

Business Mathematics

Prepared by Dr. Faisal Hasan

Lecture NO 2

جامعة الجميع الذكية

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MTH 112

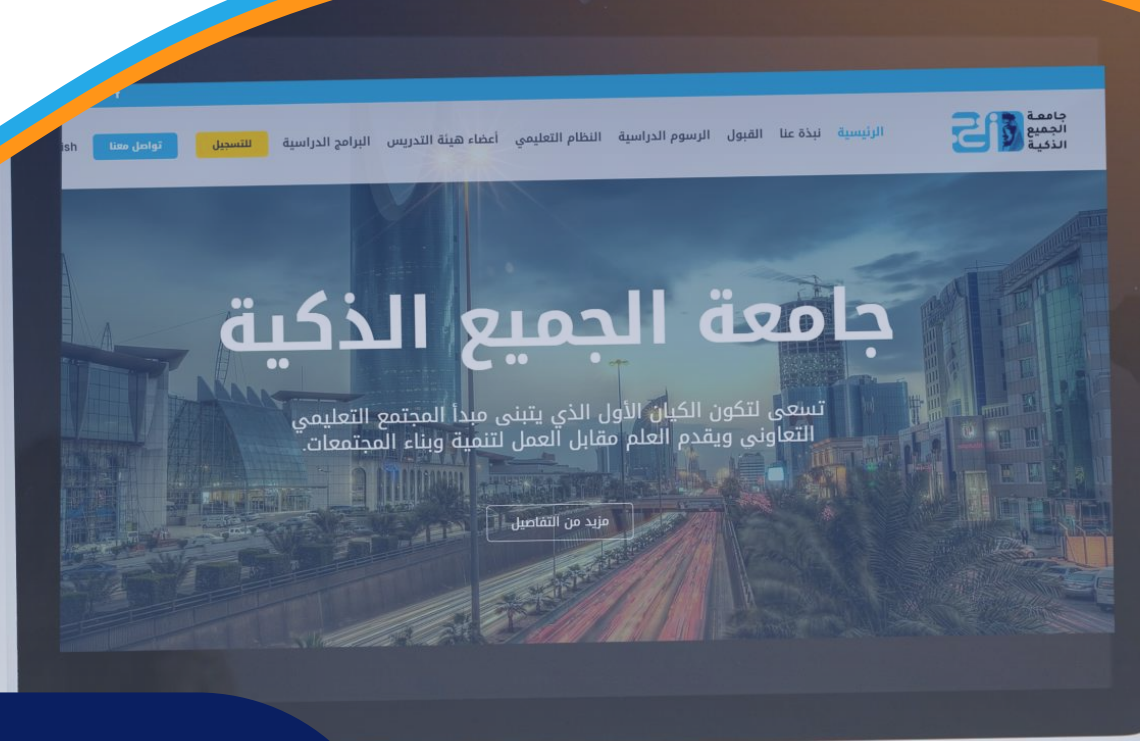
Business Mathematics



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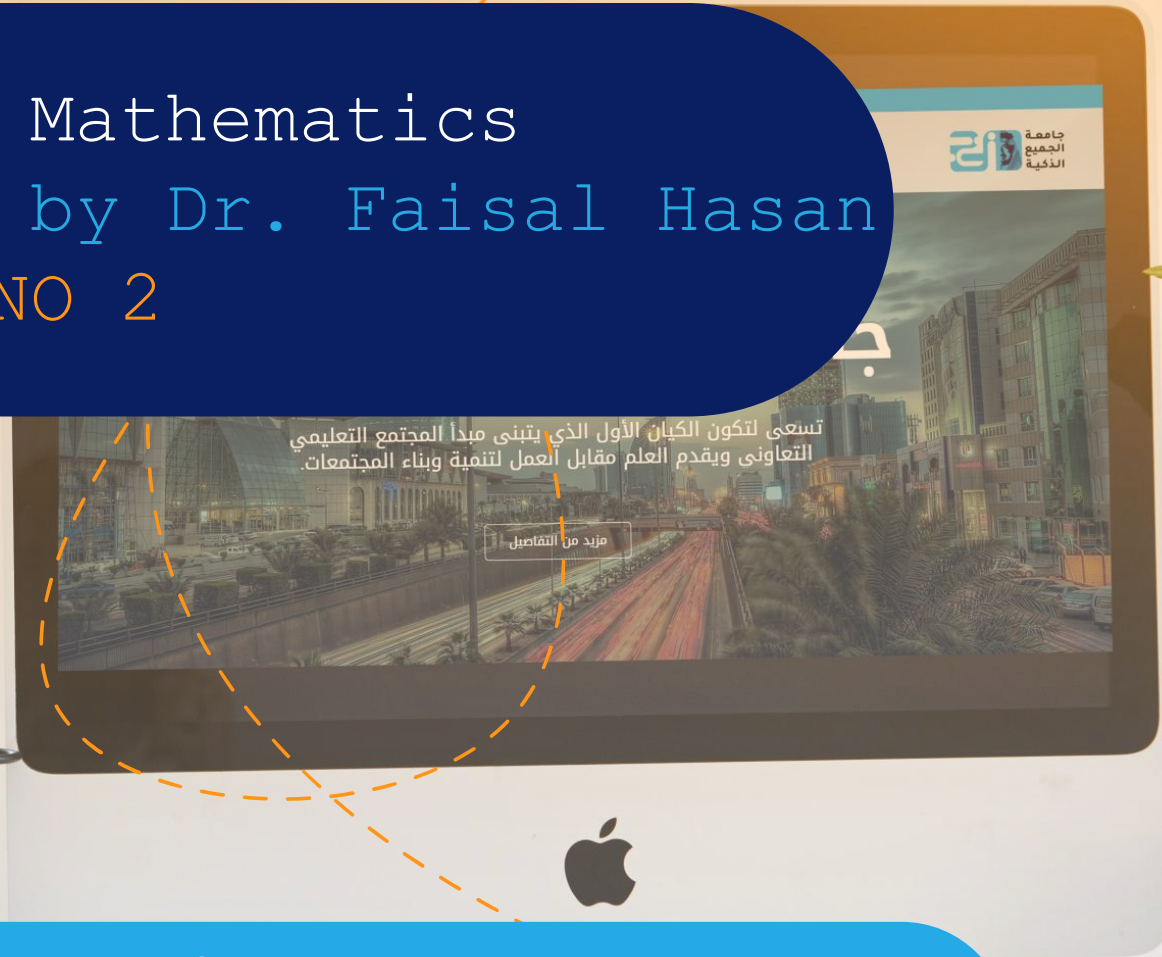
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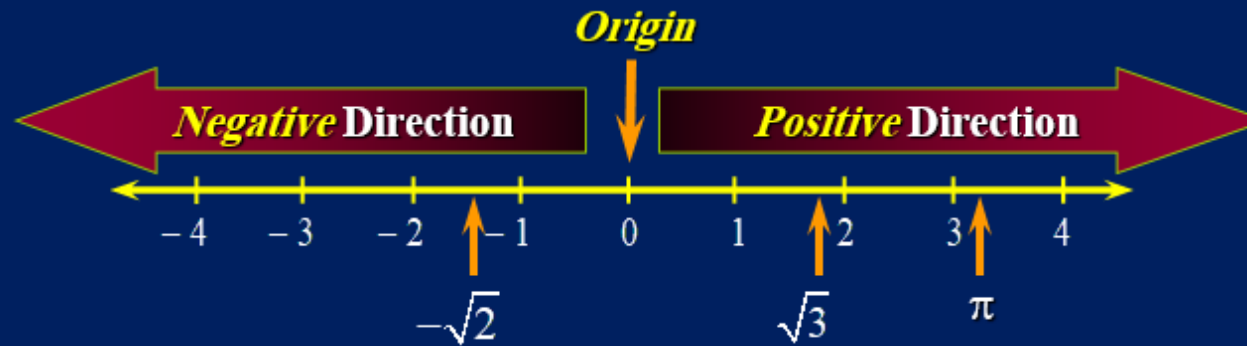
Topics

