

Applications: D, S, C, R
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Lecture 03



جامعة الجميع الذكية
EVERYONE'S SMART UNIVERSITY

MTH 112
Business Mathematics

- **At the end of this Lecture you should be familiar with:**
 - Calculate and plot the equation of a line given two points or slope and any point on the line
 - How to model (i) graphically and (ii) algebraically (writing down their equations) the following basic functions:
 - Demand and supply
 - Cost
 - Revenue

□ The equation of a line given slope and any point on the line

Since the slope of a line is the same when measured between any two points, such as the points (x_1, y_1) and (x, y) , we may write

$$\frac{\Delta y}{\Delta x} = m = \frac{y - y_1}{x - x_1}$$

The equation of a straight line with slope, m , which passes through the specific point (x_1, y_1) is

$$y - y_1 = m(x - x_1).$$

Chapter 02

The equation of a line given slope and any point on the line

□ The equation of a line given slope and any point on the line

Example:

- (a) Deduce the equation of a line having a slope of 1.7 and which passes through the point (2,5).
- (b) Plot the graph of the line.

□ The equation of a line given slope and any point on the line

Solution:

(a) The equation of the line is deduced as

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 1.7(x - 2)$$

$$y = 1.6 + 1.7x$$

This line has a slope of 1.7 and it intersects the y -axis at $y = 1.6$.

(b) H.W. at home

□ The equation of a line given two points

There are two steps involved when deducing the equation of a line when given two points, say points (x_1, y_1) and (x_2, y_2) .

Step 1: Calculate the slope of the line using equation

$$\text{Slope} = \frac{\text{Change in y-Coordinate}}{\text{Change in x-Coordinate}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Step 2: Determine the equation of the line by substituting the slope (as calculated in step 1), and either point into equation $y - y_1 = m(x - x_1)$.

□ The equation of a line given two points

Example:

- (a) A line passes through the points $(2,4)$ and $(6,1)$. Deduce the equation of the line.
- (b) Plot the graph of the line. (H.W.)

□ The equation of a line given two points

Solution: (a)

Step 1: The slope of the line is calculated by substituting the points into equation:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{6 - 2} = \frac{-3}{4} = -0.75$$

Step 2: The equation of the line is deduced by substituting this slope, and either point into equation: $y - y_1 = m(x - x_1) \rightarrow y - 4 = -0.75(x - 2) \rightarrow y = 5.5 - 0.75x$

The equation of the line has a negative slope of -0.75 and y-intercept of 5.5 .

□ Demand and supply

Demand and supply decisions by consumers, firms and the government determine the level of economic activity within an economy. As these decisions play a vital role in business and consumer activity, it is important to possess mathematical tools that enable us to analyse them.

□ The demand function

There are several variables that influence the demand for a good X . These may be expressed by the general demand function

$$Q = f(P, Y, P_s, P_c, T_a, A, \dots)$$

where

Q is the quantity demanded of good X

P is the price of good X

Y is the income of the consumer

P_s is the price of substitute goods

P_c is the price of complementary goods

T_a is the taste or fashion of the consumer

A is the level of advertising



□ The demand function

Definitions

A **substitute good** is one that can be used instead of another good, e.g., trains and buses.

A **complementary good** is one that is consumed in conjunction with another, e.g. petrol/cars.

□ The demand function

The simplest model for the demand function is written as

$$Q = f(P)$$

Therefore, quantity demanded depends on price only, so long as the other variables upon which demand depends remain constant; that is, $Y, P_s, P_c, T_a, A, \dots$ are constant.

□ The demand function

For example, the demand for good X may be given by the linear equation, $Q = 200 - 2P$. This equation describes the **law of demand**, a basic economic hypothesis which states that there is a negative relationship between quantity demanded and price, that is, when the price of a good increases, the quantity demanded will decrease, all other variables remaining constant.

□ The equation of demand function

The demand function, $P = g(Q)$, can be modelled by the simple linear equation

$$P = a - bQ$$

where a and b are constants. This is the equation of a straight line.

The general format for the equation of a straight line is $y = mx + c$ (or $y = c + mx$);

