

Computer Vision, and Digital Image Processing

Prepared by Dr. Mohamed Mostafa

AI 612

Lecture 2



The background features a series of vertical panels with rounded ends. The first panel on the left shows a person's hands typing on a laptop. The subsequent panels are semi-transparent and contain white line-art icons: a graduation cap, a play button, a target with an arrow, an open book, and a magnifying glass. The overall color palette is warm, with shades of orange and brown.

Digital Image Processing

Lecture 2

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Chapter

01

Image Interpolation

Basic Relations between Pixels



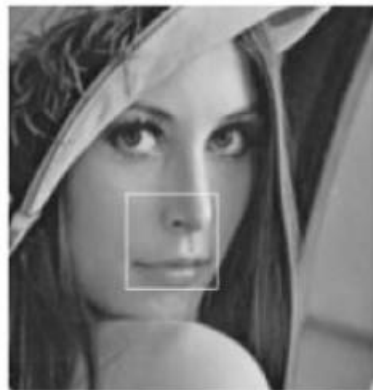
Image Interpolation

Interpolation is used in tasks such as zooming, shrinking, rotating, and geometrically correcting digital images.

Some methods used in Image Interpolation:

- Pixel Replication/Nearest Neighbor
- Bilinear Interpolation
- Bicubic Interpolation

