

Project Quality Management

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Project Quality Management

Lecture 5

PMG354

Statistical Quality Control – part 2

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Lecture

05

Objectives



At the end of this lecture, we expect to acquire a knowledge about fundamentals for statistical quality control to use in project stages and assure the conformity of outcomes



Lecture
05

Contents

- **What if the Process is Out of Control Basic Tools of Quality?**
- **Revised Control Limits for $\bar{X}$ and R**
- **Example**
- **Control Charts for variables ($\bar{X}$ mR)**
- **Attribute Control Charts**
- **Types of Attribute Control Chart**
- **Control Chart for Nonconformities (Defects) (c)**
- **Example**
- **Control Chart for Average Number of Nonconformities per Unit (U)**
- **Example**

What if the Process is Out of Control

- If points plot out of control, then the control limits must be **revised**.
- Before revising, identify **out of control points** and look for **assignable causes**.
 - If **assignable causes can be found**, then discard the point(s) and recalculate the control limits.
 - If **no assignable causes can be found** then retain the point(s) considering the trial control limits as appropriate for current control.

Revised Control Limits for X and R

- Revised center line for $\bar{x}$ and R control charts

$$\bar{\bar{x}}_{new} = \frac{\sum \bar{x} - \bar{x}_d}{m - m_d} \qquad \bar{R}_{new} = \frac{\sum R - R_d}{m - m_d}$$

where $\bar{x}_d$ = discarded subgroup means

m_d = number of discarded subgroups

R_d = discarded subgroup ranges

Revised Control Limits for X and R

$$\bar{x}_o = \bar{\bar{x}}_{new}$$

$$R_o = \bar{R}_{new}$$

$$\sigma_o = \frac{R_o}{d_2}$$

Where d_2 is a factor from the table.

Revised Control Limits for X and R

Revised $\bar{x}$ chart

$$UCL_{\bar{x}} = \bar{x}_o + A\sigma_o$$

$$LCL_{\bar{x}} = \bar{x}_o - A\sigma_o$$

Revised R Chart

$$UCL_R = D_2\sigma_o$$

$$LCL_R = D_1\sigma_o$$

Where A , D_1 and D_2 are factors from the table corresponding to subgroup size n .

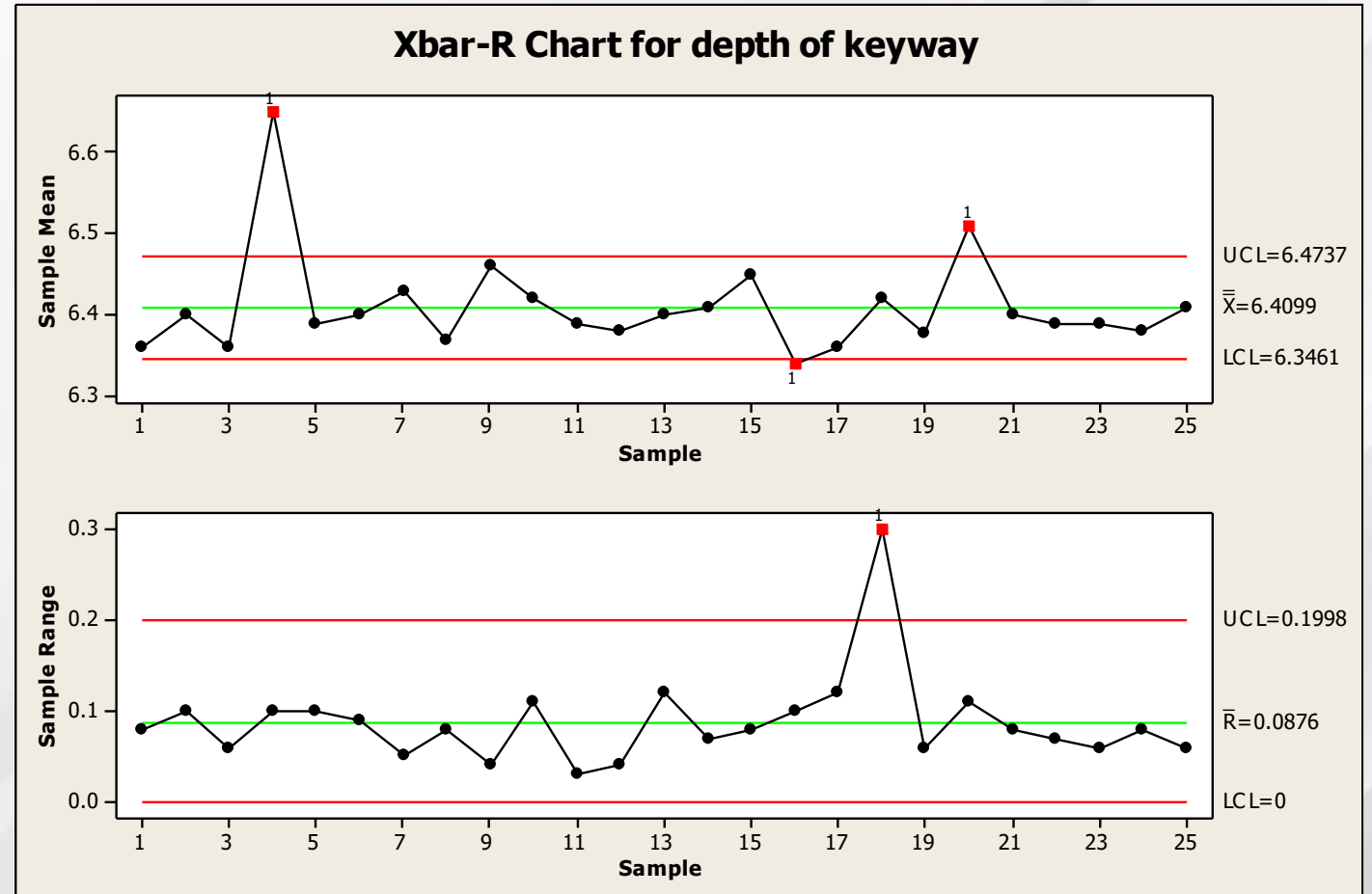
Example

- The following data represent the depth of keyway taken in millimeters. We wish to establish statistical control of the process using $\bar{x}$ and R charts. Twenty five samples, each of size 4, have been taken when we think the process is in control.
 - The depth of keyway measurements data from these samples are shown in the next table.