- Chapter 1: Mathematical preliminaries
- Chapter 2: The straight line and applications
- Chapter 3: Functions and Graphs
- Chapter 4: Limits and Continuity
- Chapter 5: Financial Mathematics
- Chapter 6: Differentiation
- Chapter 7: Methods of Differentiation
- Chapter 8: Function of several variables
- Chapter 9: Integration and applications
- Chapter 10: Linear algebra and applications

- ◆ **The Cartesian Coordinate System**
- ◆ **Straight Lines**
- ◆ **Linear Functions and Mathematical Models**
- ◆ **Intersection of Straight Lines**

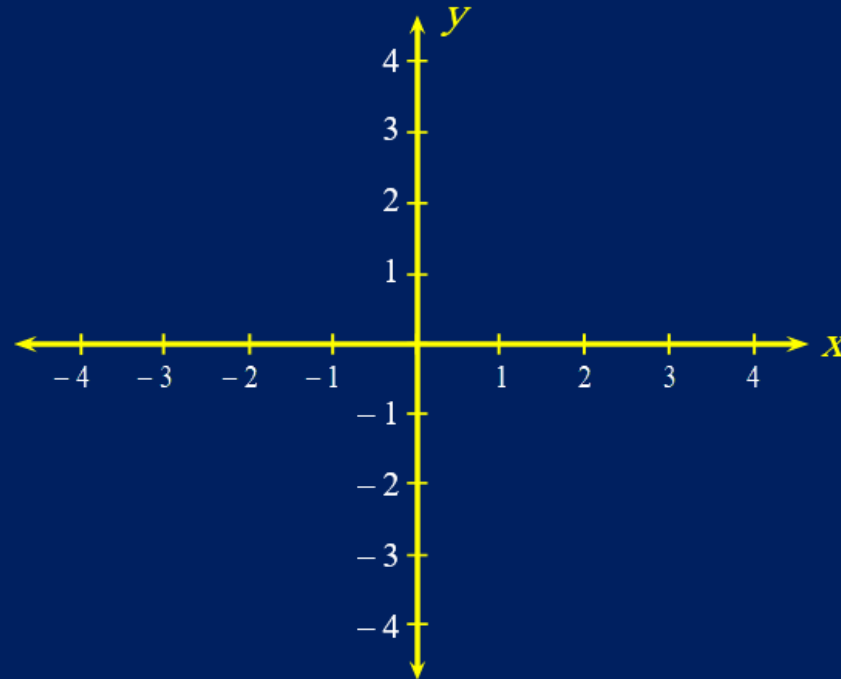
The Cartesian Coordinate System

- ◆ We can represent real numbers **geometrically** by points on a **real number**, or **coordinate**, **line**:
- ◆ This line includes **all** real numbers.
- ◆ Exactly **one** point on the line is associated with **each** real number, and vice-versa (**one dimensional space**).