4X
zooming



duplicate



bilinear



cubic

Image Interpolation

Pixel Replication/Nearest Neighbor

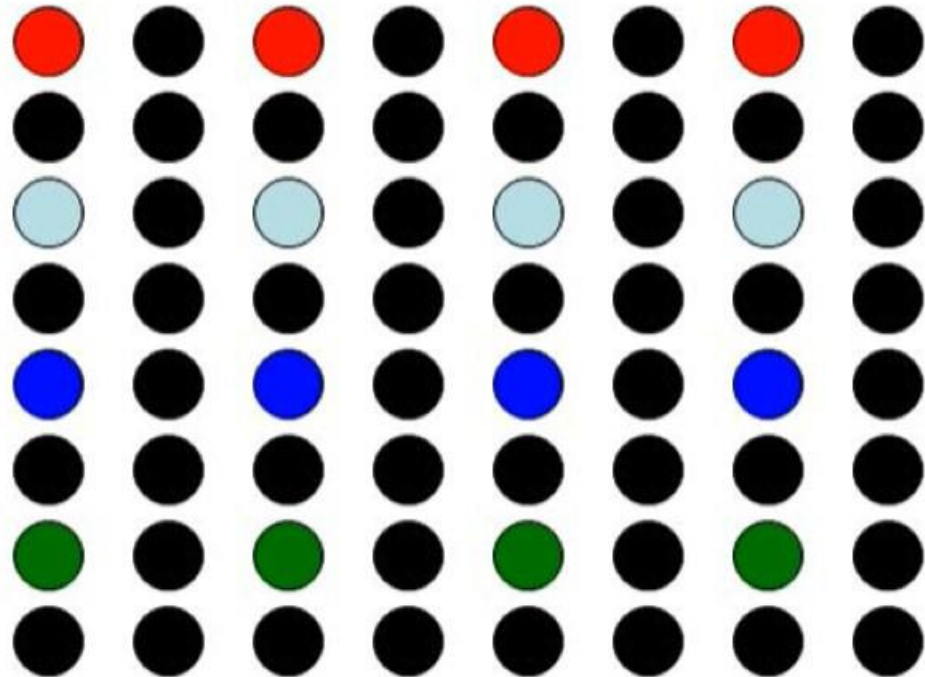
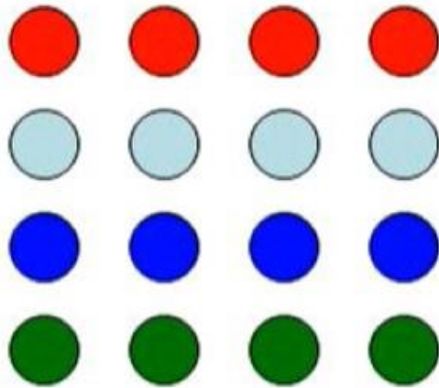
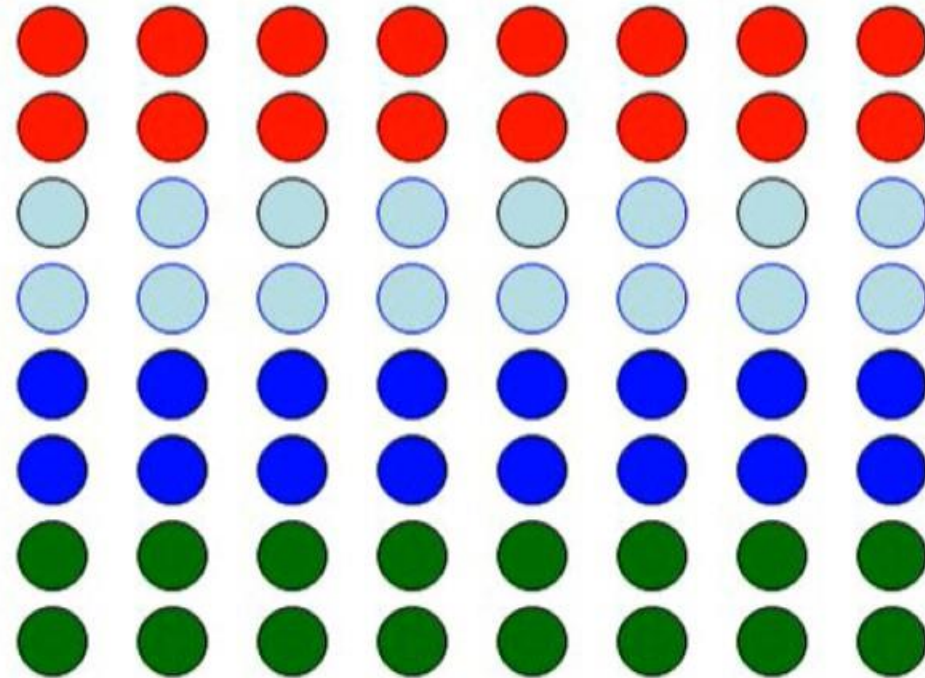
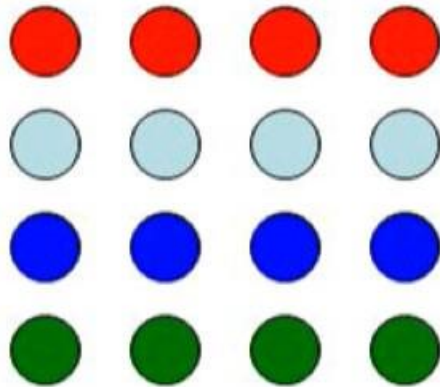


Image Interpolation

Pixel Replication/Nearest Neighbor



Checkboard Effect

Image Interpolation

Pixel Replication/Nearest Neighbor Example

30	85
45	105

2 x 2

4 x 4

30	30		
30	30	85	
45		105	

Image Interpolation

Pixel Replication/Nearest Neighbor Example

30	85
45	105

2 x 2

4 x 4

30	30	85	85
30	30	85	85
45	45	105	105
45	45	105	105

Image Interpolation

Pixel Replication/Nearest Neighbor Example

30	85
45	105

2 x 2

5 x 5

???