Sample Number	Date	Time	Measurements				Average	Range	Comment
			x ₁	x ₂	x ₃	x ₄		R _i	
1	23/12	8:50	6.35	6.40	6.32	6.37	6.36	0.08	New, temporary operator
2		11:30	6.46	6.37	6.36	6.41	6.40	0.01	
3		1:45	6.34	6.40	6.34	6.36	6.36	0.06	
4		3:45	6.69	6.64	6.68	6.59	6.65	0.10	
5		4:20	6.38	6.34	6.44	6.40	6.39	0.10	
6	27/12	8:35	6.42	6.41	6.43	6.34	6.40	0.09	
7		9:00	6.44	6.41	6.41	6.46	6.43	0.05	
8		9:40	6.33	6.41	6.38	6.36	6.37	0.08	
9		1:30	6.48	6.44	6.47	6.45	6.46	0.04	
⋮									
24		2:00	6.43	6.43	6.35	6.38	6.38	0.08	
25		4:25	6.39	6.39	6.43	6.44	6.41	0.06	
Sum							160.25	2.19	

Example

- Since the R chart indicates that the process variability is out of control, we may revise the chart.
- Also, the $\bar{X}$ chart is out of control, and need revision.
- R chart has to be revised first, and then chart



Example

- **Test Results for X-bar Chart of $X_1, \dots, X_4$**
 - TEST 1. One point more than 3.00 standard deviations from center line.
 - Test Failed at points: 4, 16, 20
 - ✖ **Sample 4 : Assignable cause; New temporary operator**
 - ✖ **Sample 16: Random Cause**
 - ✖ **Sample 20: Assignable cause; Bad material**
- **Test Results for R Chart of $X_1, \dots, X_4$**
 - TEST 1. One point more than 3.00 standard deviations from center line.
 - Test Failed at points: 18
 - ✖ **Sample 18: Assignable cause; Damage oil line**

Example

- The revised center line is

$$\bar{R}_{new} = \frac{\sum R - R_d}{m - m_d} = \frac{2.19 - 0.30}{25 - 1} = 0.0788$$

$$R_0 = \bar{R}_{new} = 0.0788$$

$$\sigma_0 = \frac{R_0}{d_2} = \frac{0.0788}{2.059} = 0.03827$$

- The value of d_2 , for sample of size $n = 4$ and using the table equal
 - $d_2 = 2.059$

Example

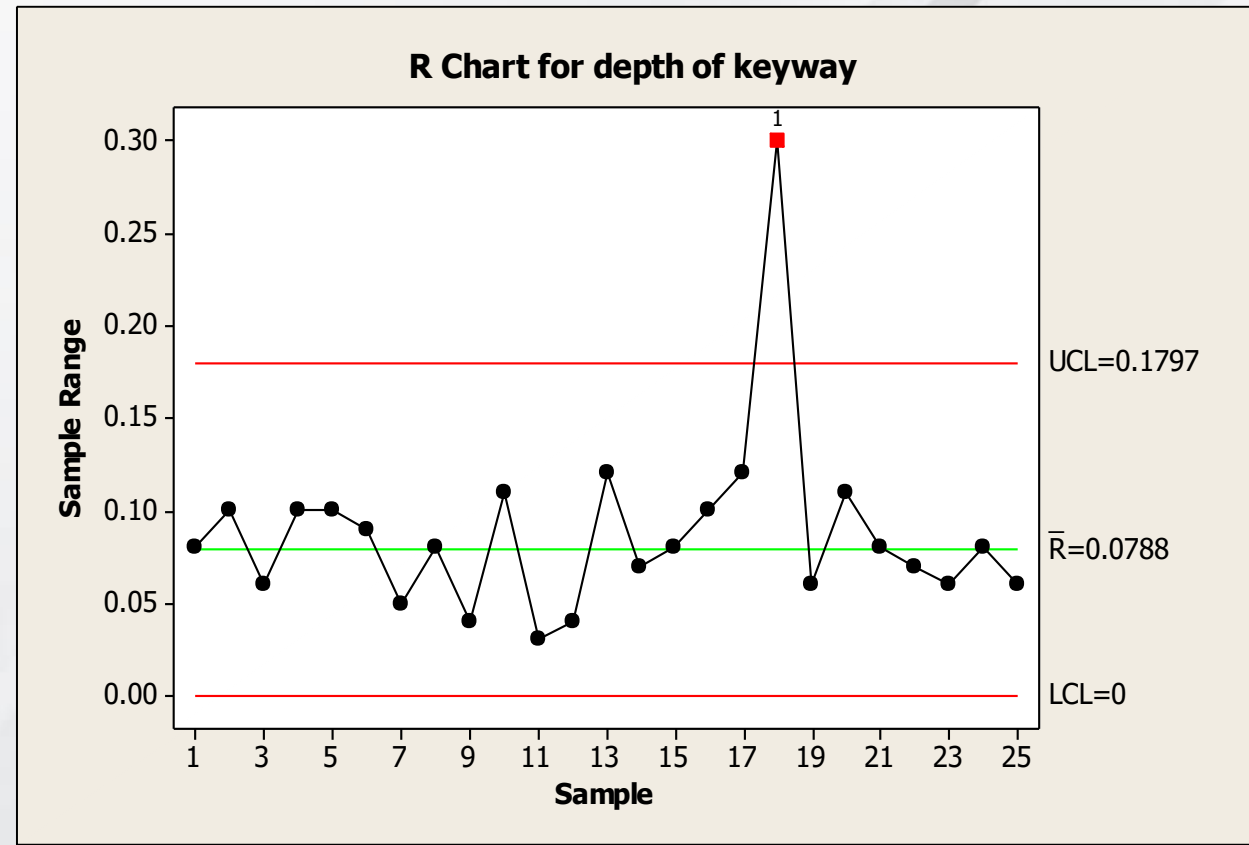
- Therefore, the Revised control limits for the R chart are

$$\begin{aligned} UCL_R &= D_2\sigma_0 \\ &= (4.698)(0.03827) = 0.1797 \end{aligned}$$

$$\begin{aligned} LCL_R &= D_1\sigma_0 \\ &= (0)(0.038) = 0 \end{aligned}$$

Example

- Now, the R chart is in control at the stated level. i.e the variability of the process is within control.
- We can revise the chart, to control the process center.



