

Business Mathematics

Prepared by Dr. Faisal Hasan

Lecture NO 9

جامعة الجميع الذكية

تسعى لتكون الكيان الأول الذي يتبنى مبدأ المجتمع التعليمي
التعاوني ويقدم العلم مقابل العمل لتنمية وبناء المجتمعات.

MTH 112

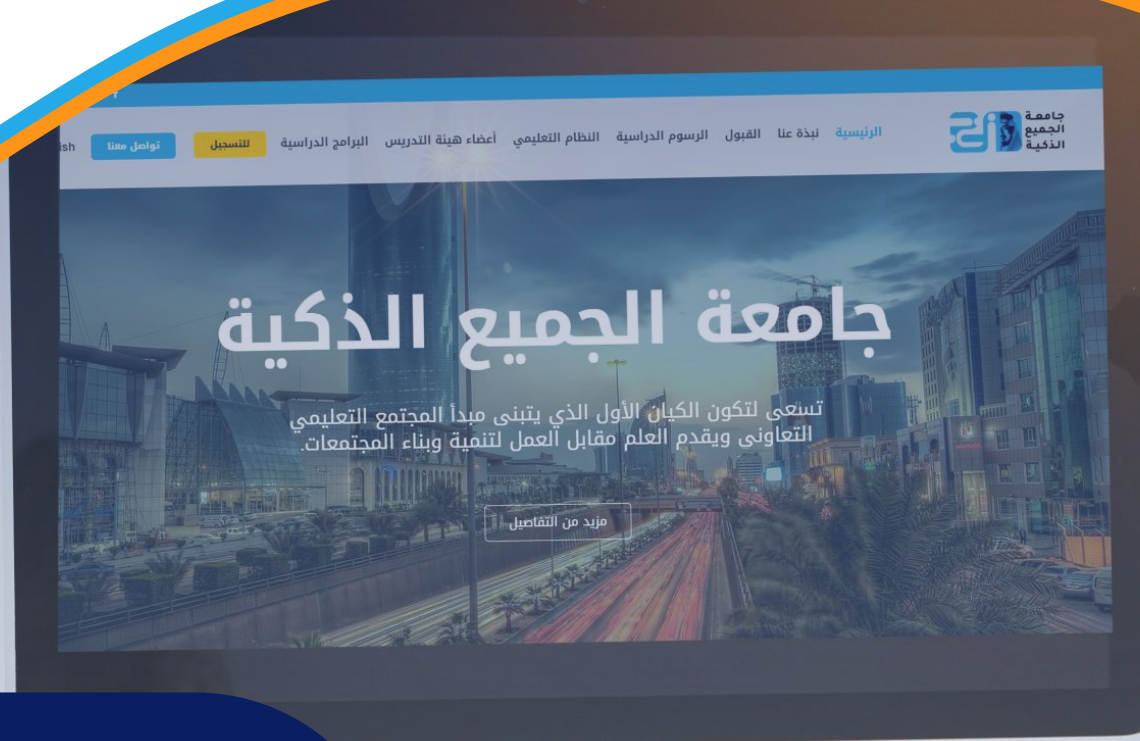
Business Mathematics



جامعة الجميع الذكية
EVERYONE'S SMART UNIVERSITY



جامعة الجميع الذكية
EVERYONE'S SMART UNIVERSITY



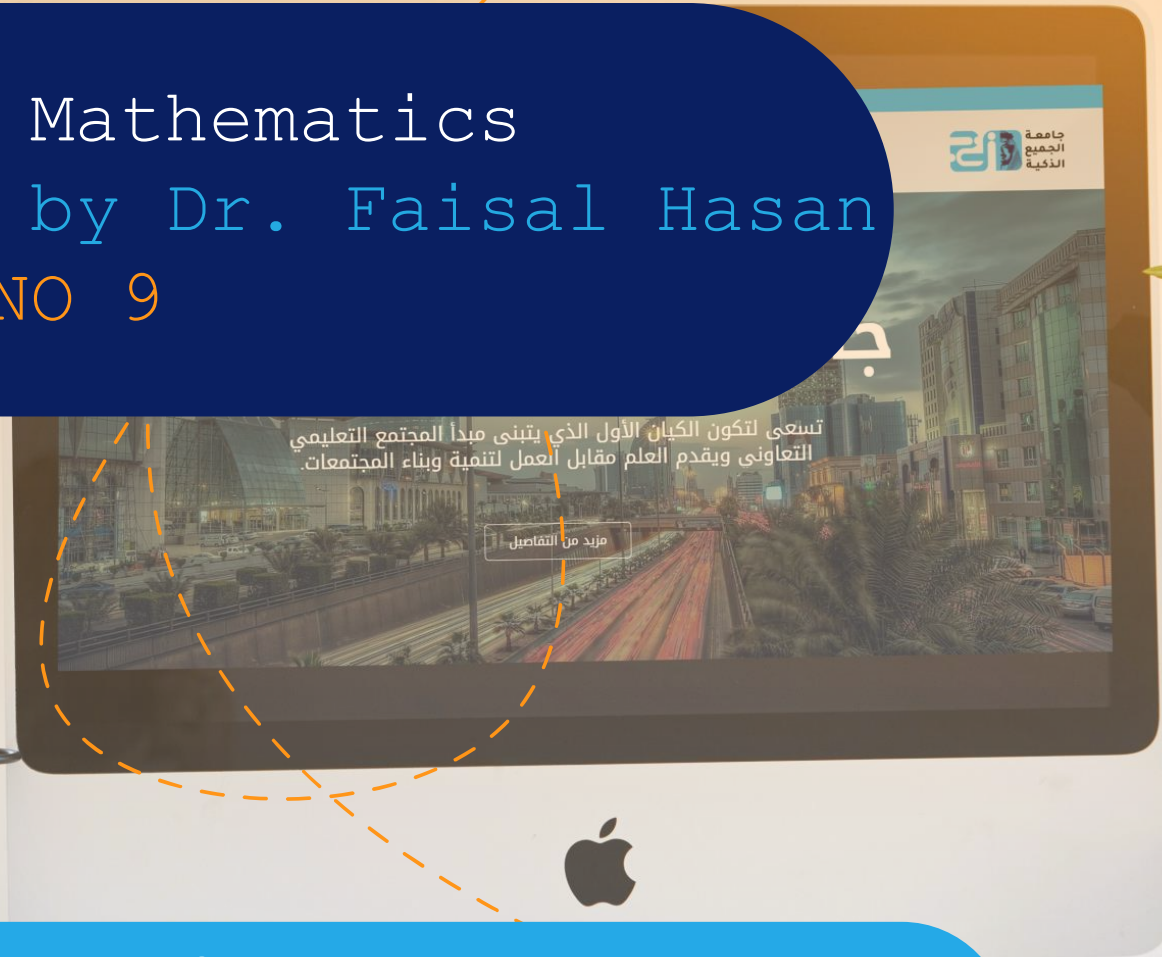
Business Mathematics
Prepared by Dr. Faisal Hasan
Lecture NO 9

MTH 112
Business Mathematics

Business Mathematics

Prepared by Dr. Faisal Hasan

Lecture NO 9



جامعة العنبر الذكية
EVERYONE'S SMART UNIVERSITY

MTH 112

Business Mathematics

Topics

- Chapter 1: Mathematical preliminaries
- Chapter 2: The straight line and applications
- Chapter 3: Functions and Graphs
- Chapter 4: Limits and Continuity
- Chapter 5: Financial Mathematics
- Chapter 6: Differentiation
- Chapter 7: Methods of Differentiation
- Chapter 8: Function of several variables
- Chapter 9: Integration and applications
- Chapter 10: Linear algebra and applications

$f'(x)$ “ f prime of x ”

Remember that

y' “ y prime”

$\frac{dy}{dx}$ “the derivative of y with respect to x ”

$\frac{d}{dx}$ is a verb. “Take the derivative with respect to x ...”



Remember that

- Differentiation is the process of computing the derivative of a function.

You may be asked to:

- Differentiate.
- Derive.
- Find the derivative of...

$$\frac{d}{dx}[c] = 0, \quad c \text{ is a constant}$$

Remember that

The derivative of a constant function is zero.