30			85	
45			105	

Image Interpolation

Nearest Neighbor Example

$$I(R \times C) = \begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) \\ (2,1) & (2,2) & (2,3) & (2,4) \\ (3,1) & (3,2) & (3,3) & (3,4) \\ (4,1) & (4,2) & (4,3) & (4,4) \end{bmatrix}$$

$$R=r \text{ \& } C=c$$

$$S_r = \frac{R}{r} = 0.5 \quad S_c = \frac{C}{c} = 0.5$$

$$J(\hat{R} \times \hat{C}) = \begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) & (1,7) & (1,8) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) & (2,7) & (2,8) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) & (3,7) & (3,8) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) & (4,7) & (4,8) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) & (5,7) & (5,8) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) & (6,7) & (6,8) \\ (7,1) & (7,2) & (7,3) & (7,4) & (7,5) & (7,6) & (7,7) & (7,8) \\ (8,1) & (8,2) & (8,3) & (8,4) & (8,5) & (8,6) & (8,7) & (8,8) \end{bmatrix}$$

$$[(r_f, c_f)] = [(S_r * \hat{r}, S_c * \hat{c})] = \begin{bmatrix} (0.5, 0.5) & (0.5, 1) & (0.5, 1.5) & (0.5, 2) & (0.5, 2.5) & (0.5, 3) & (0.5, 3.5) & (0.5, 4) \\ (1, 0.5) & (1, 1) & (1, 1.5) & (1, 2) & (1, 2.5) & (1, 3) & (1, 3.5) & (1, 4) \\ (1.5, 0.5) & (1.5, 1) & (1.5, 1.5) & (1.5, 2) & (1.5, 2.5) & (1.5, 3) & (1.5, 3.5) & (1.5, 4) \\ (2, 0.5) & (2, 1) & (2, 1.5) & (2, 2) & (2, 2.5) & (2, 3) & (2, 3.5) & (2, 4) \\ (2.5, 0.5) & (2.5, 1) & (2.5, 1.5) & (2.5, 2) & (2.5, 2.5) & (2.5, 3) & (2.5, 3.5) & (2.5, 4) \\ (3, 0.5) & (3, 1) & (3, 1.5) & (3, 2) & (3, 2.5) & (3, 3) & (3, 3.5) & (3, 4) \\ (3.5, 0.5) & (3.5, 1) & (3.5, 1.5) & (3.5, 2) & (3.5, 2.5) & (3.5, 3) & (3.5, 3.5) & (3.5, 4) \\ (4, 0.5) & (4, 1) & (4, 1.5) & (4, 2) & (4, 2.5) & (4, 3) & (4, 3.5) & (4, 4) \end{bmatrix}$$

$$[(\bar{r}, \bar{c})] = [\text{Round}[(r_f, c_f)]] = \begin{bmatrix} (1, 1) & (1, 1) & (1, 2) & (1, 2) & (1, 3) & (1, 3) & (1, 4) & (1, 4) \\ (1, 1) & (1, 1) & (1, 2) & (1, 2) & (1, 3) & (1, 3) & (1, 4) & (1, 4) \\ (2, 1) & (2, 1) & (2, 2) & (2, 2) & (2, 3) & (2, 3) & (2, 4) & (2, 4) \\ (2, 1) & (2, 1) & (2, 2) & (2, 2) & (2, 3) & (2, 3) & (2, 4) & (2, 4) \\ (3, 1) & (3, 1) & (3, 2) & (3, 2) & (3, 3) & (3, 3) & (3, 4) & (3, 4) \\ (3, 1) & (3, 1) & (3, 2) & (3, 2) & (3, 3) & (3, 3) & (3, 4) & (3, 4) \\ (4, 1) & (4, 1) & (4, 2) & (4, 2) & (4, 3) & (4, 3) & (4, 3.5) & (4, 4) \\ (4, 1) & (4, 1) & (4, 2) & (4, 2) & (4, 3) & (4, 3) & (4, 4) & (4, 4) \end{bmatrix}$$

Source:
ImageProcessingJU

Image Interpolation

Bilinear Interpolation

We use the four nearest neighbors to estimate the intensity at a given location.