Example

- The revised center line is

$$\bar{\bar{x}}_{new} = \frac{\sum \bar{x} - \bar{x}_d}{m - m_d} = \frac{160.25 - (6.65 + 6.51)}{25 - 2} = 6.3951$$

$$\bar{x}_0 = \bar{\bar{x}}_{new} = 6.3951$$

- The value of d_2 , for sample of size $n = 4$ and using the table equal
 - $d_2 = 2.059$

Example

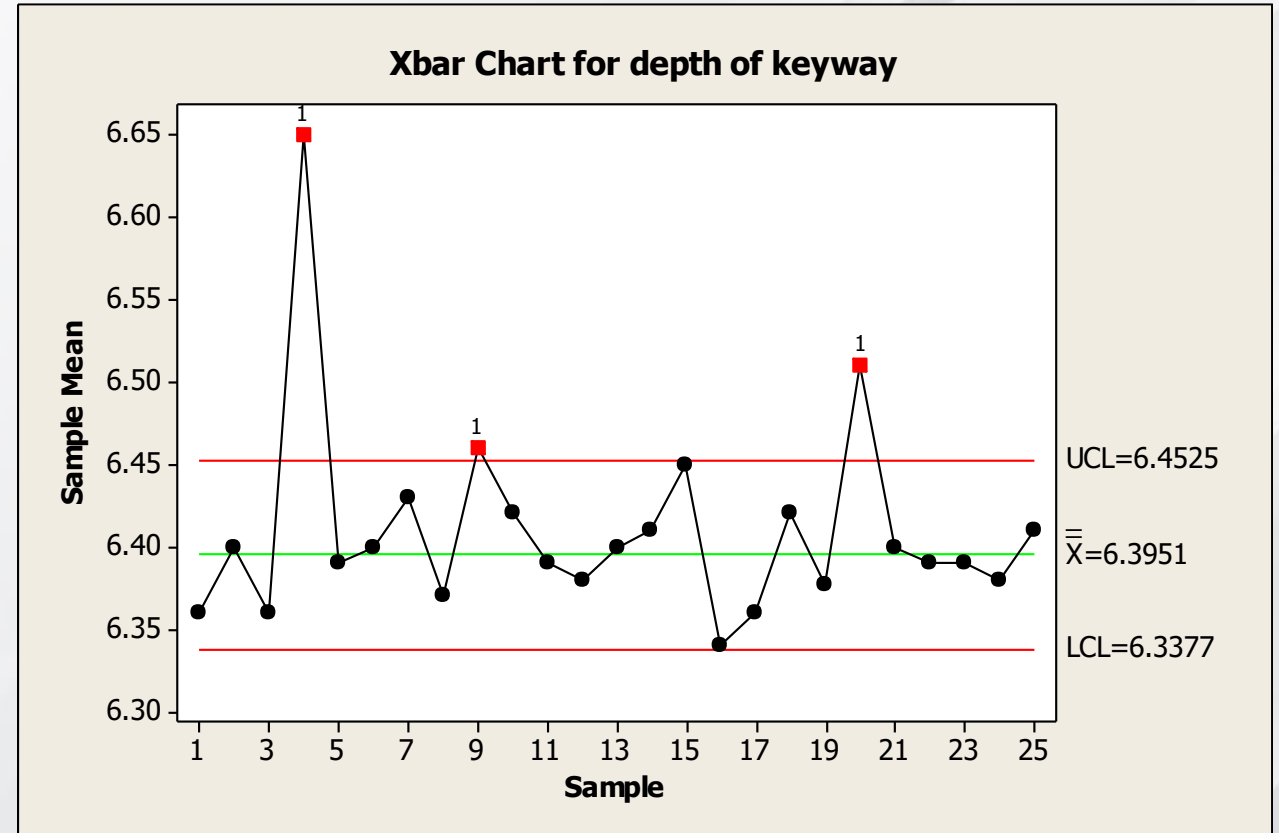
- Therefore, the Revised control limits for the $\bar{x}$ chart are

$$\begin{aligned}UCL_{\bar{x}} &= \bar{x}_0 + A\sigma_0 \\&= 6.395 + (1.5 \times 0.03827) \\&= 6.4525\end{aligned}$$

$$\begin{aligned}LCL_{\bar{x}} &= \bar{x}_0 - A\sigma_0 \\&= 6.395 - (1.5 \times 0.03827) \\&= 6.3377\end{aligned}$$

Example

- The chart is now in control at the stated level; center line and control limits.
- Note that subgroup 9 is out of control. The cause responsible for this signal should be treated as random cause.



Control Charts for variables (XmR)

- **The Shewhart Control Chart for Individual Measurements**
- What if you could not get a sample size greater than 1 ($n = 1$)? Examples include
 - Automated inspection and measurement technology is used, and every unit manufactured is analyzed.
 - The production rate is very slow, and it is inconvenient to allow samples sizes of $n > 1$ to accumulate before analysis
 - Repeated measurements on the process differ only because of laboratory or analysis error, as in many chemical processes.