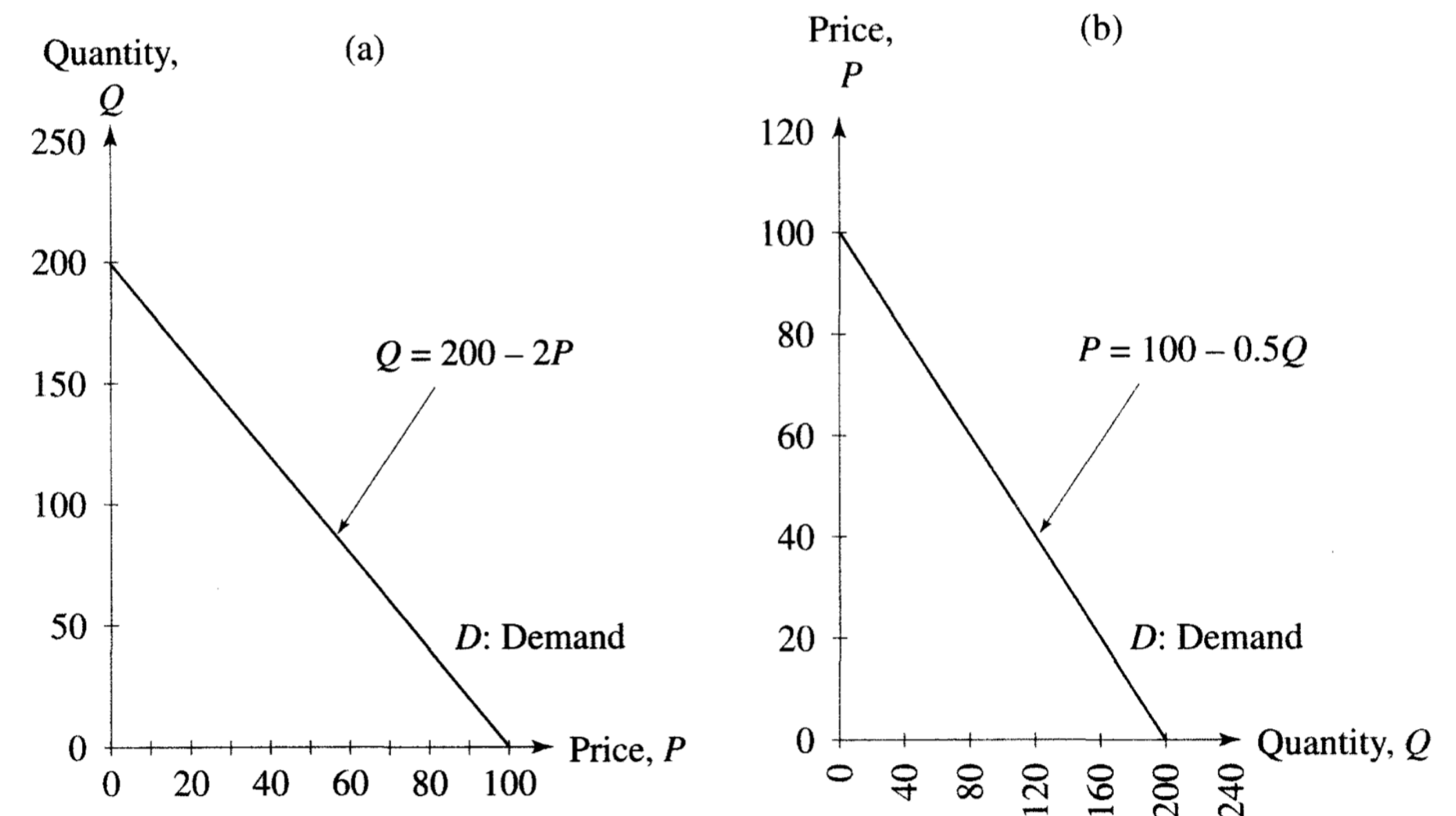


Figure 2.17 Demand functions (a) $Q = f(P)$ and (b) $P = g(Q)$

□ The demand function

Therefore the following can be deduced:

$$P = a + (-b)Q$$

.....

$$y = c + (m)x$$

$a > 0$: the **vertical intercept** of the demand function is positive ($a > 0$ denotes 'a greater than zero'). The value of a depends on the size of the terms on the RHS of the general demand function, except the price of the good itself.

$(-b) < 0$: the **slope** of the demand function is negative ($(-b) < 0$ denotes ' $-b$ less than zero').

□ The demand function

The negative slope indicates that price drops by b units for each unit increase in quantity.

Note: $P = a - bQ$ intersects the horizontal axis at the point where $P = 0$. Substituting $P = 0$ into the demand function gives

$$0 = a - bQ \rightarrow bQ = a \rightarrow Q = \frac{a}{b}$$

Therefore, the horizontal intercept occurs at $(Q = a/b, P = 0)$.

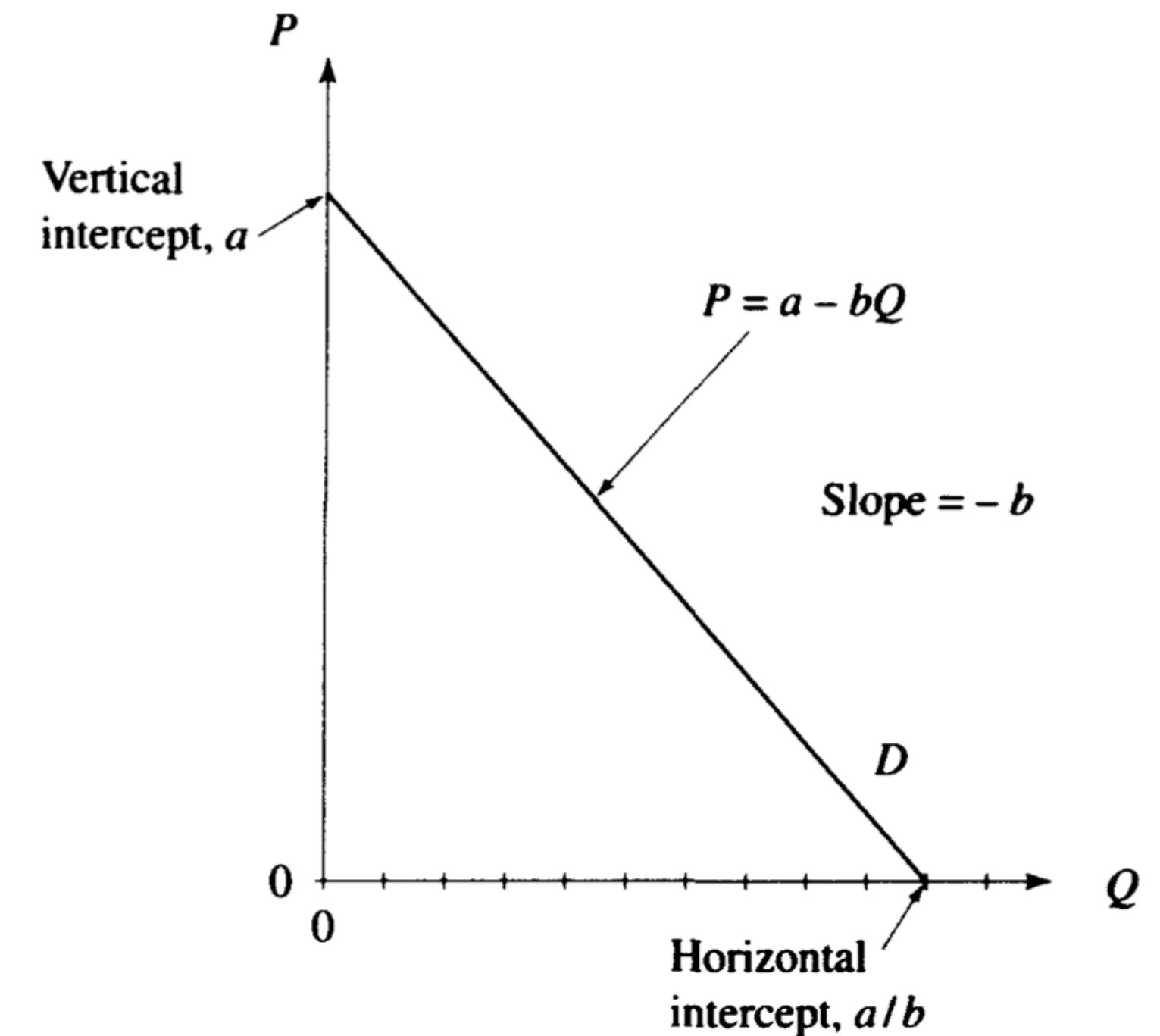


Figure 2.18 Demand function $P = a - bQ$

□ The demand function

Example:

The demand function is given by the equation $P = 100 - 0.5Q$

- (a) State and give a verbal description of the slope and intercepts.
- (b) What is the quantity demanded when $P = 5$?
- (c) Plot the demand function $P = 100 - 0.5Q$ for $0 \leq Q \leq 200$.
- (d) Find an expression for the demand function in the form $Q = f(P)$ and graph it.

□ The demand function

Solution: (a)

The slope and vertical intercept may be deduced as follows,

$$P = 100 + (-0.5)Q$$

$$P = a + (-b)Q$$

$$y = c + mx$$

The vertical intercept is 100. This means that $P = 100$ when $Q = 0$.

The slope $\Delta P / \Delta Q = -0.5$. This indicates that the price drops by 0.5 units for each successive unit increase in quantity demanded.