$$v(x, y) = ax + by + cxy + d$$

coefficients that need to be estimated

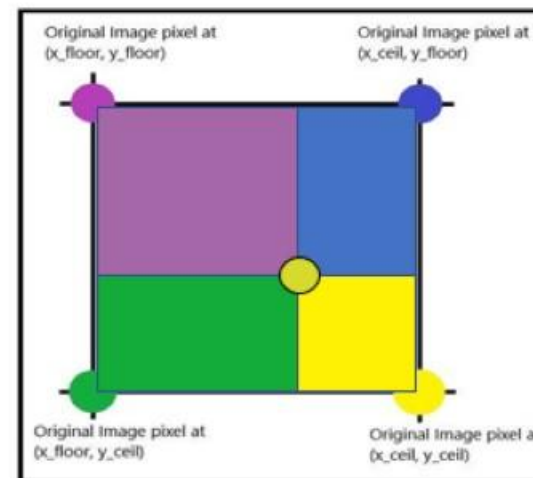
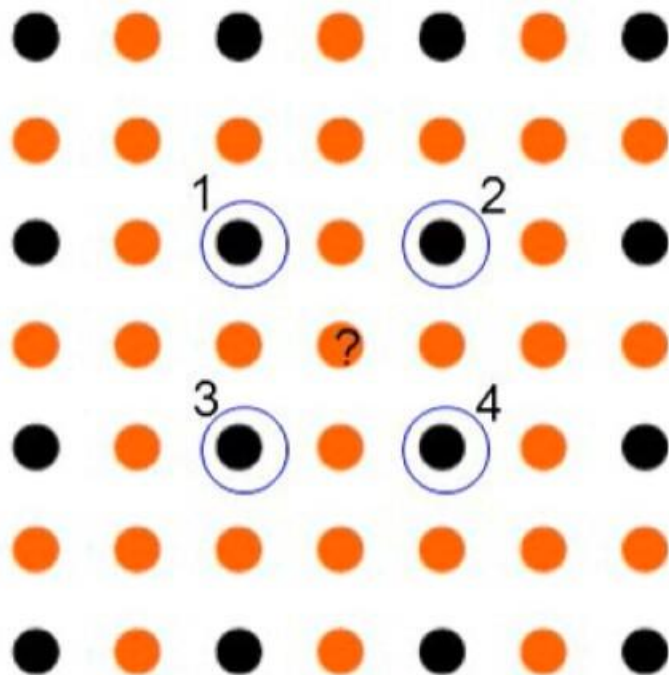


Image Interpolation

Bilinear Interpolation

We use the four nearest neighbors to estimate the intensity at a given location.



$$v(x, y) = ax + by + cxy + d$$

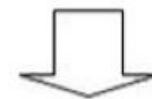
coefficients that need to be estimated

$$ax_1 + by_1 + cx_1y_1 + d = f(x_1, y_1)$$

$$ax_2 + by_2 + cx_2y_2 + d = f(x_2, y_2)$$

$$ax_3 + by_3 + cx_3y_3 + d = f(x_3, y_3)$$

$$ax_4 + by_4 + cx_4y_4 + d = f(x_4, y_4)$$



a, b, c, d

Image Interpolation

Bilinear Interpolation Example:

Suppose that 2x2 image has been enlarged by factor of 5. Use linear interpolation to find the values of p, q, r, s.

$$P = 10 * 4/5 + 40 * 1/5 = 16$$

10	40
30	20



10	p	q	r	s	40
	1/5		4/5		
30					20

Image Interpolation

Bilinear Interpolation Example:

Suppose that 2x2 image has been enlarged by factor of 5. Use linear interpolation to find the values of p, q, r, s.

$$q = 10 * 3/5 + 40 * 2/5 = 22$$

10	40
30	20



10	p	q	r	s	40
30					20

Image Interpolation

Bilinear Interpolation Example:

Suppose that 2x2 image has been enlarged by factor of 5. Use linear interpolation to find the values of p, q, r, s.