Control Charts for variables (XmR)

- Multiple measurement are taken on the same unit of product, such as measuring oxide thickness at several locations on a wafer in semiconductor manufacturing.
- In process plants, such as papermaking, measurements on some parameter such as coating thickness *across* the roll will differ very little and produce a standard deviation that is much too small if the objective is to control coating thickness *along* the roll.
- Periodic (Weekly, Monthly) administrative data
- The X and MR charts are useful for samples of sizes $n = 1$.

Control Charts for variables (XmR)

- The moving range (MR) is defined as the absolute difference between two successive observations:

$$MR_i = |x_i - x_{i-1}|$$

which will indicate possible shifts or changes in the process from one observation to the next.

Control Charts for variables (XmR)

- The X chart is the plot of the individual observations. The control limits are

$$UCL = \bar{x} + 3 \frac{\overline{MR}}{d_2}$$

$$CL = \bar{x}$$

$$LCL = \bar{x} - 3 \frac{\overline{MR}}{d_2}$$

where

$$\overline{MR} = \frac{\sum_{i=1}^m MR_i}{m}$$

Control Charts for variables (XmR)

- The control limits on the moving range chart are:

$$UCL = D_4 \overline{MR}$$

$$CL = \overline{MR}$$

$$LCL = D_3 \overline{MR}$$

Example

- The viscosity of an aircraft primer paint is an important quality characteristic.
- The product is produced in batches, and because each batch takes several hours to produce, the production rate is too slow to allow samples sizes greater than 1.
- The viscosity of the previous 15 batches are shown in the table in the next slide.
- Set up a control chart for this situation.

Example

Batch number	Viscosity, x	Moving Range MR
1	33.75	
2	33.05	0.70
3	34.00	0.95
4	33.81	0.19
5	33.46	0.35
6	34.02	0.56
7	33.68	0.34
8	33.27	0.41
9	33.49	0.22
10	33.20	0.29
11	33.62	0.42
12	33.00	0.62
13	33.54	0.54
14	33.12	0.42
15	33.84	0.72

$$\bar{x} = 33.52$$

$$\overline{MR} = 0.48$$

Example

- Using the data from the previous table, we find that the center line for the MR chart is

$$\overline{MR} = \frac{\sum_{i=1}^{14} MR_i}{14} = 0.48$$

- For samples of $n = 1$, and using the table we have
 - $D_3 = 0$
 - $D_4 = 3.267$

Example

- The trial control limits for the MR chart are

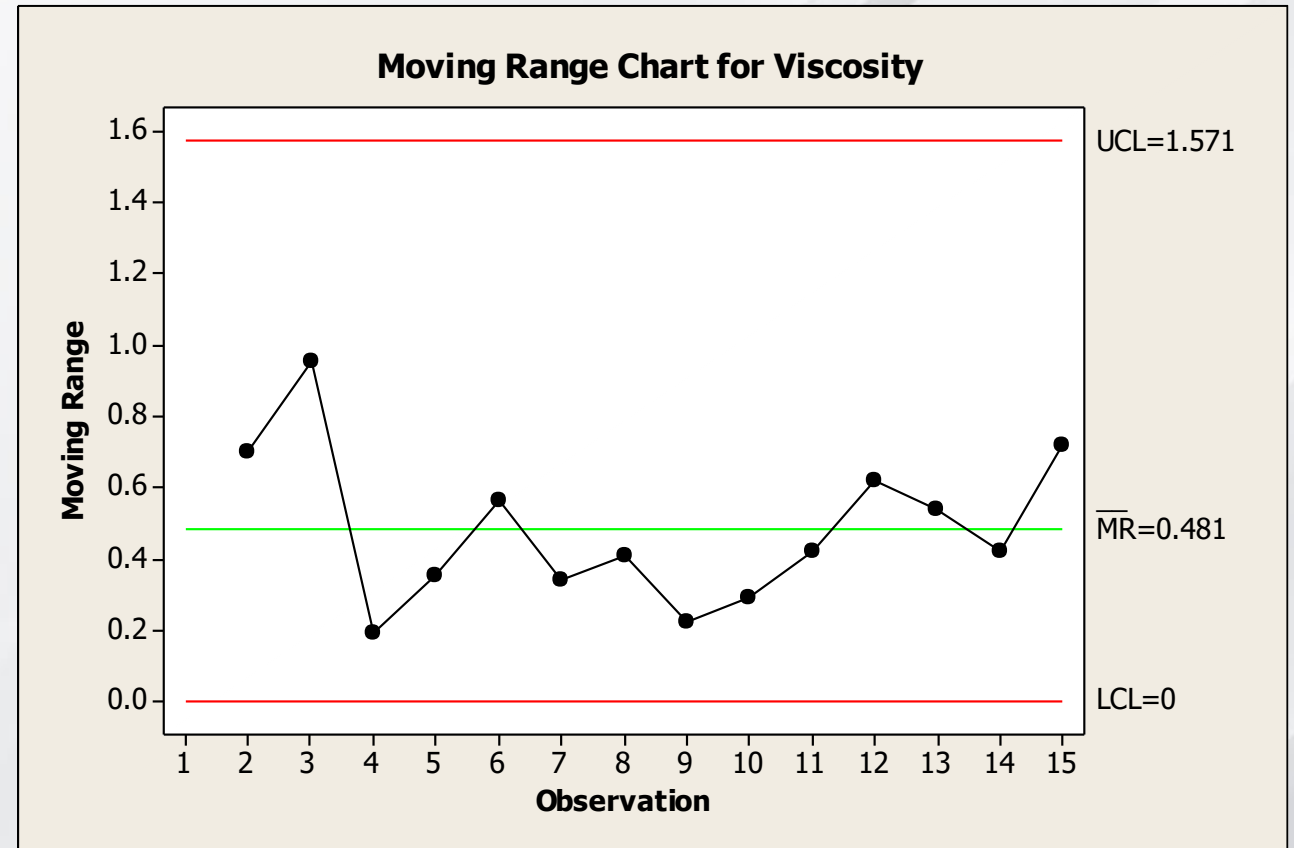
$$\begin{aligned}UCL_{MR} &= D_4 \overline{MR} \\&= (3.267)(0.48) \\&= 1.57\end{aligned}$$

$$\begin{aligned}LCL_{MR} &= D_3 \overline{MR} \\&= (0)(0.48) \\&= 0\end{aligned}$$