□ The demand function

Solution: (a)

The slope and vertical intercept may be deduced as follows,

$$P = 100 + (-0.5)Q$$

The horizontal intercept is calculated by substituting $P = 0$ into the equation of the demand function, that is,

$$P = 100 - 0.5Q \rightarrow 0 = 100 - 0.5Q \rightarrow -0.5Q = -100 \rightarrow Q = \frac{100}{0.5} = 200$$

Therefore, the horizontal intercept is at $(Q = 200, P = 0)$.

□ The demand function

Solution: (c)

The quantity demanded when $P = 5$ is calculated by substituting $P = 5$ into the demand function

$$P = 100 - 0.5Q$$

$$5 = 100 - 0.5Q$$

$$0.5Q = 100 - 5$$

$$0.5Q = 95$$

$$Q = \frac{95}{0.5} = 190$$

□ The demand function

Solution: (c) Method A

To plot the demand function over the range, $0 \leq Q \leq 200$, choose various quantity values within this range. In **Table 2.3** equal intervals of 40 units of quantity are used. Substitute these quantity values into the equation of the demand function to derive corresponding values for P . Plot these points and graph the demand function as in Figure 2.19.

Table 2.3 Demand schedule

Quantity	Price
0	100
40	80
80	60
120	40
160	20
200	0

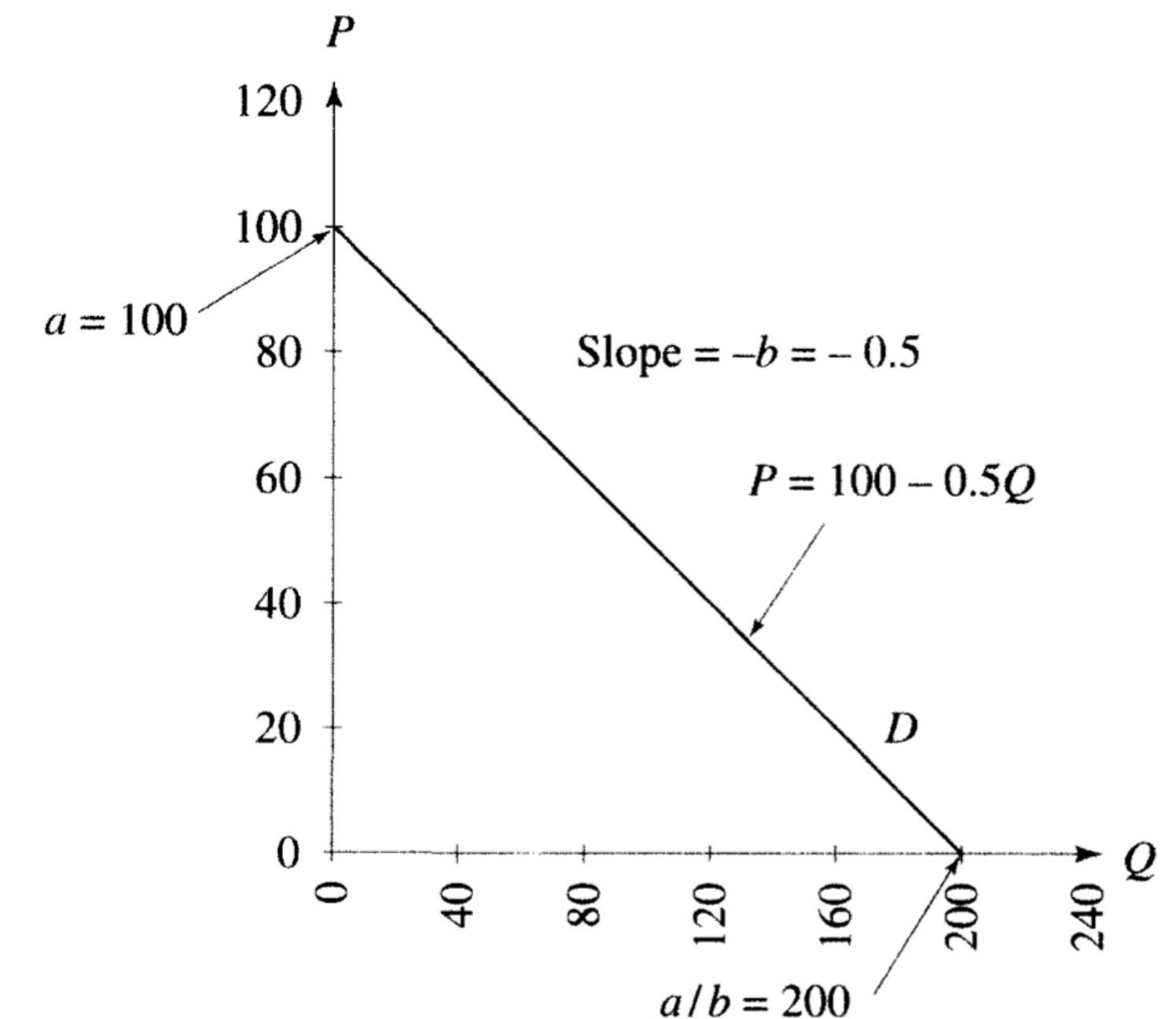


Figure 2.19 Demand function $P = 100 - 0.5Q$

□ The demand function

Solution: (c) Method B

Plot the horizontal and vertical intercepts calculated in part (a).

Draw a line through these points.