**Final
Solution**

10	40
30	20



10	16	22	28	34	40
30					20

Image Interpolation

Bilinear Interpolation Example:

Suppose that 2x2 image has been enlarged by factor of 5. Use linear interpolation to find the values of p, q, r, s.

**What is t, u and
are required**

10	40
30	20



10					40
		t			
			u		
30					20

Image Interpolation

Bilinear Interpolation Example:

- Scale the current image by factor of 2

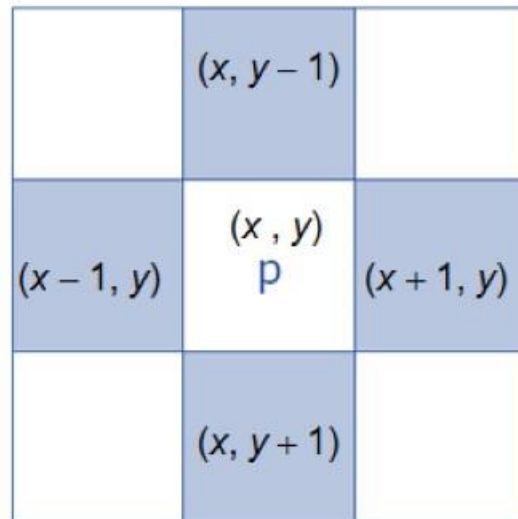
30	85
45	105

Take home: Practice

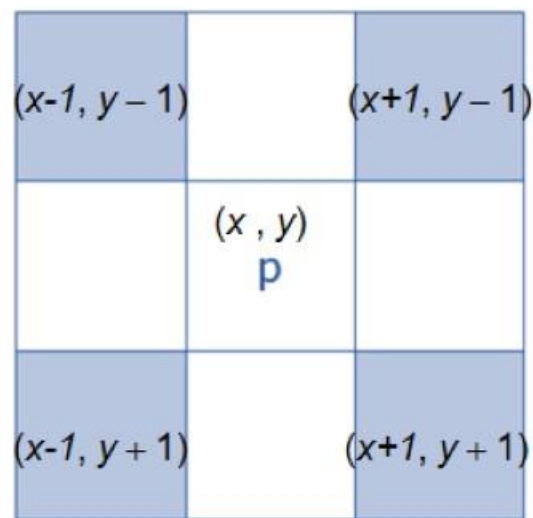
30			85
45			105

Basic Relations between Pixels

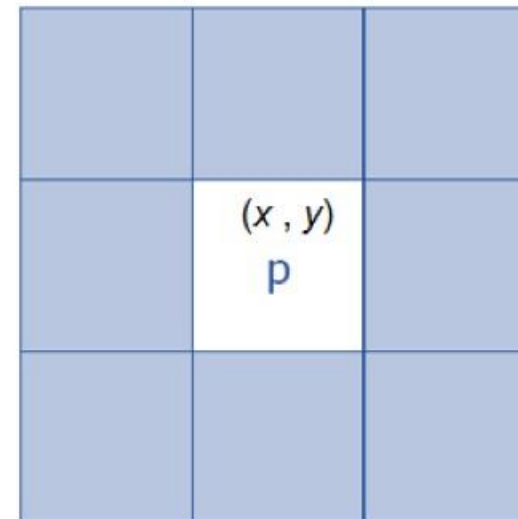
Neighbors of a Pixel