Remember that

$$\frac{d}{dx}[x^N] = Nx^{N-1}, \quad N \text{ is any real number}$$

$$\frac{d}{dx}[x] = 1$$



Remember that

$$\frac{d}{dx}[c (f (x))] = c (f '(x)), \quad c \text{ is a constant}$$

- The derivative of a constant times a function is equal to the constant times the derivative of the function.

Remember that

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

The derivative of a sum is the sum of the derivatives.

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

The derivative of a difference is the difference of the derivatives.

Remember that

The derivative of the product of two differentiable functions is given by

$$y = f(x) \cdot g(x)$$

$$\frac{dy}{dx} = y' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

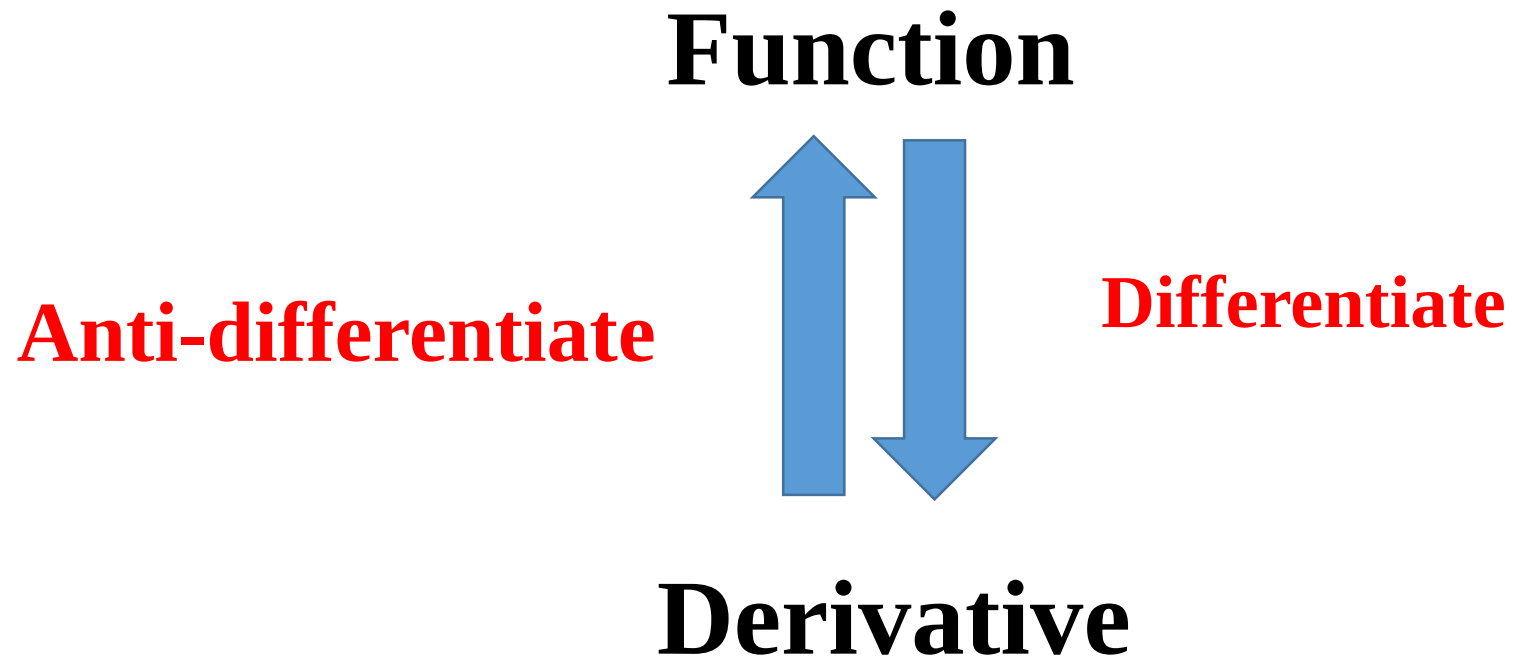
$$\frac{dy}{dx} = y' = f' \cdot g + f \cdot g'$$