Example

- Here, no points exceed the upper control limits, we can consider that the process is in control



Example

- For the control chart of individual measurement, the parameters are

$$UCL = \bar{x} + 3 \frac{\overline{MR}}{d_2} = 33.52 + 3 \frac{0.48}{1.128} = 34.80$$

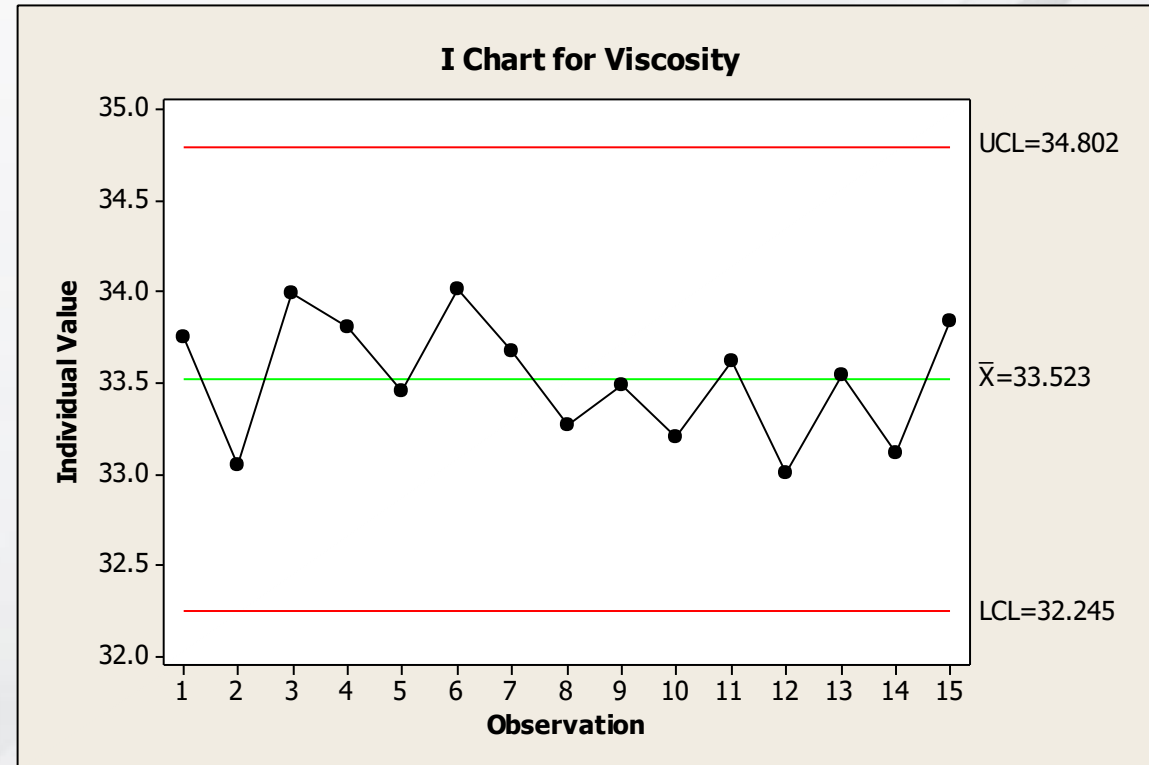
$$\bar{x} = 33.52$$

$$LCL = \bar{x} - 3 \frac{\overline{MR}}{d_2} = 33.52 - 3 \frac{0.48}{1.128} = 32.24$$

- If a moving range of $n=1$ observations is used, then
 - $d_2 = 1.128$

Example

- There is no indication of an out of control condition.