4-adjacency: $N_4(p)$



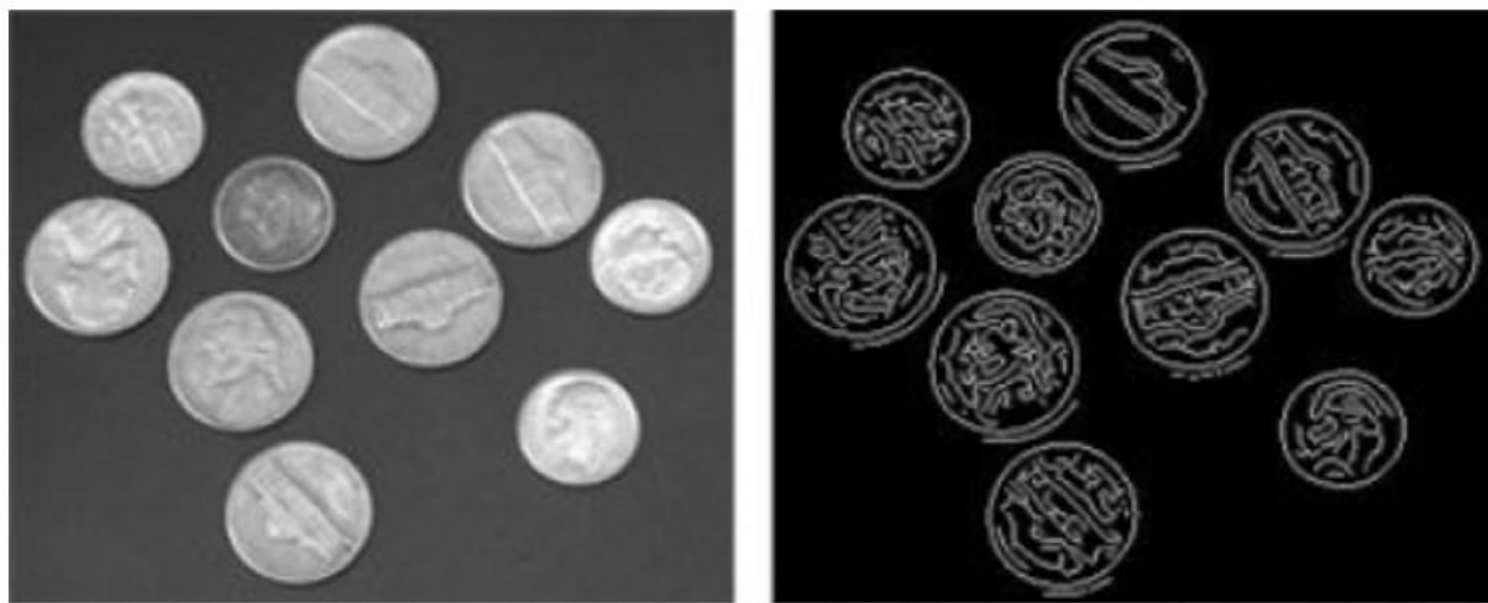
4-adjacency: $N_D(p)$



8-adjacency: $N_4(p) \cup N_D(p)$

Basic Relations between Pixels

Neighbors of a Pixel



Edge Detection Algorithm Like the Sobel Operator

Basic Relations between Pixels

Adjacency

Two pixels that are neighbors and have the same gray level are adjacent;

Types: **4-adjacency**, **8-adjacency**, **m-adjacency**.

Binary Image

0	0	0	0
1	0	1	0
0	1	1	0
0	0	0	0

4-adjacency

intensity value, $V = \{1\}$

Basic Relations between Pixels

Adjacency

Two pixels that are neighbors and have the same gray level are adjacent;
Types: **4-adjacency**, **8-adjacency**, **m-adjacency**.