Remember that

- The derivative of the quotient of two differentiable functions is given by

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \quad (g(x) \neq 0)$$

- **Integration or Anti-derivative of the Function**



$$f(x) = x^3$$

$$f(x) = x^3 + 4$$

$$f(x) = x^3 - 7$$

Differentiate

$$f'(x) = 3x^2$$

Anti-differentiate ???

$$f'(x) = 3x^2$$

Anti-differentiate will be

$$f(x) = x^3 + C$$

C Is the constant of integration

We can write $\int (3x^2) dx = x^3 + C$

$\int$ Is the integration sign

dx Means that integration with respect to x

C Is the constant of integration

$(3x^2)$ Is the derivative of x^3

In general

$$\int f'(x) dx = f(x) + C$$

$\int$ Is the integration sign

dx Means that integration with respect to x

C Is the constant of integration

$f'(x)$ Is the derivative of $f(x)$

Find $\int (4x^3) dx$

$$\int (4x^3) dx = x^4 + C$$

In general

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$



Basic Integration Rules

Differentiation Formula

$$\frac{d}{dx}[C] = 0$$

$$\frac{d}{dx}[kx] = k$$

$$\frac{d}{dx}[kf(x)] = kf'(x)$$

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

Integration Formula

$$\int 0 \, dx = C$$

$$\int k \, dx = kx + C$$

$$\int kf(x) \, dx = k \int f(x) \, dx$$

$$\int [f(x) \pm g(x)] \, dx = \int f(x) \, dx \pm \int g(x) \, dx$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1 \quad \text{Power Rule}$$

$$\int kx^n dx = \frac{kx^{n+1}}{n+1} + C$$



Find

$$\int 3x \, dx$$



$$\begin{aligned}\int 3x \, dx &= 3 \int x \, dx \\ &= 3 \int x^1 \, dx \\ &= 3 \left(\frac{x^2}{2} \right) + C \\ &= \frac{3}{2} x^2 + C\end{aligned}$$



Find

$$\int \frac{1}{x^2} dx$$



$$\begin{aligned} F(x) &= \int \frac{1}{x^2} dx \\ &= \int x^{-2} dx \\ &= \frac{x^{-1}}{-1} + C \\ &= \frac{-1}{x} + C \end{aligned}$$



Find

$$\int \sqrt{x} \, dx$$



$$\begin{aligned}\int \sqrt{x} \, dx \\&= \int x^{\frac{1}{2}} \, dx \\&= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C \\&= \frac{2}{3} x^{\frac{3}{2}} + C\end{aligned}$$



Find

$$\int \frac{1}{\sqrt{x}} dx$$



$$\begin{aligned}\int \frac{1}{\sqrt{x}} dx &= \int x^{-\frac{1}{2}} dx \\&= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C \\&= 2x^{\frac{1}{2}} + C \\&= 2\sqrt{x} + C\end{aligned}$$

Find

$$\int \frac{1}{\sqrt[3]{x}} dx$$



$$\int \frac{1}{\sqrt[3]{x}} dx = \int x^{-\frac{1}{3}} dx$$

$$= \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + C$$

$$= \frac{3}{2} x^{\frac{2}{3}} + C$$



Find

$$\int (x^6 + 5x - 4) dx$$



$$\int (x^6 + 5x - 4) dx$$

$$= \int (x^6) dx + \int (5x) dx + \int (-4) dx$$

$$= \frac{x^7}{7} + \frac{5x^2}{2} - 4x + C$$

Find

$$\int \left(10x^4 + \frac{1}{x^2} + 6\sqrt{x} - 3 \right) dx$$



$$\int \left(10x^4 + \frac{1}{x^2} + 6\sqrt{x} - 3 \right) dx =$$

$$\int 10x^4 dx + \int x^{-2} dx + \int 6x^{\frac{1}{2}} dx - \int 3 dx$$

$$= \frac{10x^5}{5} + \frac{x^{-1}}{-1} + \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} - 3x + C$$

$$= 2x^5 - \frac{1}{x} + 4\sqrt{x^3} - 3x + C$$

$$\int f'(x) dx = f(x) + C$$

$$\int_a^b f'(x) dx = [f(x)]_a^b + \cancel{C}$$

$$= f(b) - f(a)$$



Find

$$\int_0^2 3x \, dx$$



$$\begin{aligned}\int_0^2 3x \, dx &= 3 \int_0^2 x \, dx \\&= \left[3 \frac{x^2}{2} \right]_0^2 \\&= \frac{3}{2} [x^2]_0^2 \\&= \frac{3}{2} ((2^2) - (0^2)) = 6\end{aligned}$$