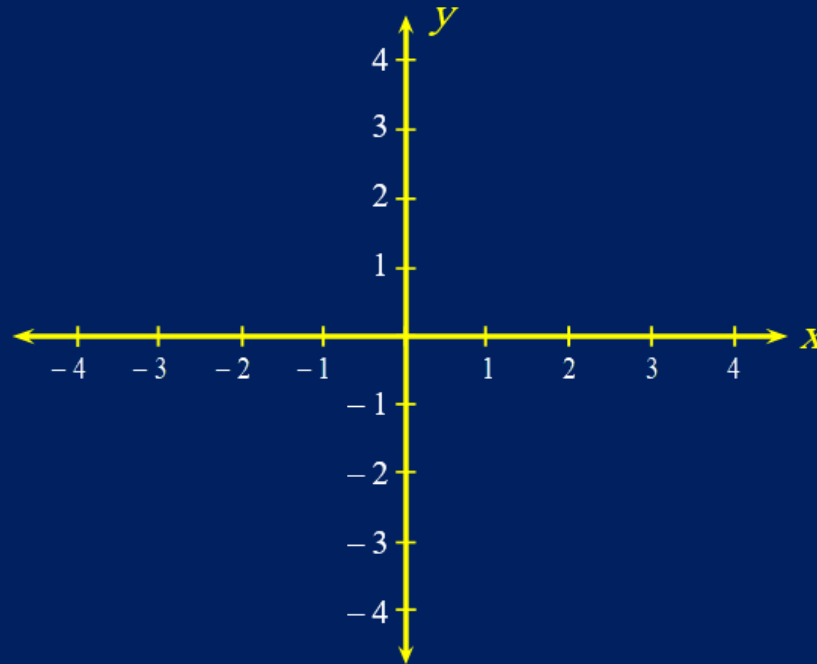
The Cartesian Coordinate System

- ◆ The Cartesian coordinate system extends this concept to a plane (two dimensional space) by adding a vertical axis.



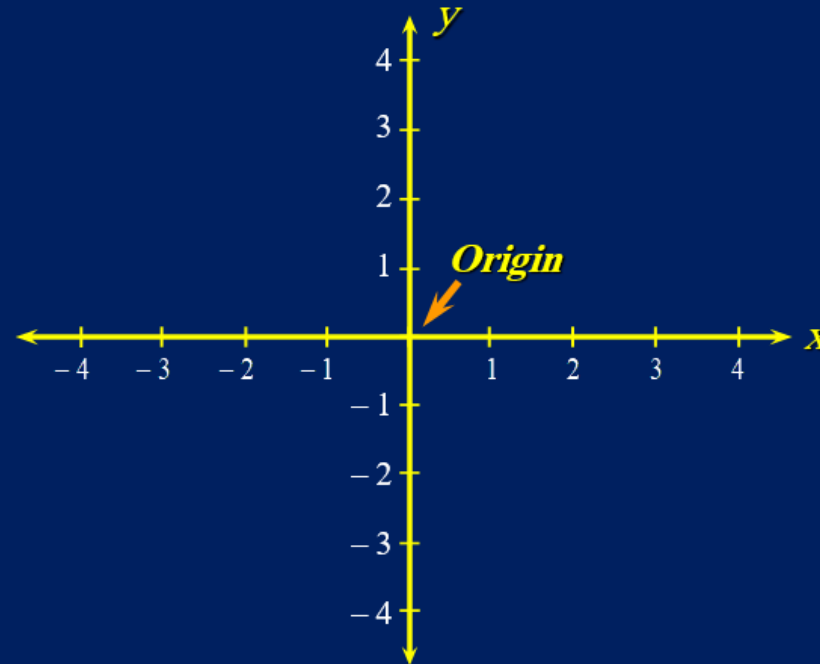
The Cartesian Coordinate System

- ◆ The **horizontal** line is called the x -axis, and the **vertical** line is called the y -axis.



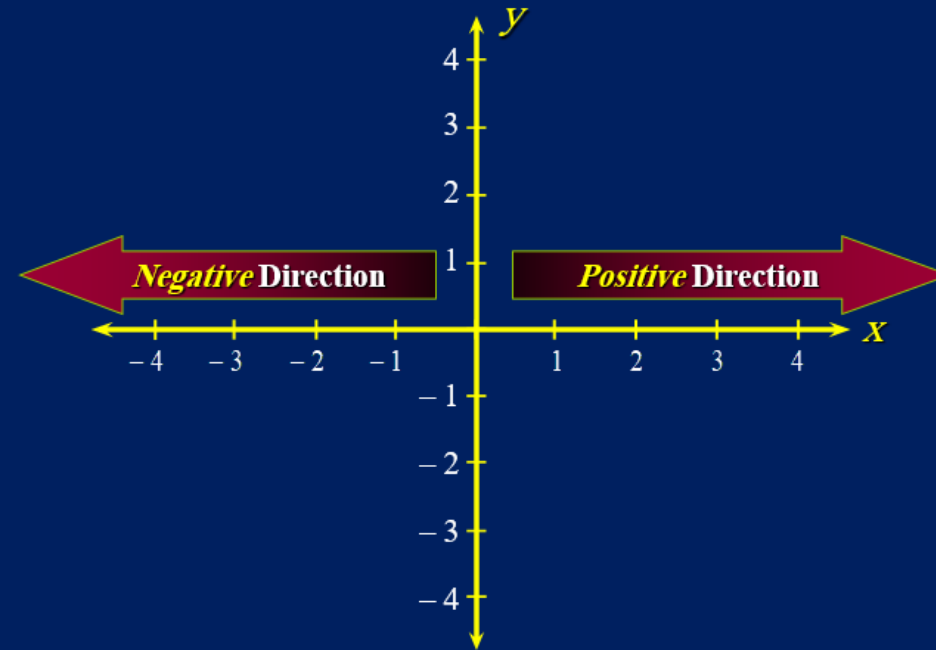
The Cartesian Coordinate System

- ◆ The point where these two lines **intersect** is called the **origin**.