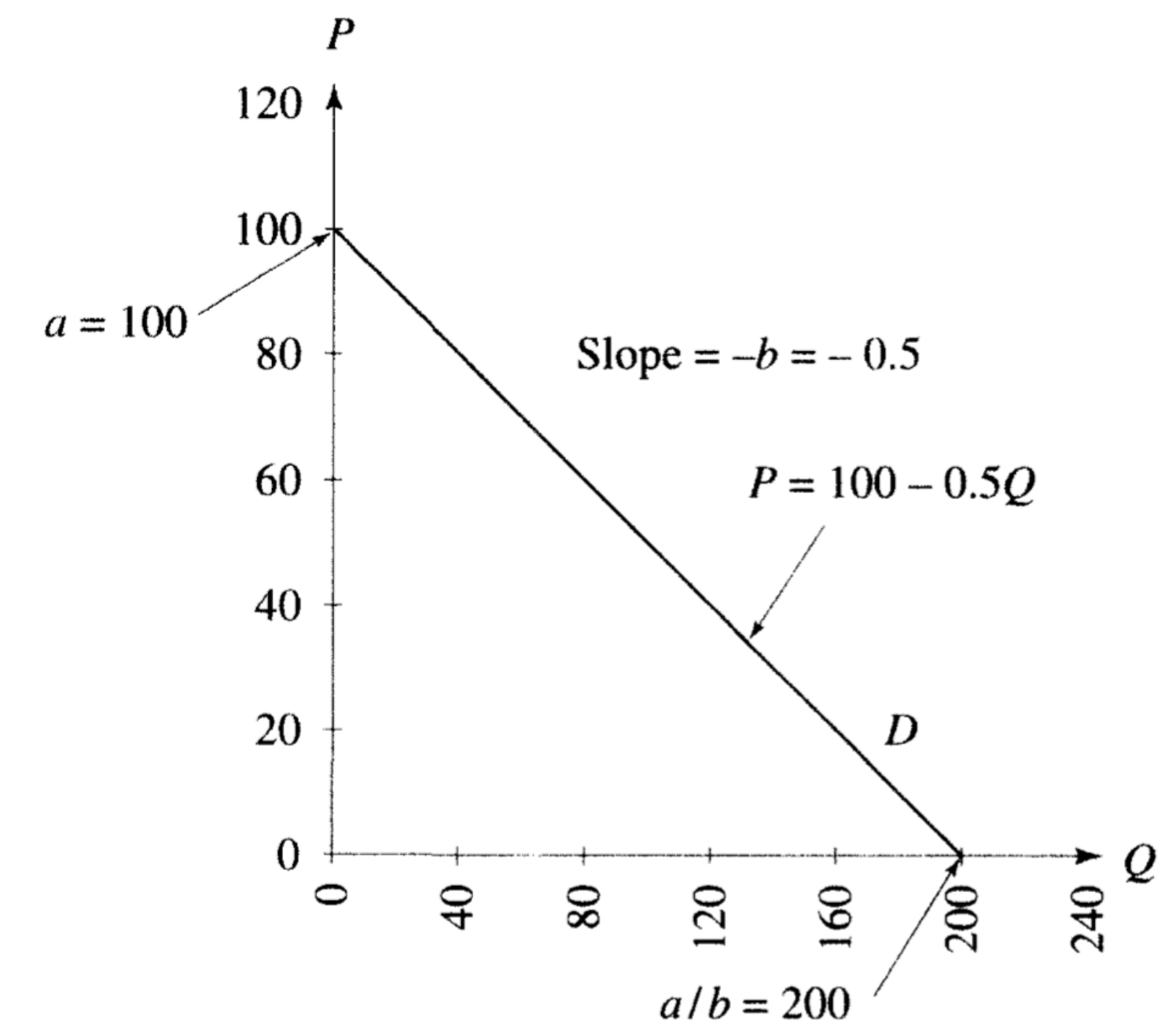


Figure 2.19 Demand function $P = 100 - 0.5Q$

□ The demand function

Solution: (d)

The demand function, $Q = f(P)$ is graphed in **Figure 2.20**. Its equation is derived as

$$P = 100 - 0.5Q \text{ or in general form as } P = a - bQ$$

$$0.5Q = 100 - P$$

$$Q = 200 - 2P$$

$$bQ = a - P$$

$$Q = \frac{a}{b} - \frac{1}{b}P$$

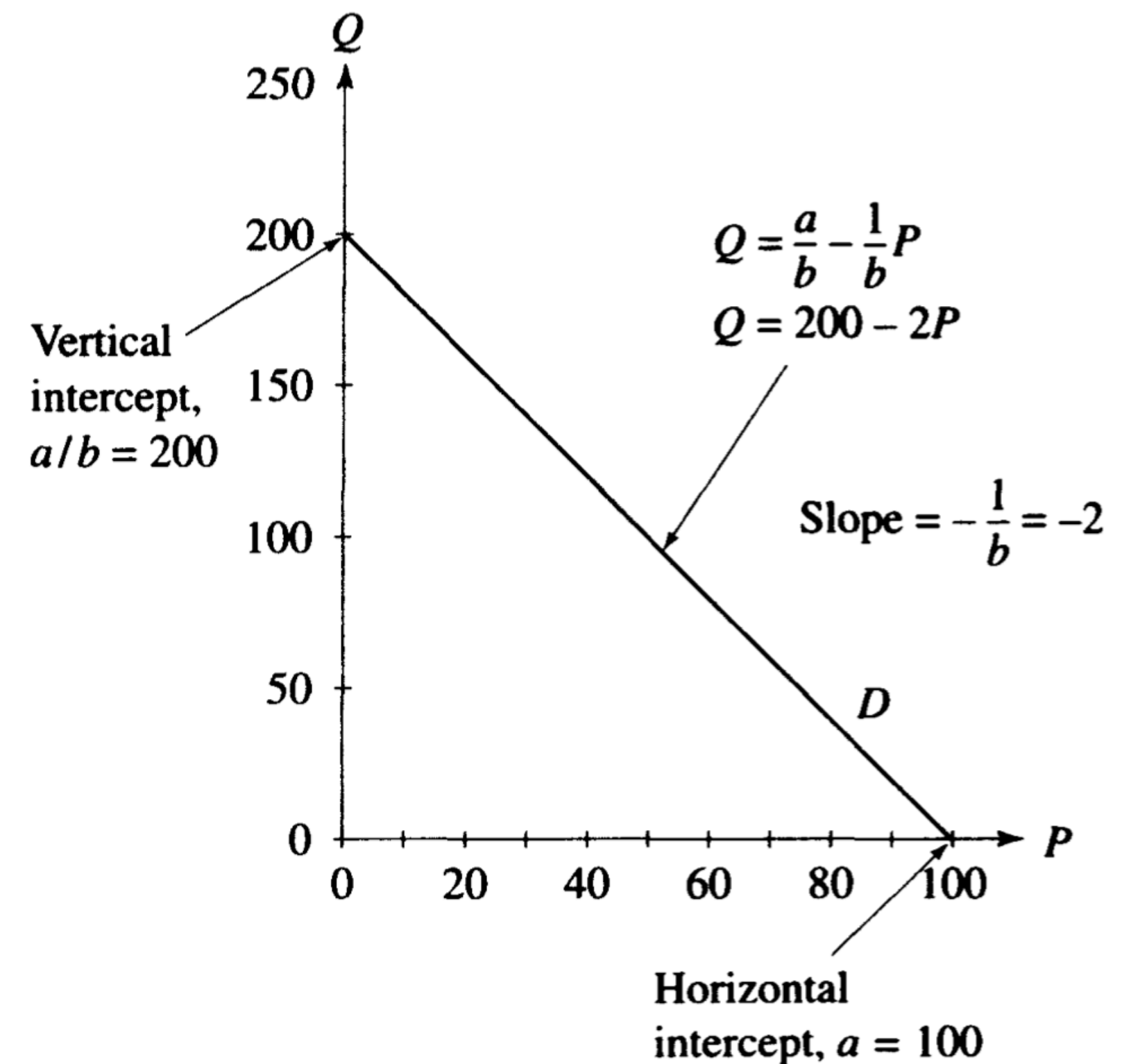


Figure 2.20 Demand function $Q = 200 - 2P$

□ The supply function

There are several variables that influence the supply of good X. These may be expressed by the general supply function

$$Q = f(P, C, P_0, T_e, N, O, \dots)$$

where

Q is the quantity supplied of good X

P is the price of the good itself

C is the cost of production

P_0 is the price of other goods

T_e is the available technology

N is the number of producers in the market

O is other factors, e.g. tax/subsidies



□ The supply function

The equation of the supply function

The supply function $P = h(Q)$ can be modelled by the simple linear equation

$$P = c + dQ$$

where c and d are constants.

