

# Mathematics

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## Lecture 8

جامعة الجميع الذكية

MAT 112

Methods and Applications of Integration



جامعة الجميع الذكية  
EVERYONE'S SMART UNIVERSITY

- To develop and apply the formula for integration by parts.
- To show how to integrate a proper rational function.
- To illustrate the use of the table of integrals.
- To develop the concept of the average value of a function.
- To define and evaluate improper integrals.



Integration by Parts

Integration with initial Conditions ( One Condition )

Various Examples for different methods of Integration

Extra Practice

## Formula for Integration by Parts

$$\int u \, dv = uv - \int v \, du$$

**Example 1 – Integration by Parts**

Find  $\int \frac{\ln x}{\sqrt{x}} dx$  by integration by parts.

Solution:

$$\begin{aligned} \text{Let } u = \ln x \text{ and } dv &= \frac{1}{\sqrt{x}} dx \\ du &= \frac{1}{x} dx & v &= \int x^{-1/2} dx = 2x^{1/2} \end{aligned}$$

$$\begin{aligned} \text{Thus, } \int \frac{\ln x}{\sqrt{x}} dx &= (\ln x)(2\sqrt{x}) - \int (2x^{1/2})\left(\frac{1}{x} dx\right) \\ &= 2\sqrt{x}[\ln(x) - 2] + C \end{aligned}$$

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$$\frac{1}{\frac{1}{2}}$$

$$= 1 \div \frac{1}{2}$$

$$= 1 * 2 = 2$$



**Example – Integration by Parts where  $u$  is the Entire Integrand**

Determine  $\int \ln y \, dy$ .

Solution: Let  $u = \ln y$  and  $dv = dy$

$$du = \left(\frac{1}{y}\right) dy \quad v = y$$

$$\begin{aligned} \text{Thus, } \int \ln y \, dy &= (\ln y)(y) - \int y \left(\frac{1}{y} dy\right) \\ &= y \ln y - y + C \\ &= y[\ln y - 1] + C \end{aligned}$$

## Integration with initial Conditions

- Use initial conditions to find the constant,  $C$ .

### Example 1 - Initial-Condition Problem

If  $y$  is a function of  $x$  such that  $y' = 8x - 4$  and  $y(2) = 5$ , find  $y$ .

Solution: We find the integral,

$$y = \int (8x - 4) dx = (8) \frac{x^2}{2} - 4x + C = 4x^2 - 4x + C$$

Using the condition,  $5 = 4(2)^2 - 4(2) + C$

$$C = -3$$

The equation is  $y = 4x^2 - 4x - 3$

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## \*\* EXTRA PRACTISE

Find (y) subject to the given condition:

$$1) y' = 3x - 4 \qquad y(-1) = \frac{13}{2}$$

$$y = \int (3x - 4) dx = \frac{3x^2}{2} - 4x + C$$

$$\text{Using } y(-1) = \frac{13}{2} \text{ gives}$$

$$\frac{13}{2} = \frac{3(-1)^2}{2} - 4(-1) + C$$

$$\frac{13}{2} = \frac{11}{2} + C$$

$$\text{Thus } C = 1, \text{ so } y = \frac{3x^2}{2} - 4x + 1.$$



$$y' = \frac{9}{8\sqrt{x}}$$

$$y(16) = 10$$

$$\begin{aligned} y &= \int \frac{9}{8\sqrt{x}} dx \\ &= \frac{9}{8} \int x^{-1/2} dx \\ &= \frac{9}{8} \cdot \frac{x^{1/2}}{\frac{1}{2}} + C \\ &= \frac{9\sqrt{x}}{4} + C \end{aligned}$$

$$y(16) = 10 \text{ implies } 10 = \frac{9\sqrt{16}}{4} + C, \quad 10 = 9 + C,$$

$$C = 1. \text{ Thus } y = \frac{9\sqrt{x}}{4} + 1.$$

$$y' = -x^2 + 2x \quad y(2) = 1$$

$$y = \int (-x^2 + 2x) dx = -\frac{x^3}{3} + x^2 + C$$

$$y(2) = 1 \text{ implies } 1 = -\frac{8}{3} + 4 + C, \text{ so } C = -\frac{1}{3}.$$

$$\text{Thus } y = -\frac{x^3}{3} + x^2 - \frac{1}{3}.$$

**Find the integrals:**

$$\int (x + 5)^7 dx$$

$$\text{Let } u = x + 5 \Rightarrow du = 1dx = dx$$

$$\int (x + 5)^7 [dx] = \int u^7 du = \frac{u^8}{8} + C = \frac{(x + 5)^8}{8} + C$$

$$\int 2x(x^2 + 3)^5 dx$$

$$\text{Let } u = x^2 + 3 \Rightarrow du = 2x \, dx$$

$$\begin{aligned} \int 2x(x^2 + 3)^5 dx &= \int (x^2 + 3)^5 [2x \, dx] = \int u^5 du = \frac{u^6}{6} + C \\ &= \frac{(x^2 + 3)^6}{6} + C \end{aligned}$$

$$\int \frac{4}{x} dx$$

$$\int \frac{4}{x} dx = 4 \int \frac{1}{x} dx = 4 \ln|x| + C$$



- For area under the graph from limit  $a \rightarrow b$ ,

$$\int_a^b f(x) dx$$

- $x$  is called the **variable of integration** and  $f(x)$  is the **integrand**.