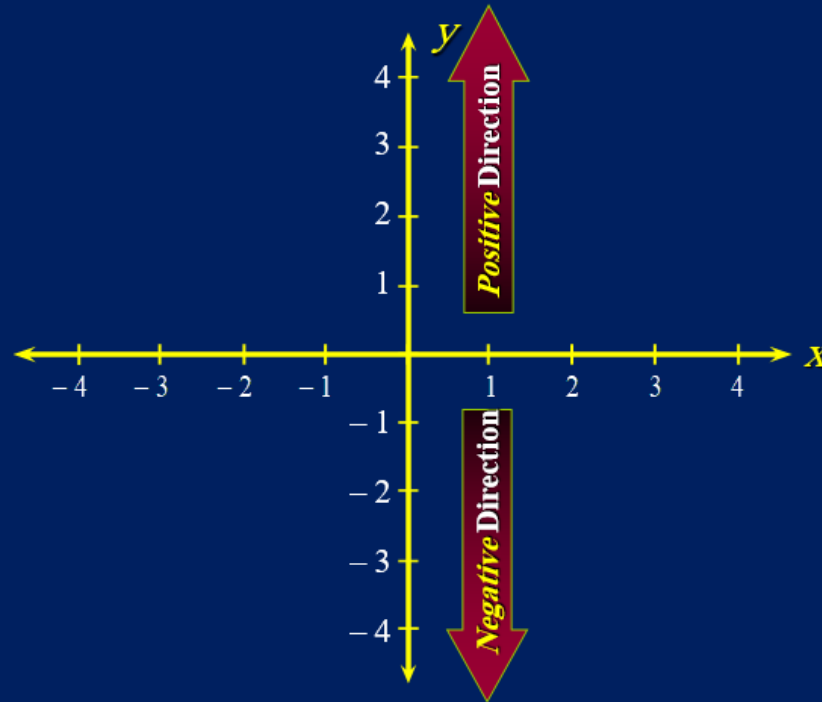
The Cartesian Coordinate System

- ◆ In the x -axis, **positive numbers** are to the **right** and **negative numbers** are to the **left** of the origin.



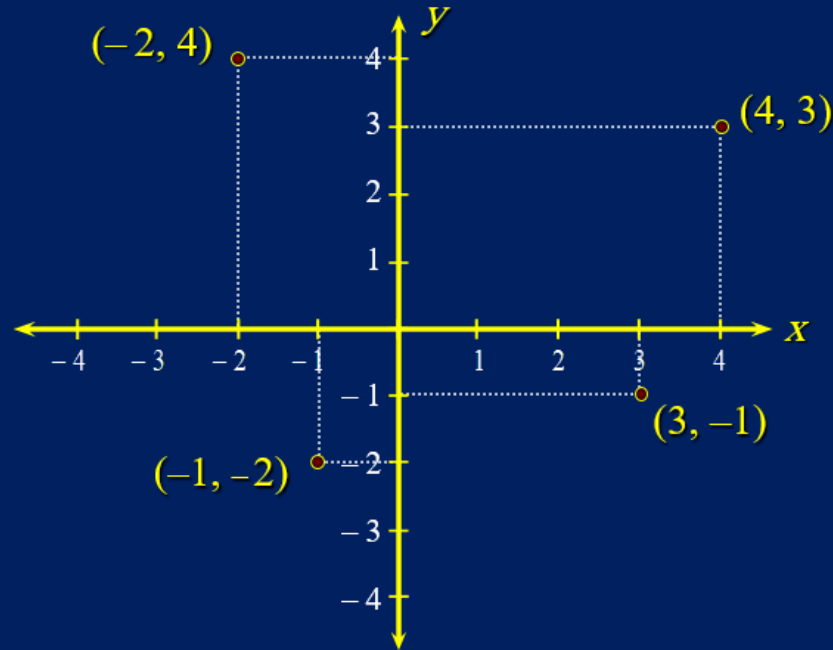
The Cartesian Coordinate System

- ◆ In the y -axis, **positive numbers** are **above** and **negative numbers** are **below** the origin.



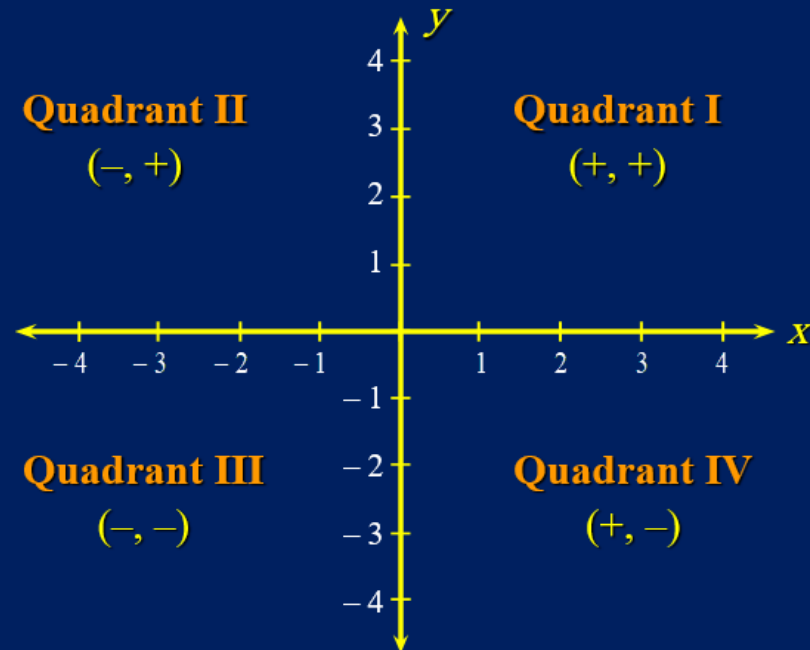
The Cartesian Coordinate System

- ◆ A **point** in the **plane** can now be represented uniquely in this coordinate system by an **ordered pair of numbers** (x, y) .