$c > 0$: the vertical intercept is positive. Its value depends on the size of the terms on the RHS of the general supply function except the price of the good itself.

$d > 0$: the slope of the supply function is positive. Price increases by d units for every unit increase in quantity supplied.

□ The supply function

The equation of the supply function

The supply function is graphically represented in Figure 2.21.

Note: $P = c + dQ$ intersects the horizontal axis at the point where $P = 0$.

Substituting $P = 0$ into the supply function gives

$$P = c + dQ$$

$$0 = c + dQ$$

$$-c = dQ$$

$$-\frac{c}{d} = Q$$

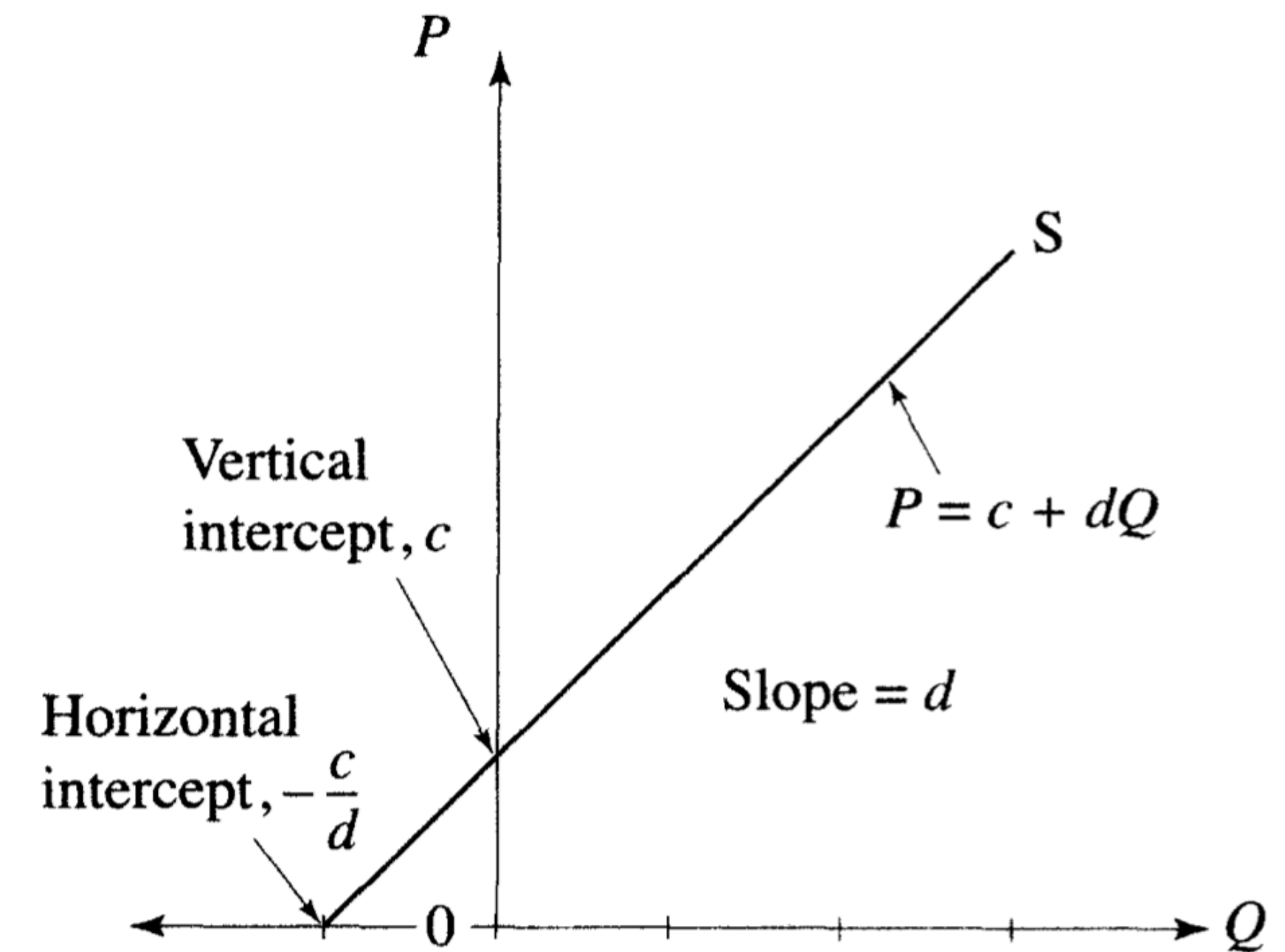


Figure 2.21 Supply function $P = c + dQ$

□ The supply function

Example:

The supply function is given by $P = 10 + 0.5Q$.

- (a) State and verbally describe the slope and intercepts.
- (b) Plot the supply function, $P = 10 + 0.5Q$ for $0 \leq Q \leq 100$.

□ The supply function

Solution: (a)

The vertical intercept is $c = 10$. This means that the firm will supply no units at $P \leq 10$. The slope $\Delta P / \Delta Q = 0.5$. This means that price increases by 0.5 units for every successive unit increase in quantity supplied.

The horizontal intercept is calculated by substituting $P = 0$ into the equation of the supply function, that is,

$$\begin{aligned} P &= 10 + 0.5Q \\ -10 &= 0.5Q \\ -\frac{10}{0.5} &= Q \rightarrow Q = -20 \end{aligned}$$

□ The supply function

Solution: (a)

Therefore, the horizontal intercept occurs at $(Q = -c/d, P = 0)$. A minus quantity is not economically meaningful; therefore, many economic textbooks only illustrate the supply function in the positive quadrant only.

□ The supply function

Solution: (b) Method A

To plot the supply function over the range, $0 \leq Q \leq 100$, choose various quantity values within this range. In **Table 2.4** equal intervals of 20 units of quantity are used. Substitute these quantity values into the equation of the supply function to derive corresponding values for P . Plot these points and graph the supply function as in **Figure 2.22**.