Binary Image

0	0	0	0
1	0	1	0
0	1	1	0
0	0	0	0

8-adjacency

intensity value, $V = \{1\}$

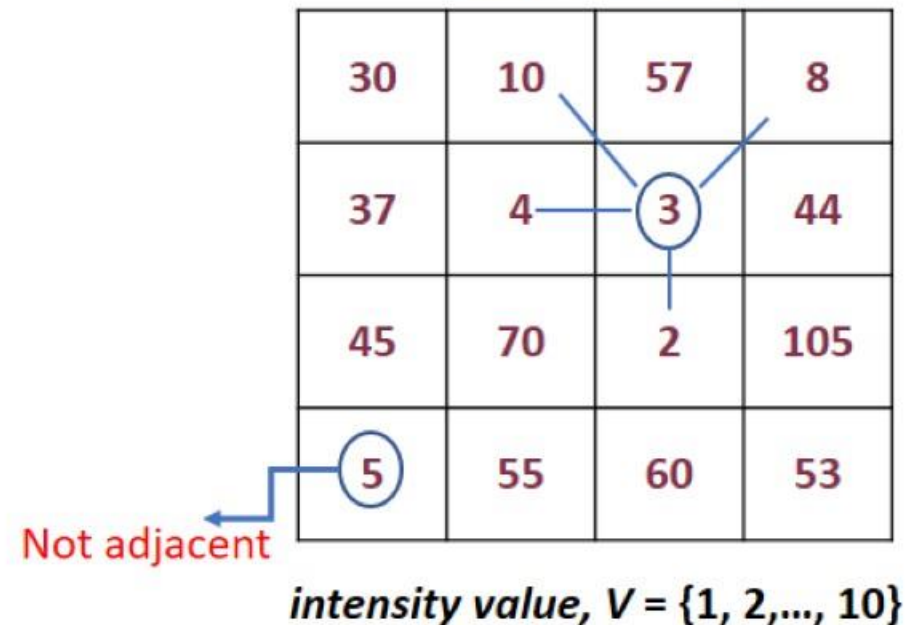
Basic Relations between Pixels

Adjacency

Two pixels that are neighbors and have the same gray level are adjacent;

Types: **4-adjacency**, **8-adjacency**, **m-adjacency**.

Gray-scale Image



Basic Relations between Pixels

Adjacency *m-adjacency.*

Binary Image

0	1	1
0	1	0
0	0	1

0	1	1
0	1	0
0	0	1

0	1	1
0	1	0
0	0	1

intensity value, $V = \{1\}$

No Unique Path (ambiguity)

Basic Relations between Pixels

Adjacency

m-adjacency.

q is in $N_D(p)$ or the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V .

Binary Image

0	1	1
0	1	0
0	0	1

4-Adjacency

8-Adjacency

intensity value, $V = \{1\}$

Basic Relations between Pixels

Adjacency

m-adjacency.

q is in $N_D(p)$ or the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V .

Binary Image

0	1	1
0	1	0
0	0	1

4-Adjacency **v**

8-Adjacency **x**

intensity value, $V = \{1\}$

Basic Relations between Pixels

Adjacency

m-adjacency.

q is in $N_D(p)$ or the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V .

Binary Image

0	0	1
0	1	0
0	0	1

4-Adjacency **x**

8-Adjacency **v**

intensity value, $V = \{1\}$

Basic Relations between Pixels

Adjacency

Practice

Consider the two image subsets, S_1 and S_2 , shown in the following figure. For $V=\{1\}$, determine whether these two subsets are

- (a) 4-adjacent,
- (b) 8-adjacent, or
- (c) m-adjacent.

	S_1				S_2				
0	0	0	0	0	0	0	1	1	0
1	0	0	1	0	0	1	0	0	1
1	0	0	1	0	1	1	0	0	0
0	0	1	1	1	0	0	0	0	0
0	0	1	1	1	0	0	1	1	1

Basic Relations between Pixels

Adjacency

- (a) S_1 and S_2 are not 4-connected since q is not in the set $N_4(p)$,
- (b) S_1 and S_2 **are 8-connected**, since q is in the set $N_8(p)$,
- (c) S_1 and S_2 **are m-adjacent** since:
 - i. q is in the set $N_D(p)$, and
 - ii. the set of $N_4(p) \cap N_4(q)$ is empty.