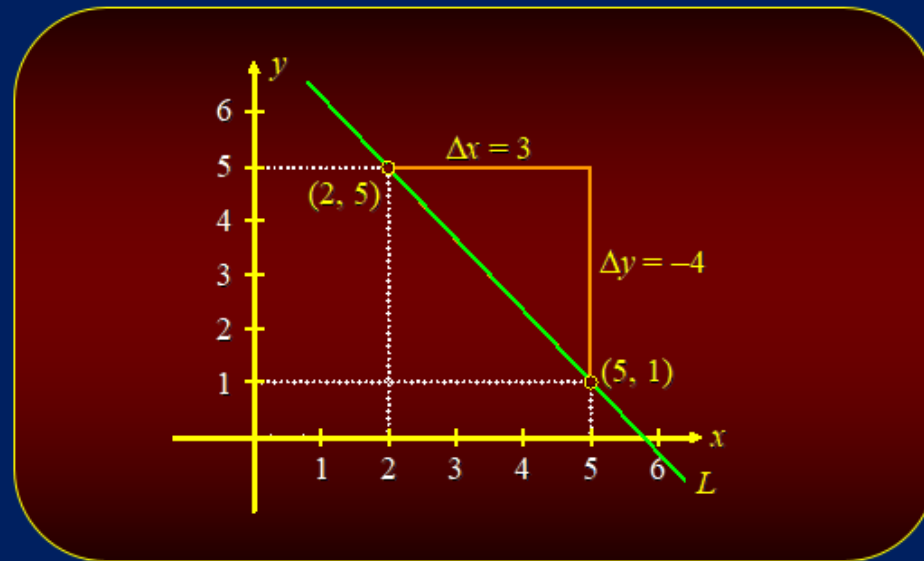


The Cartesian Coordinate System

- ◆ The axes divide the plane into **four quadrants** as shown below.

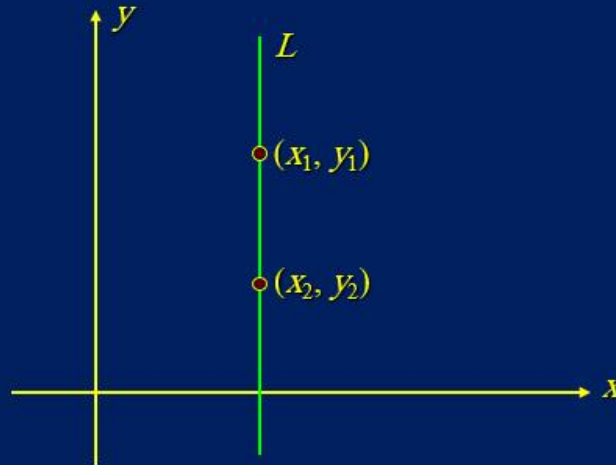


Straight Lines



Slope of a Vertical Line

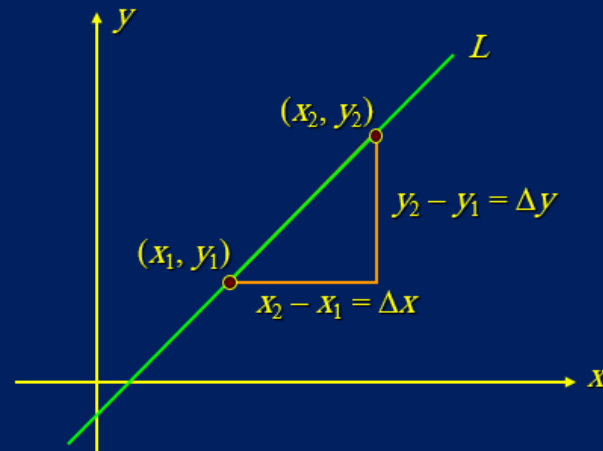
- ◆ Let L denote the unique straight line that passes through the two distinct points (x_1, y_1) and (x_2, y_2) .
- ◆ If $x_1 = x_2$, then L is a **vertical line**, and the **slope** is **undefined**.



Slope of a Nonvertical Line

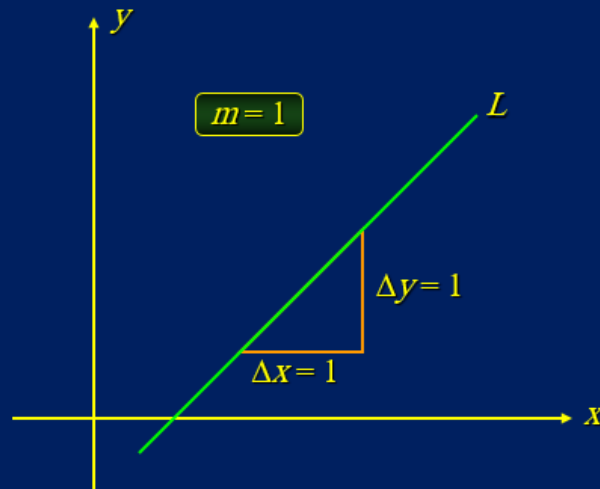
- ◆ If (x_1, y_1) and (x_2, y_2) are two distinct points on a **nonvertical line** L , then the **slope** m of L is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



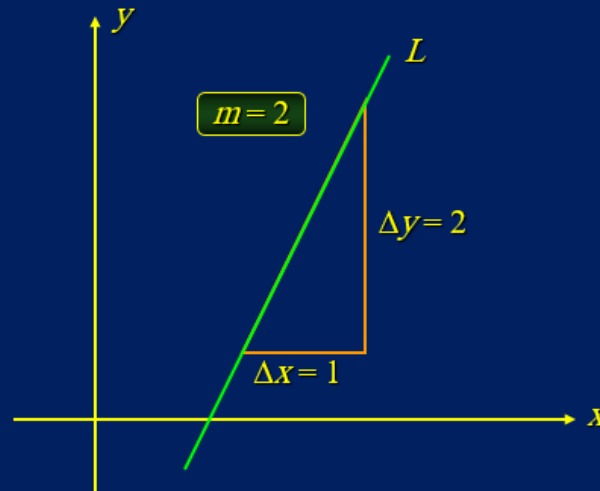
Slope of a Nonvertical Line

◆ If $m > 0$, the line **slants upward** from left to right.