Table 2.4 Supply schedule

Quantity	Price
0	10
20	20
40	30
60	40
80	50
100	60

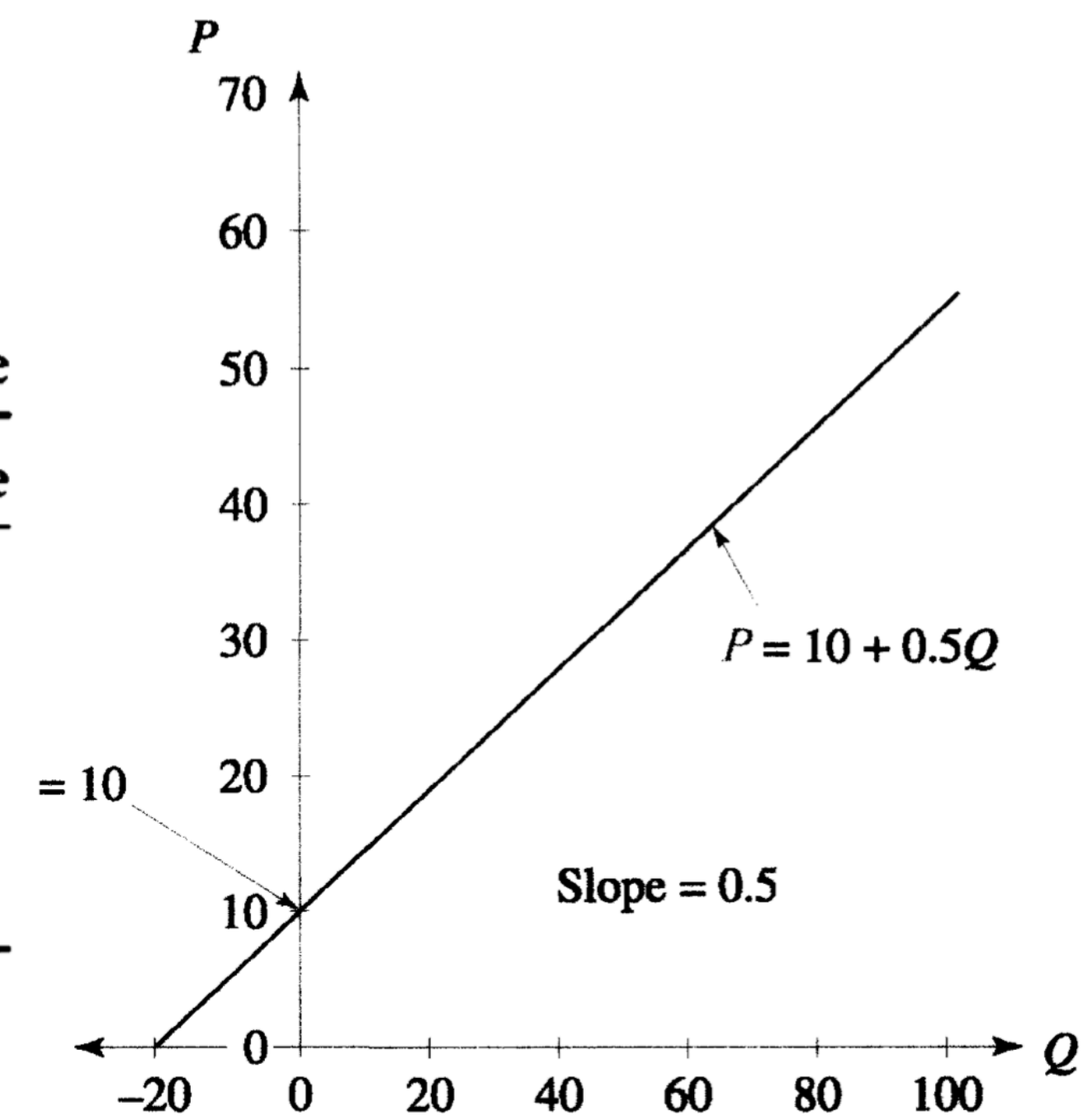


Figure 2.22 Supply function $P = 10 + 0.5Q$

□ The supply function

Solution: (b) Method B

Plot the horizontal and vertical intercepts calculated in part (a). Draw the supply function by joining these points.