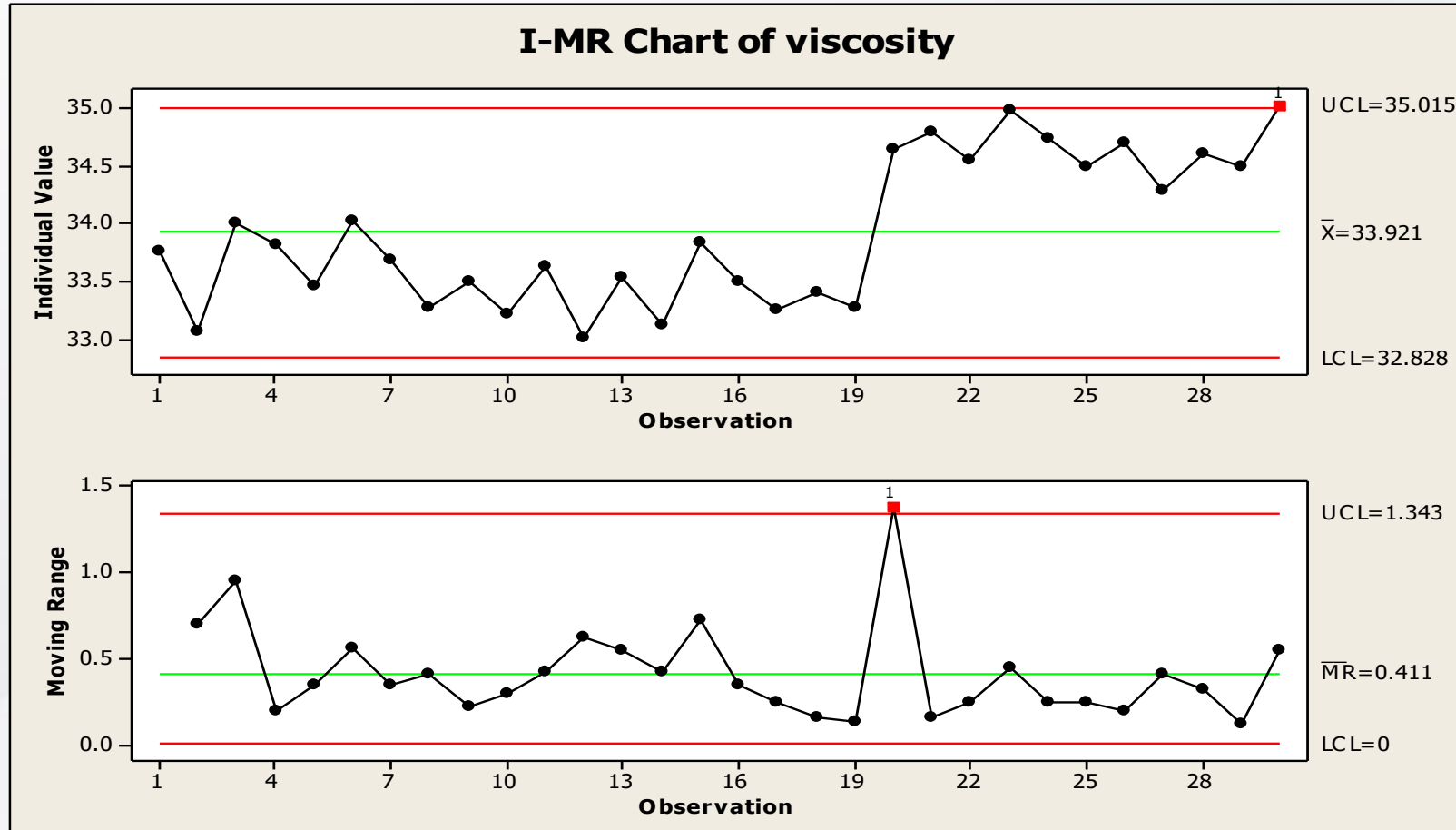
Example

- X Charts can be interpreted similar to MR charts. MR charts cannot be interpreted the same as X or R charts.
- Since the MR chart plots data that are “correlated” with one another, then looking for patterns on the chart does not make sense.
- More emphasis should be placed on interpretation of the X chart.

Example

Batch number	Viscosity, x	Moving Range MR
16	33.50	0.34
17	33.25	0.25
18	33.40	0.15
19	33.27	0.13
20	34.65	1.38
21	34.80	0.15
22	34.55	0.25
23	35.00	0.45
24	34.75	0.25
25	34.50	0.25
26	34.70	0.20
27	34.29	0.41
28	34.61	0.32
29	34.49	0.12
30	35.03	0.54

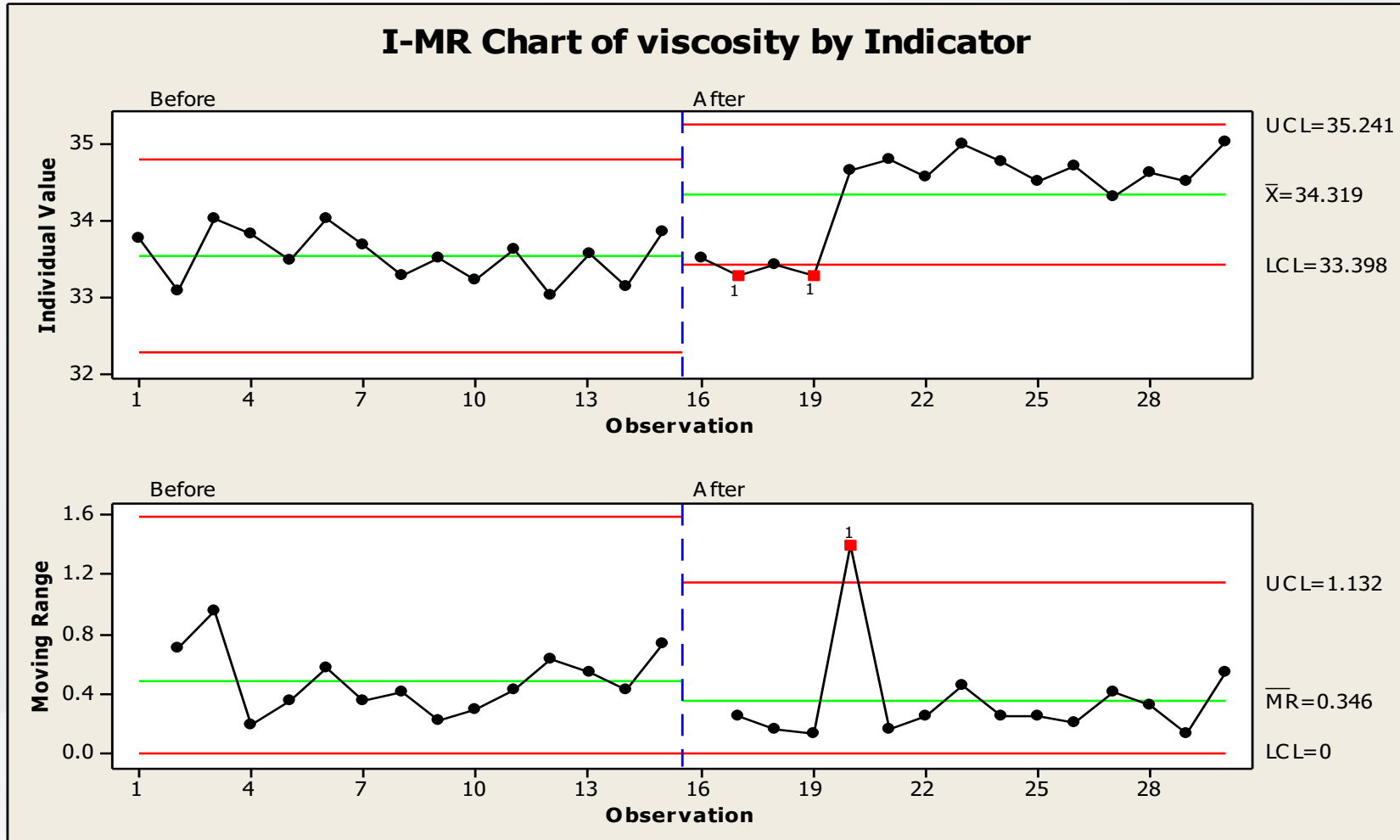
Example



Example

- Interpretation of the chart, an upward shift in mean viscosity has occurred around batch 20 or 21, Since there is an obvious “shift in process level” pattern on the chart for individual.
- Note that the moving range chart also reacts to this level shift with a single large spike at sample 20.
- This spike on the moving range chart sometimes helpful in identifying exactly where a process shift in the mean has occurred.