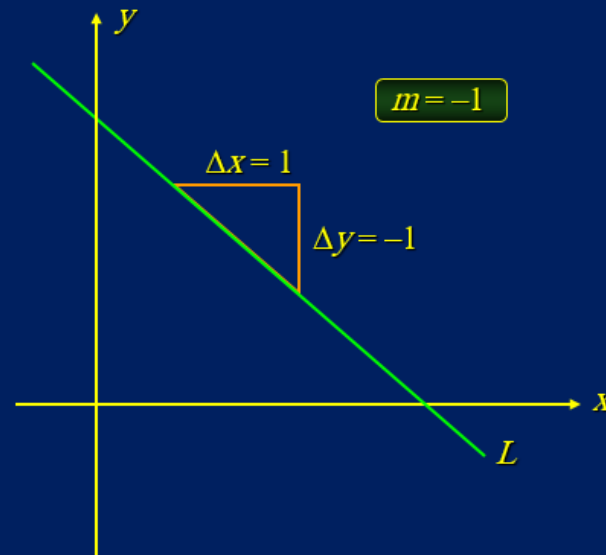
Slope of a Nonvertical Line

◆ If $m > 0$, the line **slants upward** from left to right.



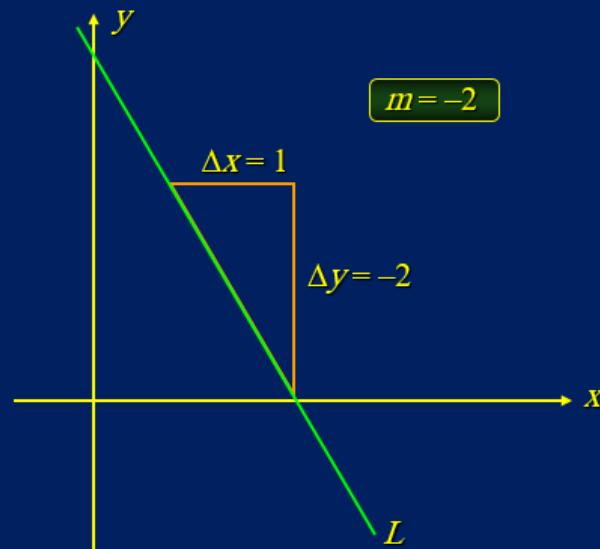
Slope of a Nonvertical Line

◆ If $m < 0$, the line **slants downward** from left to right.



Slope of a Nonvertical Line

◆ If $m < 0$, the line **slants downward** from left to right.

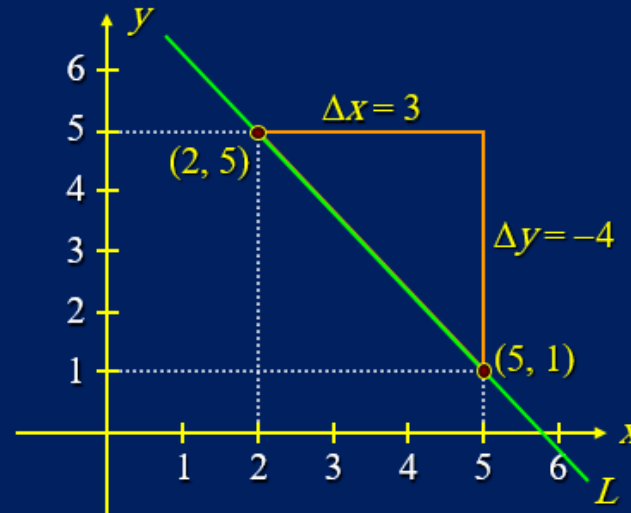


Examples

- ◆ Sketch the straight line that passes through the point $(2, 5)$ and has slope $-4/3$.

Solution

1. Plot the **point** $(2, 5)$.
2. A **slope** of $-4/3$ means that if x **increases** by 3, y **decreases** by 4.
3. Plot the **resulting point** $(5, 1)$.
4. Draw a **line** through the two points.



Examples

- ◆ Find the **slope** m of the line that goes through the **points** $(-1, 1)$ and $(5, 3)$.

Solution

- ◆ Choose (x_1, y_1) to be $(-1, 1)$ and (x_2, y_2) to be $(5, 3)$.
- ◆ With $x_1 = -1, y_1 = 1, x_2 = 5, y_2 = 3$, we find

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{5 - (-1)} = \frac{2}{6} = \frac{1}{3}$$

Examples

- ◆ Find the **slope** m of the line that goes through the **points** $(-2, 5)$ and $(3, 5)$.