Table 2.4 Supply schedule

Quantity	Price
0	10
20	20
40	30
60	40
80	50
100	60

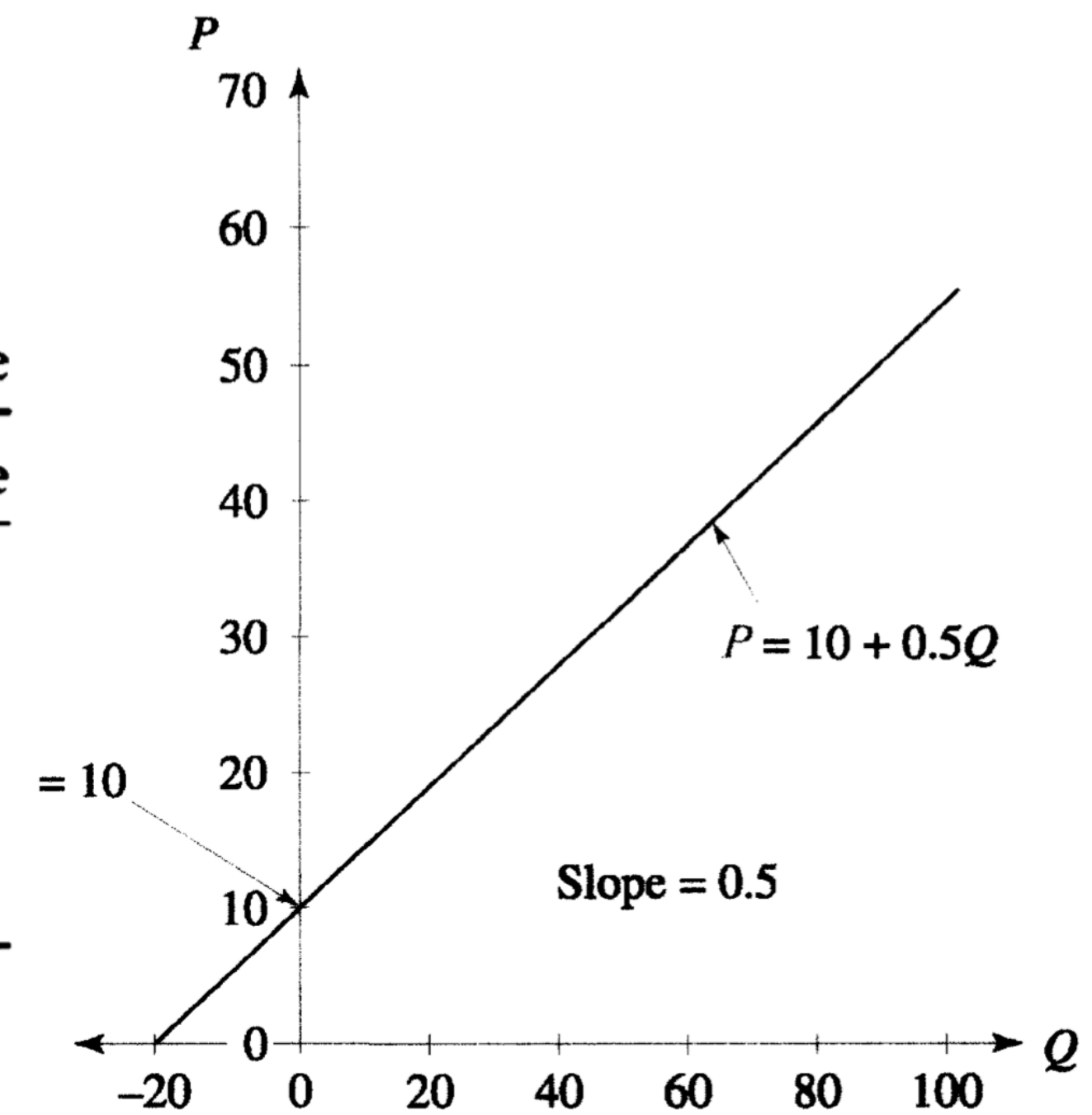


Figure 2.22 Supply function $P = 10 + 0.5Q$

□ Cost

Firms incur costs when they employ inputs, such as capital and labour, in order to produce goods for sale in the market. The total cost of producing a good will normally consist of:

- (i) Fixed costs, FC : costs that are fixed irrespective of the level of output, e.g. rent on premises.
- (ii) Variable costs, VC : costs which vary with the level of output, e.g. each extra unit of a good produced will require additional units of raw materials, labour, etc.

□ Cost

Total cost, , is therefore the sum of fixed costs and variable costs:

$$TC = FC + VC$$

These costs can be modelled by a linear cost function, such as

$$TC = 20 + 4Q$$

where $FC = 20$ (the vertical intercept) and $VC = 4Q$, where 4 = slope of the line.

□ Cost

Example:

A firm has fixed production costs of £10 and variable production costs of £2 per unit produced.

- (a) Write down the equation of the total cost function.
- (b) Graph the total cost function.

□ Cost

Solution: (a)

$FC = £10$; while $VC = £2Q$, since to produce

$$1 \text{ unit} \quad VC = 2(1)$$

$$2 \text{ unit} \quad VC = 2(2)$$

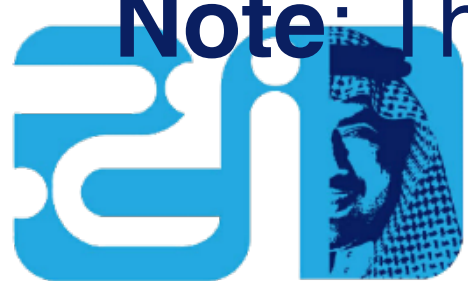
$$Q \text{ unit} \quad VC = 2(Q)$$

that is, the total variable costs incurred in producing Q units of this good is $£2Q$. Since

$TC = FC + VC$, the total cost incurred in producing Q units is

$$TC = 10 + 2Q$$

Note: This is the same as the line $y = 10 + 2x$, where $y \equiv TC$ and $x \equiv Q$.



□ Cost

Solution: (b)

Graphically, the TC function is a straight line, with costs measured on the vertical axis and units of the good produced on the horizontal axis. Since the vertical intercept = 10 (level of FC) and the slope = 2 are known, the total cost function may be plotted. Alternatively, total costs may be plotted by calculating at least two points on the line and joining these points. A number of points are outlined in **Table 2.5** and then graphed in **Figure 2.25**.

□ Cost

Solution: (b)

Table 2.5 Fixed, variable and total costs

Q	Variable costs (at £2 per unit)	$TC = FC + VC$ ($y = 10 + 2Q$)	Point (Q, TC) (x, y)
0	0	$TC = 10 + 0 = 10$	(0, 10)
1	$1(2) = 2$	$TC = 10 + 2 = 12$	(1, 12)
2	$2(2) = 4$	$TC = 10 + 4 = 14$	(2, 14)
3	$3(2) = 6$	$TC = 10 + 6 = 16$	(3, 16)
4	$4(2) = 8$	$TC = 10 + 8 = 18$	(4, 18)
5	$5(2) = 10$	$TC = 10 + 10 = 20$	(5, 20)
6	$6(2) = 12$	$TC = 10 + 12 = 22$	(6, 22)