Example



Attribute Control Charts

- Data that can be classified into one of several categories or classifications is known as **attribute** data.
- Classifications such as conforming and nonconforming are commonly used in quality control.
- Another example of attributes data is the count of defects.

Types of Attribute Control Chart

- Charts for nonconformities (based on the Poisson distribution).
 - C chart (Number of defects or nonconformities) - in case of the sample contains one unit for inspection
 - U chart (Nonconformities per unit) – in case the sample contains more than one inspection unit.

Control Chart for Nonconformities (Defects) (c)

- **Standard Given:**
$$UCL = c + 3\sqrt{c}$$
$$CL = c$$
$$LCL = c - 3\sqrt{c}$$
- **No Standard Given:**
$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$
$$CL = \bar{c}$$
$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

Example

- The data in the following table presents the number of nonconformities observed in 26 successive samples of 100 printed circuit boards.
- For reason of convenience, each sample represent one inspection unit. The inspection unit is defined as 100 boards.
- Set up a c chart to test whether the process exhibit control.

Example

Sample Number	Number of Nonconformities	Sample Number	Number of Nonconformities	Sample Number	Number of Nonconformities
1	21	12	24	23	16
2	24	13	16	24	19
3	16	14	19	25	17
4	12	15	10	26	15
5	15	16	17	Sum	516
6	5	17	13		
7	28	18	22		
8	20	19	18		
9	31	20	39		
10	25	21	30		
11	20	22	24		

Example

- Since the 26 samples contain 516 nonconformities, we estimate $\bar{c}$ by:

$$\bar{c} = \frac{516}{26} = 19.85$$

- Therefore, the trial control limits are given by:

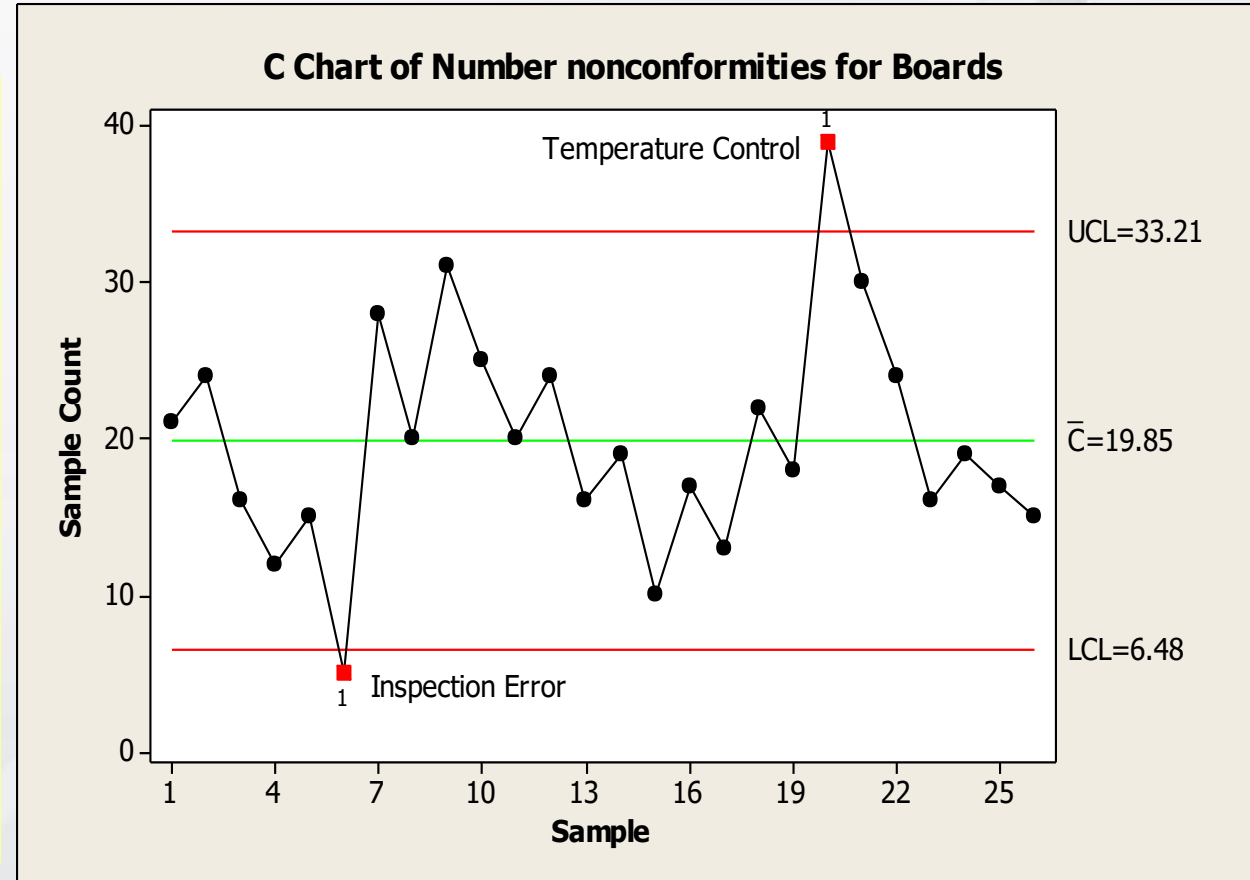
$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 19.85 + 3\sqrt{19.85} = 33.22$$

$$CL = \bar{c} = 19.85$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}} = 19.85 - 3\sqrt{19.85} = 6.48$$

