (Moore)

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Sequential Circuits

Design Sequential circuits

- The procedure for designing synchronous sequential circuit is:
 1. Translate the design specifications to the state diagram for the circuit
 2. Assign binary values to the states
 3. Obtain the binary coded state table

Sequential Circuits

Design Sequential circuits

3. Choose the type of flip-flops to be used
4. Derive the simplified flip-flop input equations and output equations
5. Draw the logic diagram

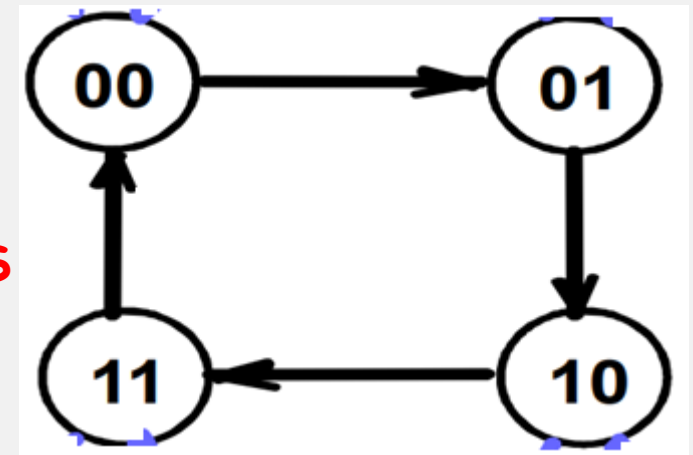
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Example 1: Design 2-bit Counter using D-Flip Flops.

1. Translate the design specifications to the state diagram for the circuit
2. Assign binary values to the states
3. Obtain the binary coded state table

Since Number of states = $2^{\text{number of Flip Flops}}$
4 = $2^{\text{Number of Flip FLOPS}}$

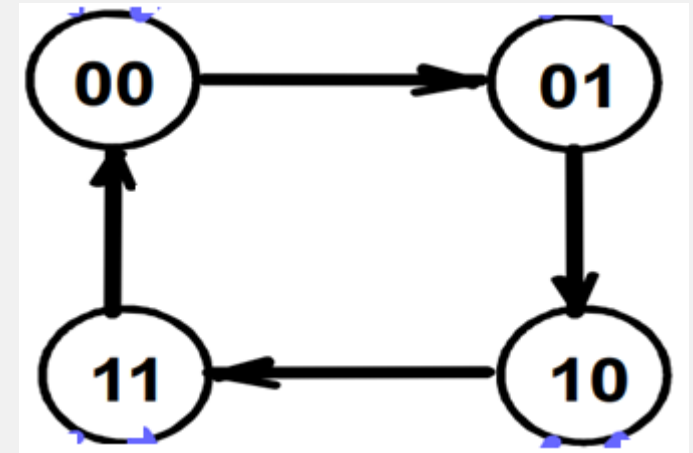
So, Number of Flip[Flops = 2
We Use D-type Flip Flop



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3. Obtain the binary coded state table

Present State		Next State	
A	B	A	B
0	0	0	1
0	1	1	0
1	0	1	1
1	1	0	0



Sequential Circuits

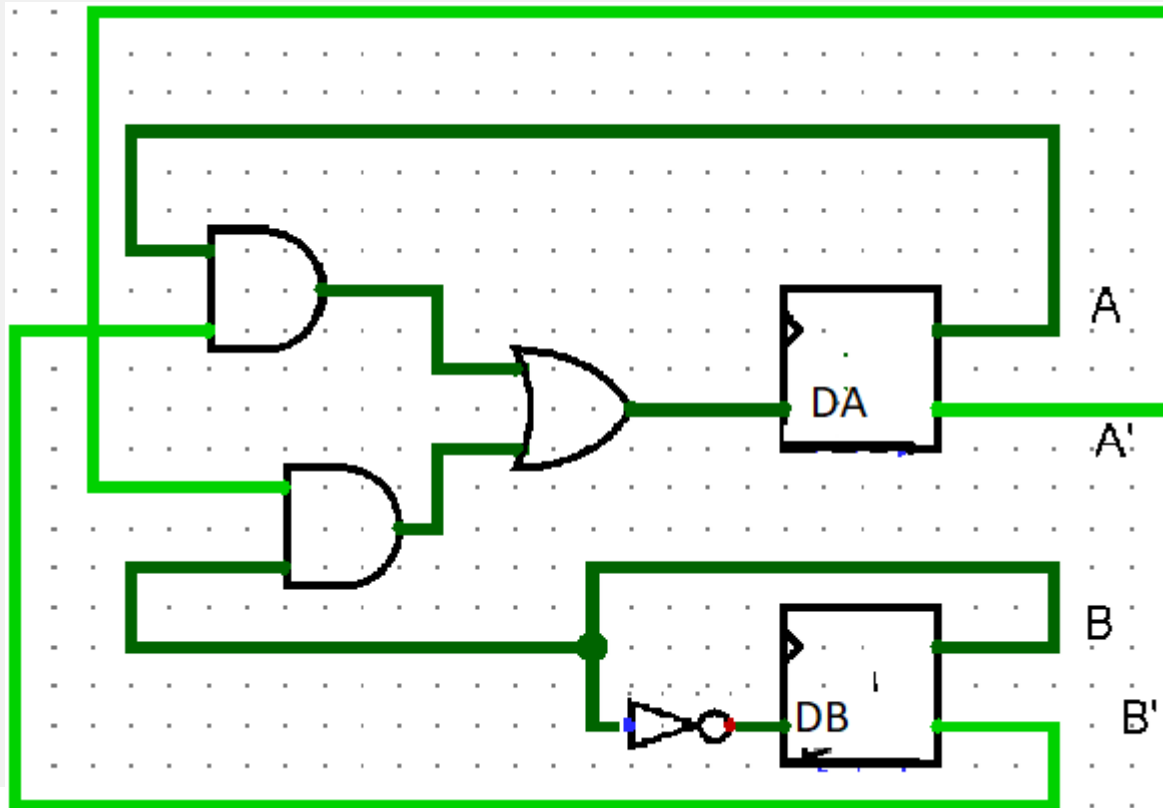
3. Choose the type of flip-flops to be used (D-type)
4. Derive the simplified flip-flop input equations and output equations

$$A(t+1) = A.B' + A'.B$$

$$B(T+1) = B'$$

Present State		Next State	
A	B	A	B
0	0	0	1
0	1	1	0
1	0	1	1
1	1	0	0

5. Draw the logic diagram



Present State		Next State	
A	B	A	B
0	0	0	1
0	1	1	0
1	0	1	1
1	1	0	0

End of lecture 9

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