Solution

- ◆ Choose (x_1, y_1) to be $(-2, 5)$ and (x_2, y_2) to be $(3, 5)$.
- ◆ With $x_1 = -2, y_1 = 5, x_2 = 3, y_2 = 5$, we find

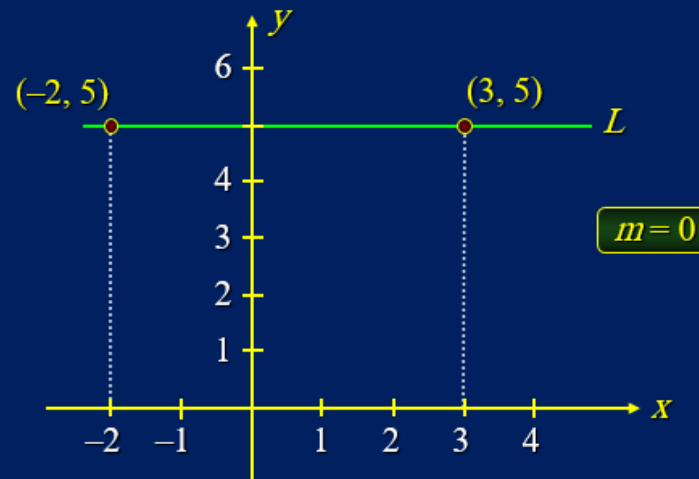
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 5}{3 - (-2)} = \frac{0}{5} = 0$$

Examples

- ◆ Find the **slope** m of the line that goes through the **points** $(-2, 5)$ and $(3, 5)$.

Solution

- ◆ The slope of a **horizontal line** is **zero**:



Parallel Lines

- ◆ Two distinct lines are **parallel** if and only if their **slopes are equal** or their **slopes are undefined**.

Example

- ◆ Let L_1 be a line that passes through the points $(-2, 9)$ and $(1, 3)$, and let L_2 be the line that passes through the points $(-4, 10)$ and $(3, -4)$.
- ◆ Determine whether L_1 and L_2 are parallel.

Solution

- ◆ The **slope** m_1 of L_1 is given by

$$m_1 = \frac{3 - 9}{1 - (-2)} = -2$$

- ◆ The **slope** m_2 of L_2 is given by

$$m_2 = \frac{-4 - 10}{3 - (-4)} = -2$$

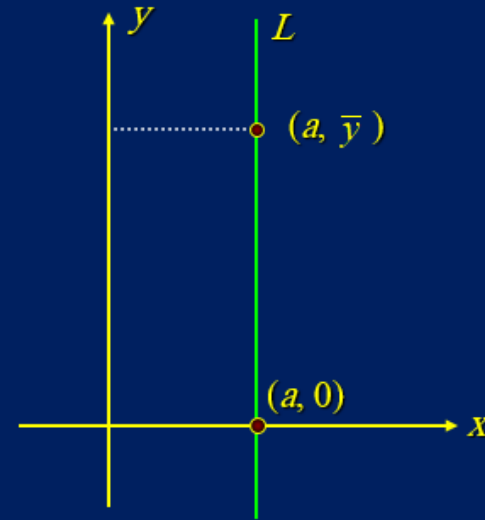
- ◆ Since $m_1 = m_2$, the lines L_1 and L_2 are in fact **parallel**.



Equations of Lines

- ◆ Let L be a **straight line parallel** to the y -axis.
- ◆ Then L **crosses** the x -axis at some **point** $(a, 0)$, with the **x -coordinate** given by $x = a$, where a is a real number.
- ◆ Any other point on L has the form $(a, \bar{y})$, where $\bar{y}$ is an appropriate number.
- ◆ The **vertical line** L can therefore be described as

$$x = a$$



Equations of Lines

- ◆ Let L be a **nonvertical line** with a slope m .
- ◆ Let (x_1, y_1) be a **fixed point** lying on L , and let (x, y) be a **variable point** on L distinct from (x_1, y_1) .
- ◆ Using the slope formula by letting $(x, y) = (x_2, y_2)$, we get

$$m = \frac{y - y_1}{x - x_1}$$

- ◆ Multiplying both sides by $x - x_1$ we get

$$y - y_1 = m(x - x_1)$$



Point-Slope Form

- ◆ An equation of the line that has slope m and passes through point (x_1, y_1) is given by

$$y - y_1 = m(x - x_1)$$

Examples

- ◆ Find an equation of the line that passes through the point $(1, 3)$ and has slope 2.

Solution

- ◆ Use the **point-slope form**

$$y - y_1 = m(x - x_1)$$

- ◆ Substituting for **point** $(1, 3)$ and **slope** $m = 2$, we obtain

$$y - 3 = 2(x - 1)$$

- ◆ **Simplifying** we get

$$2x - y + 1 = 0$$



Examples

- ◆ Find an equation of the line that passes through the points $(-3, 2)$ and $(4, -1)$.

Solution

- ◆ The slope is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 2}{4 - (-3)} = -\frac{3}{7}$$

- ◆ Substituting in the **point-slope form** for point $(4, -1)$ and slope $m = -3/7$, we obtain

$$y + 1 = -\frac{3}{7}(x - 4)$$

$$7y + 7 = -3x + 12$$

$$3x + 7y - 5 = 0$$



Perpendicular Lines

- ◆ If L_1 and L_2 are two distinct nonvertical lines that have slopes m_1 and m_2 , respectively, then L_1 is **perpendicular** to L_2 (written $L_1 \perp L_2$) if and only if

$$m_1 = -\frac{1}{m_2}$$

Example

- ◆ Find the equation of the line L_1 that passes through the point $(3, 1)$ and is perpendicular to the line L_2 described by

$$y - 3 = 2(x - 1)$$

Solution

- ◆ L_2 is described in **point-slope form**, so its **slope** is $m_2 = 2$.
- ◆ Since the lines are **perpendicular**, the **slope** of L_1 must be