Example

- Two points plot outside the control limits, samples 6 and 20.
- The process is out of control.
- The point must be investigated to see whether an assignable cause can be determined



Example

- After dropping samples 6 and 20, the estimate of c is now:

$$\bar{c} = \frac{472}{24} = 19.67$$

- The revised control limits are given by:

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 19.67 + 3\sqrt{19.67} = 32.97$$

$$CL = \bar{c} = 19.67$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}} = 19.67 - 3\sqrt{19.67} = 6.36$$

Control Chart for Average Number of Nonconformities per Unit (U)

- If we find x total nonconformities in a sample of n inspection units, then the **average number of nonconformities** per inspection unit is $\bar{u} = x/n$.
- The control limits for the average number of nonconformities is

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$$

$$CL = \bar{u}$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$$

$$\text{Where } \bar{u} = \frac{\sum_{i=1}^m u_i}{m}$$

Example

- A personal computer manufacture wishes to establish a control chart for nonconformities per unit on the final assembly line.
- The sample size is selected as five computers.
- Data on the number of nonconformities in 20 samples of 5 computers each are shown in the next table.

Example

Sample Number, i	Sample Size n	Total Number of Nonconformities, x_i	Average Number of Nonconformities per unit, $u_i = x_i/n$
1	5	10	2.0
2	5	12	2.4
3	5	8	1.6
4	5	14	2.8
5	5	10	2.0
6	5	16	3.2
7	5	11	2.2
8	5	7	1.4
9	5	10	2.0
10	5	15	3.0
11	5	9	1.8
⋮	⋮	⋮	⋮
19	5	7	1.4
20	5	5	1.0
Sum		193	38.6

Example

- From the data, we would estimate the average number of nonconformities per unit to be:

$$\bar{u} = \frac{\sum_{i=1}^{20} u_i}{20} = \frac{38.60}{20} = 1.93$$

- Therefore, the trial control limits are given by:

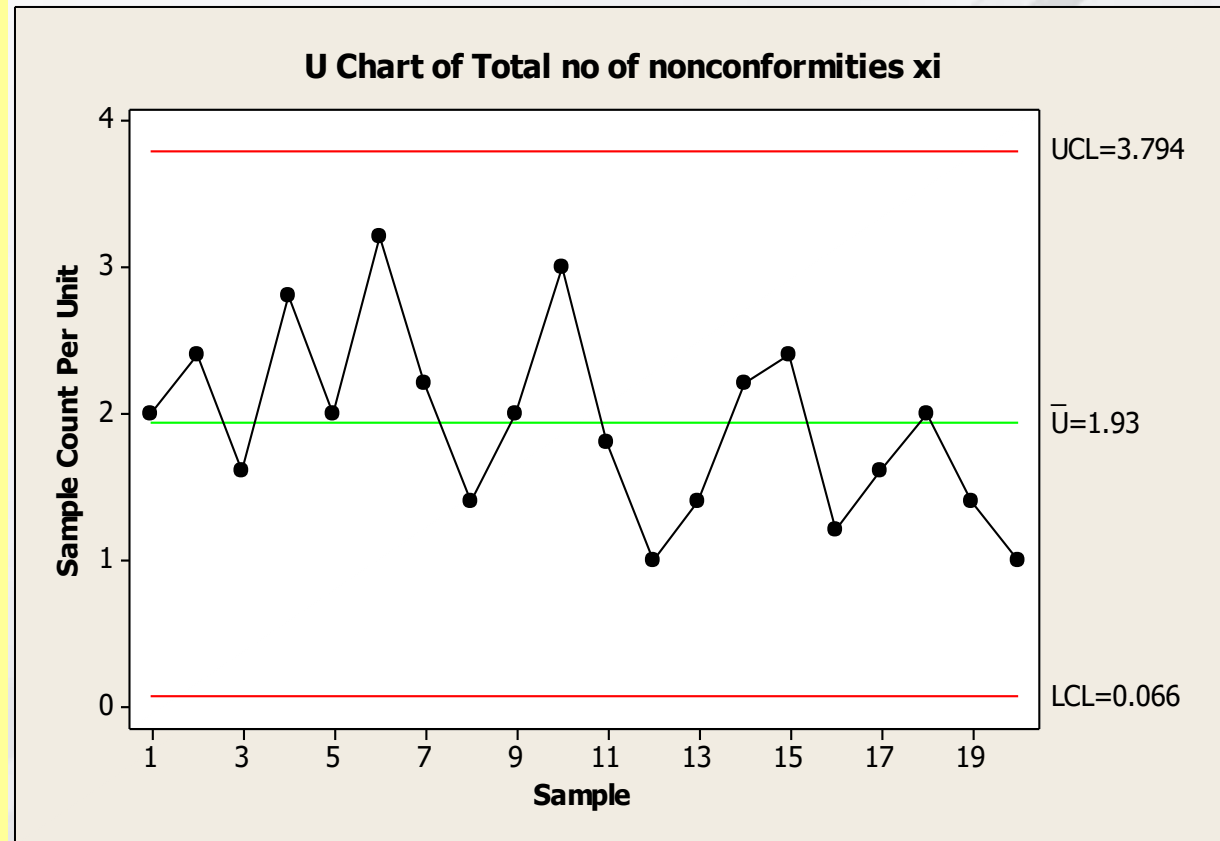
$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 1.93 + 3\sqrt{\frac{1.93}{5}} = 3.79$$

$$CL = \bar{u} = 1.93$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 1.93 - 3\sqrt{\frac{1.93}{5}} = 0.07$$

Example

- The preliminary data do not exhibit lack of statistical control.
- The trial control limits should be adopted for current control purposes.
- The average number of nonconformities is unacceptably high.
- Management intervention are required.





Thank You