$$\int_3^4 \frac{3}{(x+3)^2} dx$$



$$\begin{aligned}\int_3^4 \frac{3}{(x+3)^2} dx &= 3 \int_3^4 (x+3)^{-2} dx \\ &= 3 \cdot \frac{(x+3)^{-1}}{-1} \Big|_3^4 \\ &= -3 \left[ \frac{1}{4+3} - \frac{1}{3+3} \right] \\ &= -3 \left( \frac{1}{7} - \frac{1}{6} \right) \\ &= \frac{1}{14}\end{aligned}$$

$$\int_0^1 e^5 dx$$

$$\int_0^1 e^5 dx = e^5 x \Big|_0^1 = e^5 - 0 = e^5$$



# Lecture 08

## Example

$$\int_{\frac{1}{2}}^{\frac{3}{2}} (x^2 + x + 1) dx$$



$$\int_{1/2}^{3/2} (x^2 + x + 1) dx$$

$$= \left( \frac{x^3}{3} + \frac{x^2}{2} + x \right) \bigg|_{1/2}^{3/2}$$

$$= \frac{15}{4} - \frac{2}{3} = \frac{37}{12}$$



# Lecture 08

## \*\* EXTRA PRACTISE

$$\int_2^3 \frac{3}{x^2} dx$$



$$\begin{aligned}\int_2^3 \frac{3}{x^2} dx &= 3 \int_2^3 x^{-2} dx \\&= 3 \cdot \frac{x^{-1}}{-1} \Big|_2^3 \\&= -\frac{3}{x} \Big|_2^3 \\&= \frac{-3}{3} - \left( \frac{-3}{2} \right) \\&= -1 + \frac{3}{2} \\&= \frac{1}{2}\end{aligned}$$



$$\int_{-2}^{-1} (3w^2 - w - 1) dw$$



$$\int_{-2}^{-1} (3w^2 - w - 1) dw$$

$$= \left( w^3 - \frac{w^2}{2} - w \right) \Big|_{-2}^{-1}$$

$$= -\frac{1}{2} - (-8) = \frac{15}{2}$$



$$\int_2^8 -5x \, dx$$

$$\int_2^8 -5x \, dx$$

$$= -5 \cdot \frac{x^2}{2} \Big|_2^8 = -160 - (-10) = -150$$



$$\int_{-3}^1 (2x - 3) dx$$

$$\int_{-3}^1 (2x - 3) dx$$

$$= (x^2 - 3x) \Big|_{-3}^1$$

$$= -2 - 18 = -20$$



$$\int_4^1 (2t - 3t^2) dt$$

$$\int_4^1 (2t - 3t^2) dt$$

$$= (t^2 - t^3) \Big|_4^1$$

$$= 0 - (-48) = 48$$



$$\int_0^3 5dx$$

$$\int_0^3 5 dx = 5x \Big|_0^3$$

$$= 5(3) - 5(0)$$

$$= 15 - 0 = 15$$



$$\int 5x^{-7} dx$$

$$\begin{aligned}\int 5x^{-7} dx &= 5 \int x^{-7} dx = 5 \cdot \frac{x^{-7+1}}{-7+1} + C \\ &= 5 \cdot \frac{x^{-6}}{-6} + C = -\frac{5}{6x^6} + C\end{aligned}$$

$$\int 5x^{24} dx$$

$$\begin{aligned}\int 5x^{24} dx &= 5 \int x^{24} dx = 5 \cdot \frac{x^{24+1}}{24+1} + C \\ &= 5 \cdot \frac{x^{25}}{25} + C = \frac{x^{25}}{5} + C\end{aligned}$$

$$\int x^8 dx$$

$$\int x^8 dx = \frac{x^{8+1}}{8+1} + C = \frac{x^9}{9} + C$$



$$\int \frac{5}{x^7} dx$$

$$\begin{aligned}\int \frac{5}{x^7} dx &= 5 \int x^{-7} dx = 5 \cdot \frac{x^{-7+1}}{-7+1} + C \\ &= \frac{5x^{-6}}{-6} + C = -\frac{5}{6x^6} + C\end{aligned}$$



$$\int \frac{1}{t^{7/4}} dt$$

$$\begin{aligned}\int \frac{1}{t^{7/4}} dt &= \int t^{-7/4} dt = \frac{t^{-7/4+1}}{-\frac{7}{4}+1} + C = \frac{t^{-3/4}}{-\frac{3}{4}} + C \\ &= -\frac{4}{3t^{3/4}} + C\end{aligned}$$

$$\int (e^x + 3x^2 + 2x) dx$$

$$\begin{aligned}\int (e^x + 3x^2 + 2x) dx &= \int e^x dx + 3 \int x^2 dx + 2 \int x dx \\ &= e^x + 3 \cdot \frac{x^{2+1}}{2+1} + 2 \cdot \frac{x^{1+1}}{1+1} + C \\ &= e^x + x^3 + x^2 + C\end{aligned}$$

$$\int x^3(x^2 + 5x + 2)dx$$

$$\begin{aligned}\int x^3(x^2 + 5x + 2)dx &= \int (x^5 + 5x^4 + 2x^3)dx \\ &= \frac{x^6}{6} + 5 \cdot \frac{x^5}{5} + 2 \cdot \frac{x^4}{4} + C \\ &= \frac{x^6}{6} + x^5 + \frac{x^4}{2} + C\end{aligned}$$

Integration by Parts

Integration with initial Conditions ( One Condition )

Definite Integral

Various Examples for different methods of Integration