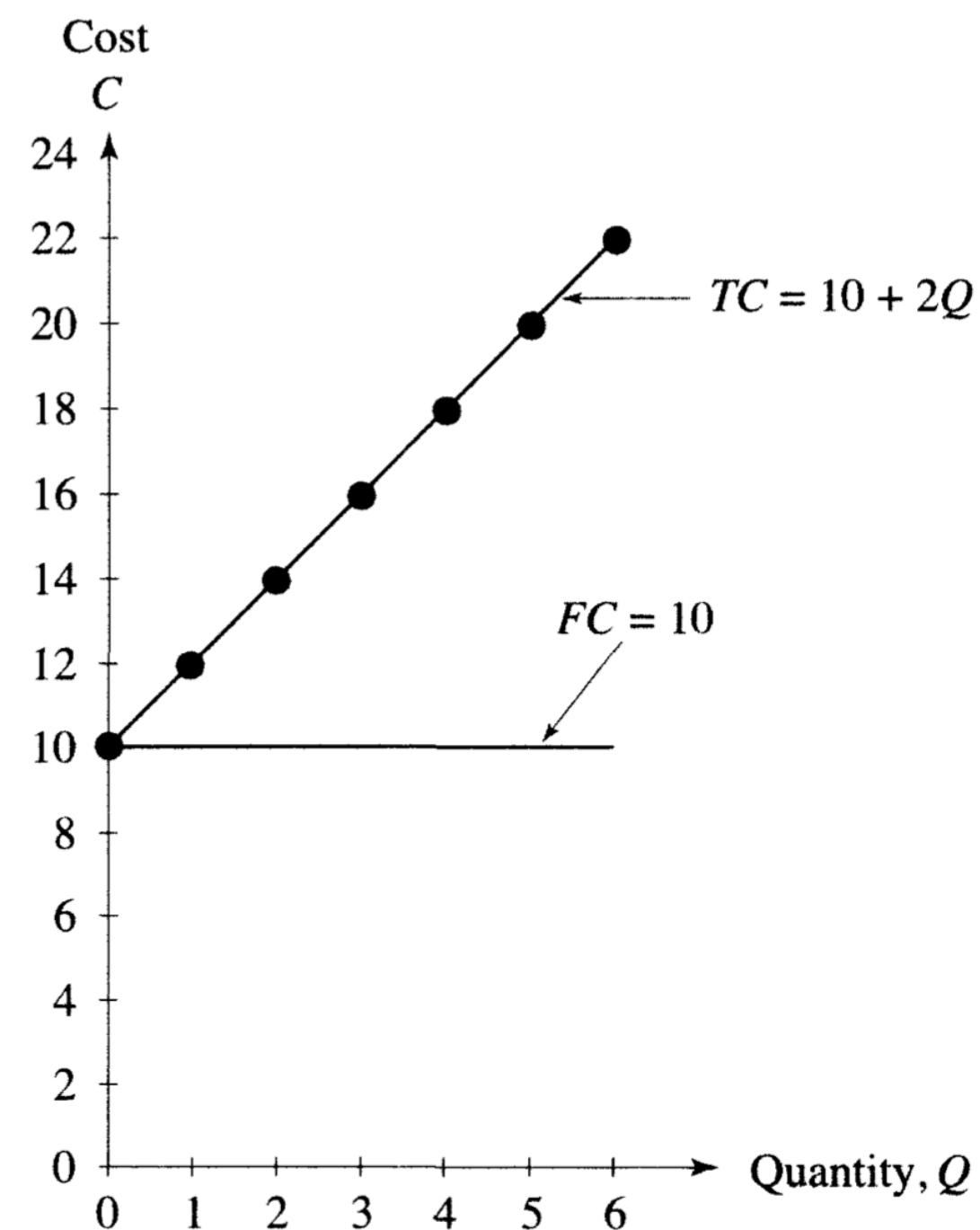


Figure 2.25 Linear total cost function

□ Revenue

A firm receives revenue when it sells output. The total revenue, TR , received is the price of the good, P , multiplied by the number of units sold, Q , that is,

$$TR = PQ$$

Total revenue may be modelled by a straight line if the price of each unit sold is the same, that is, each firm is a price-taker, denoted by a horizontal demand function.

□ Revenue

A price-taking firm is known as a perfectly competitive firm. In this case, the total revenue function would take the form

$$TR = P_0Q$$

where P_0 is the constant price per unit of the good and is represented by the vertical intercept of a horizontal demand function. For example, if $P_0 = 10$, then the total revenue function is

$$TR = 0 + 10Q = 10Q$$

$$y = c + mx = mx$$

This is a line through the origin (0,0) since the vertical intercept, $c = 0$.

□ Revenue

Example:

Suppose that each chicken snack box is sold for £3.50 irrespective of the number of units sold.

- (a) Write down the equation of the total revenue function.
- (b) Graph the total revenue function.

□ Revenue

Solution:

(a) Total revenue is price multiplied by the number of units sold, that is,

$$TR = 3.5Q$$

Note that price is constant at £3.50 irrespective of the value of Q .

□ Revenue

Solution:

(b) Total revenue is represented graphically by a straight line, with slope = 3.5 and intercept = 0. It is graphed by calculating values of TR for any values of Q , for example $Q = 0, 2, 4, 6$. These points are outlined in **Table 2.6** and then plotted in **Figure 2.26**.

Table 2.6 Total revenue

Q	$TR = PQ = 3.5Q$	Point (Q, TR)
0	$TR = 3.5(0) = 0$	(0, 0)
2	$TR = 3.5(2) = 7$	(2, 7)
4	$TR = 3.5(4) = 14$	(4, 14)
6	$TR = 3.5(6) = 21$	(6, 21)

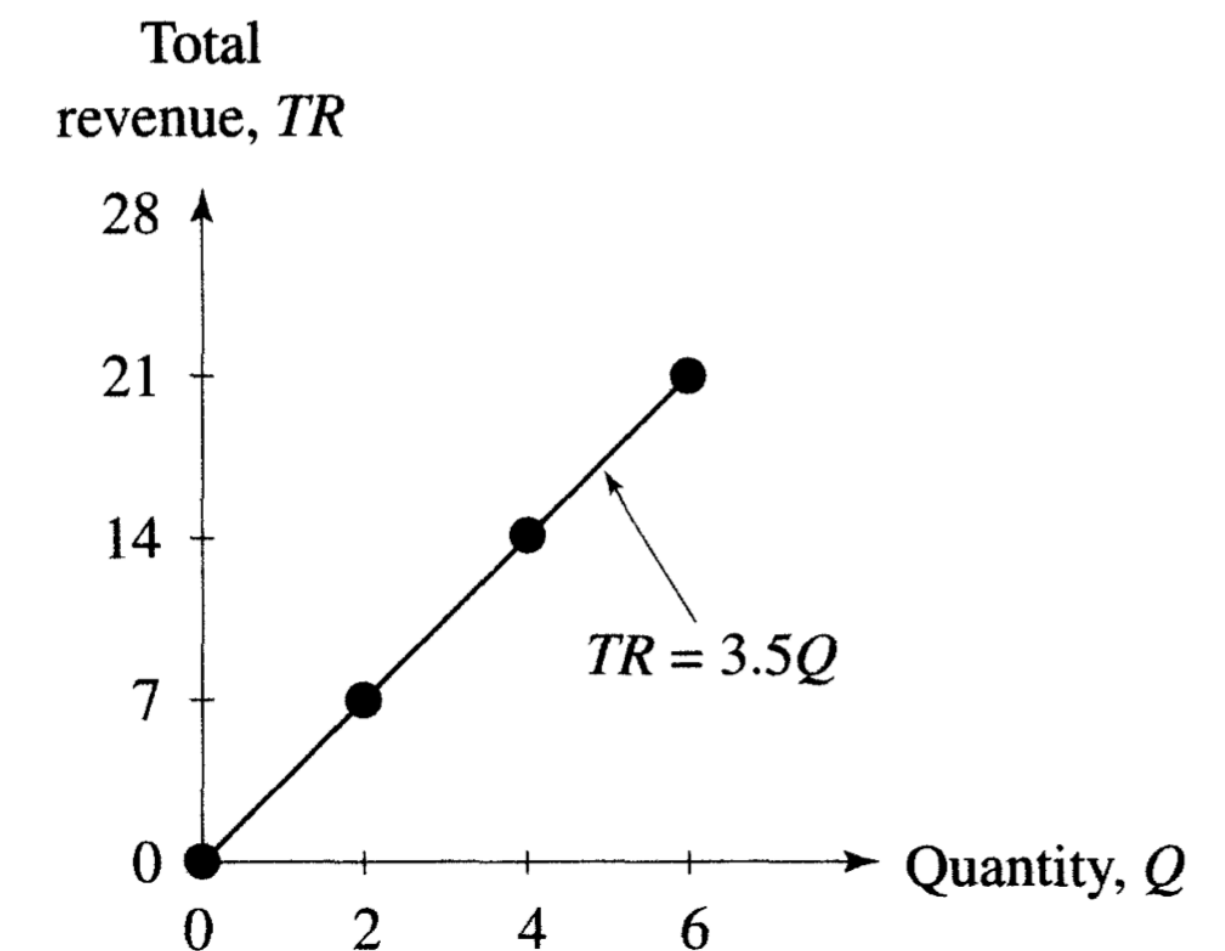


Figure 2.26 Linear total revenue function

Thank you

Thank you