$$m_1 = -1/2$$

- ◆ Using the **point-slope form** of the equation for L_1 we obtain

$$y - 1 = -\frac{1}{2}(x - 3)$$

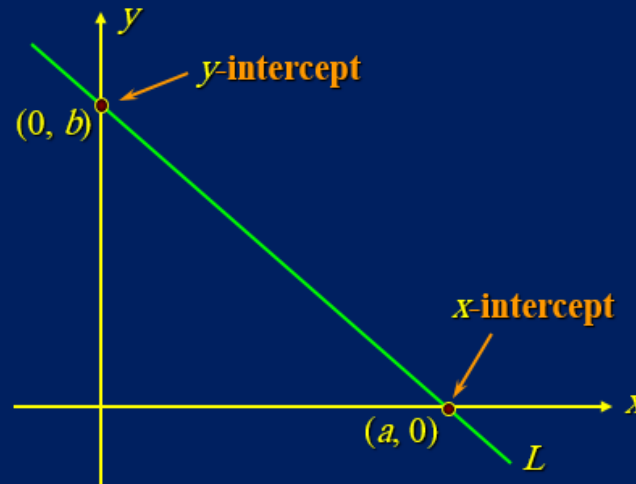
$$2y - 2 = -x + 3$$

$$x + 2y - 5 = 0$$



Crossing the Axis

- ◆ A straight line L that is **neither horizontal nor vertical** cuts the x -axis and the y -axis at, say, points $(a, 0)$ and $(0, b)$, respectively.
- ◆ The numbers a and b are called the **x -intercept** and **y -intercept**, respectively, of L .



Slope-Intercept Form

- ◆ An equation of the line that has **slope** m and **intersects** the y -axis at the **point** $(0, b)$ is given by

$$y = mx + b$$

Examples

- ◆ Find the equation of the line that has **slope 3** and **y-intercept** of -4 .

Solution

- ◆ We substitute $m=3$ and $b=-4$ into $y=mx+b$ and get

$$y=3x-4$$

Examples

- ◆ Determine the **slope** and **y-intercept** of the line whose equation is $3x - 4y = 8$.

Solution

- ◆ Rewrite the given equation in the **slope-intercept form**.

$$3x - 4y = 8$$

$$-4y = 8 - 3x$$

$$y = \frac{3}{4}x - 2$$

- ◆ Comparing to $y = mx + b$, we find that $m = \frac{3}{4}$ and $b = -2$.
- ◆ So, the **slope** is $\frac{3}{4}$ and the **y-intercept** is -2 .

Applied Example

- ◆ Suppose an art object **purchased** for \$50,000 is expected to **appreciate in value** at a **constant rate** of \$5000 per year for the next 5 years.
- ◆ Write an **equation** predicting the value of the art object for any given year.
- ◆ What will be its **value** 3 years after the purchase?

Solution

- ◆ Let $x = \text{time (in years) since the object was purchased}$
 $y = \text{value of object (in dollars)}$
- ◆ Then, $y = 50,000$ when $x = 0$, so the **y -intercept** is $b = 50,000$.
- ◆ Every year the value rises by 5000, so the **slope** is $m = 5000$.
- ◆ Thus, the equation must be $y = 5000x + 50,000$.
- ◆ After 3 years the **value of the object** will be \$65,000:
$$y = 5000(3) + 50,000 = 65,000$$

General Form of a Linear Equation

◆ The equation

$$Ax + By + C = 0$$

where A , B , and C are **constants** and A and B are not both zero, is called the **general form of a linear equation** in the **variables** x and y .

General Form of a Linear Equation

- ◆ An equation of a **straight line** is a **linear equation**; conversely, every **linear equation** represents a **straight line**.

Example

◆ Sketch the straight line represented by the equation

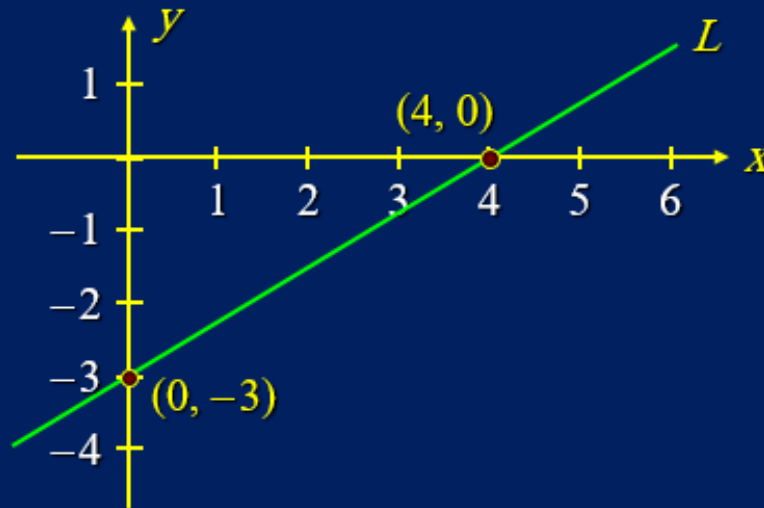
$$3x - 4y - 12 = 0$$



Solution

- ◆ Since every straight line is **uniquely determined** by **two distinct points**, we need find only two such points through which the line passes in order to sketch it.
- ◆ For convenience, let's compute the **x -** and **y -intercepts**:
 - ◆ Setting $y=0$, we find $x=4$; so the **x -intercept** is 4.
 - ◆ Setting $x=0$, we find $y=-3$; so the **y -intercept** is -3 .
- ◆ Thus, **the line goes through the points** $(4, 0)$ and $(0, -3)$.

◆ Graph the line going through the points $(4, 0)$ and $(0, -3)$.



Equation of Straight Lines

Vertical line: $x = a$

Horizontal line: $y = b$

Point-slope form: $y - y_1 = m(x - x_1)$

Slope-intercept form: $y = mx + b$

General Form: $Ax + By + C = 0$