Find

$$\int_1^3 \frac{1}{x^2} dx$$



$$\begin{aligned}\int_1^3 \frac{1}{x^2} dx &= \int_1^3 x^{-2} dx \\&= \left[\frac{x^{-1}}{-1} \right]_1^3 \\&= \left[\frac{-1}{x} \right]_1^3 = \left(-\frac{1}{3} \right) - \left(-\frac{1}{1} \right) \\&= \frac{2}{3}\end{aligned}$$

Find

$$\int_0^4 \sqrt{x} \, dx$$



$$\begin{aligned} & \int_0^4 \sqrt{x} \, dx \\ &= \int_0^4 x^{\frac{1}{2}} \, dx \\ &= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4 \\ &= \frac{2}{3} \left[x^{\frac{3}{2}} \right]_0^4 = \frac{2}{3} \left(\left(4^{\frac{3}{2}} \right) - \left(0^{\frac{3}{2}} \right) \right) = \frac{16}{3} \end{aligned}$$

Find

$$\int_4^9 \frac{1}{\sqrt{x}} dx$$



$$\begin{aligned}\int_4^9 \frac{1}{\sqrt{x}} dx &= \int_4^9 x^{-\frac{1}{2}} dx \\&= \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right]_4^9 = \left[2x^{\frac{1}{2}} \right]_4^9 \\&= 2 \left[\sqrt{x} \right]_4^9 = 2 \left[(\sqrt{9}) - (\sqrt{4}) \right] \\&= 2[3 - 2] = 2\end{aligned}$$

Find

$$\int_{-2}^3 (x^6 + 5x - 4) dx$$



$$\begin{aligned} & \int_{-2}^3 (x^6 + 5x - 4) dx \\ &= \left[\frac{x^7}{7} + \frac{5x^2}{2} - 4x \right]_{-2}^3 \\ &= \left[\frac{(3)^7}{7} + \frac{5(3)^2}{2} - 4(3) \right] - \left[\frac{(-2)^7}{7} + \frac{5(-2)^2}{2} - 4(-2) \right] \\ &= \frac{4525}{14} \end{aligned}$$

$$\int_a^a f'(x) dx = [f(x)]_a^a$$

$$= f(a) - f(a) = 0$$

$$\int_a^a f'(x) dx = 0$$



Find

$$\int_3^3 (x^6 + 5x - 4) dx$$

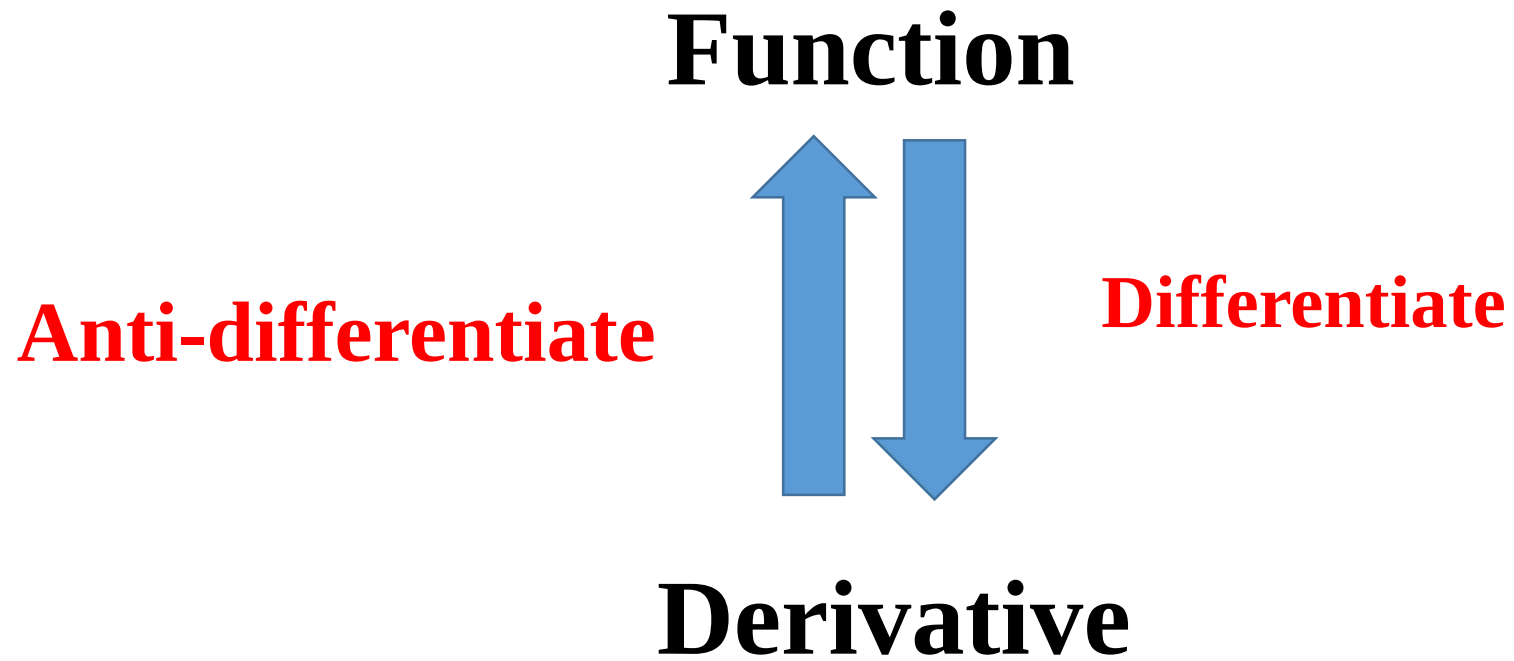


Without solution

$$\int_3^3 (x^6 + 5x - 4) dx = 0$$



- **Integration or Anti-derivative of the Function**



$$f(x) = x^3$$

$$f(x) = x^3 + 4$$

$$f(x) = x^3 - 7$$

Differentiate

$$f'(x) = 3x^2$$

Anti-differentiate ???

$$f'(x) = 3x^2$$

Anti-differentiate will be

$$f(x) = x^3 + C$$

C Is the constant of integration

We can write $\int (3x^2) dx = x^3 + C$

$\int$ Is the integration sign

dx Means that integration with respect to x

C Is the constant of integration

$(3x^2)$ Is the derivative of x^3

In general

$$\int f'(x) dx = f(x) + C$$

$\int$ Is the integration sign

dx Means that integration with respect to x

C Is the constant of integration

$f'(x)$ Is the derivative of $f(x)$

Find $\int (4x^3) dx$

$$\int (4x^3) dx = x^4 + C$$



In general

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$



Basic Integration Rules

Differentiation Formula

$$\frac{d}{dx}[C] = 0$$

$$\frac{d}{dx}[kx] = k$$

$$\frac{d}{dx}[kf(x)] = kf'(x)$$

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

Integration Formula

$$\int 0 \, dx = C$$

$$\int k \, dx = kx + C$$

$$\int kf(x) \, dx = k \int f(x) \, dx$$

$$\int [f(x) \pm g(x)] \, dx = \int f(x) \, dx \pm \int g(x) \, dx$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

Power Rule

$$\int kx^n dx = \frac{kx^{n+1}}{n+1} + C$$



Find

$$\int \sqrt{x^3} \, dx$$



Find

$$\int \frac{1}{\sqrt{x^5}} dx$$



Find

$$\int (x^9 + 5x^2 - 14) dx$$



Find

$$\int_1^3 \frac{1}{x^3} dx$$



Find

$$\int_1^2 (x^3 + 5x - 4) dx$$



Find

$$\int_1^1 (x^3 + 5x - 4) dx$$