Solution

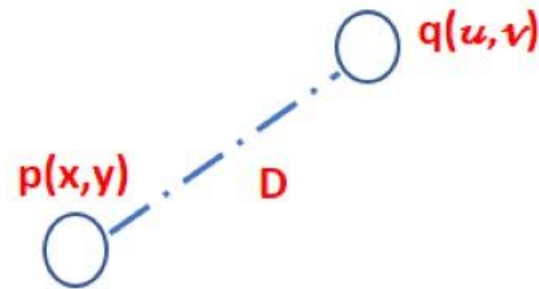
	S_1					S_2				
0	0	0	0	0		0	0	1	1	0
1	0	0	1	0		0	1	0	0	1
1	0	0	1	0		1	1	0	0	0
0	0	1	1	1	p	0	0	0	0	0
0	0	1	1	1		0	0	1	1	1

Basic Relations between Pixels

Distance Measures

- **Euclidean distance between p and q**

$$D_e(p, q) = \left[(x - u)^2 + (y - v)^2 \right]^{\frac{1}{2}}$$



- **City-block distance between p and q (D_4)**

$$D_4(p, q) = |x - u| + |y - v|$$

- **Chessboard distance between p and q (D_8)**

$$D_8(p, q) = \max(|x - u|, |y - v|)$$

Basic Relations between Pixels

Distance Measures

Euclidean distance
(2-norm)

$2\sqrt{2}$	$\sqrt{5}$	2	$\sqrt{5}$	$2\sqrt{2}$
$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$	$\sqrt{5}$
2	1	0	1	2
$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$	$\sqrt{5}$
$2\sqrt{2}$	$\sqrt{5}$	2	$\sqrt{5}$	$2\sqrt{2}$

D_4 distance
(city-block distance)

4	3	2	3	4
3	2	1	2	3
2	1	0	1	2
3	2	1	2	3
4	3	2	3	4

D_8 distance
(checkboard distance)

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Basic Relations between Pixels

Distance Measures

- ***m-adjacency between p and q***
 - The D_m distance between two points is defined as the shortest m -path between the points.
 - The distance between two pixels will depend on the values of the pixels along the path, as well as the values of their neighbors.

Note:

The Path length is the number of the pixels involved within the path

Basic Relations between Pixels

Distance Measures

- Practice**

An image segment is shown below. Let V be the set of gray levels values used to define connectivity in the image. Capture D_4 , D_8 , and D_m distances between pixels 'p' and 'q' for:

1. $V = \{2, 3\}$
2. $V = \{2, 3, 6\}$

p 3	2	7	6
7	4	3	2
1	0	2	0
5	1	7	6 q

Basic Relations between Pixels

Distance Measures

- *Coordinates of $p(x,y) = (0, 0)$*
- *Coordinates of $q(u,v) = (3, 3)$*

$$D_4(p, q) = |x - u| + |y - v|$$

- $D_4(p, q) = |0 - 3| + |0 - 3| = \mathbf{6 \text{ Units}}$

$$D_8(p, q) = \max(|x - u|, |y - v|)$$

- $D_8(p, q) = \max(|0 - 3|, |0 - 3|) = \mathbf{3 \text{ Units}}$

^p 3	2	7	6
7	4	3	2
1	0	2	3
5	1	7	6 ^q

Basic Relations between Pixels

Distance Measures

- m-distance depends on the path values*

1. let's consider the $V = \{2, 3\}$

p (3)	2	7	6
7	4	3	2
1	0	2	3
5	1	7	6 ^q

- Question :** Why we can't go from 2 to 6 since it is in the V set !!!

Basic Relations between Pixels

Distance Measures

- m-distance depends on the path values*

1. let's consider the $V = \{2, 3, 6\}$

p 3	2	7	6
7	4	3	2
1	0	2	3
5	1	7	q 6

- Answer:** The shorts path between p and q = 5 Units

Basic Relations between Pixels

Distance Measures

- *D4-path between q and p*

1. let's consider the $\{2, 3, 6\}$

p 3	2	7	6
7	4	3	2
1	0	2	3
5	1	7	6 q

- **Answer:** *There is no path between p and q dose not exist*

Note that the $D4$ and $D8$ distances between p and q are independent of any paths

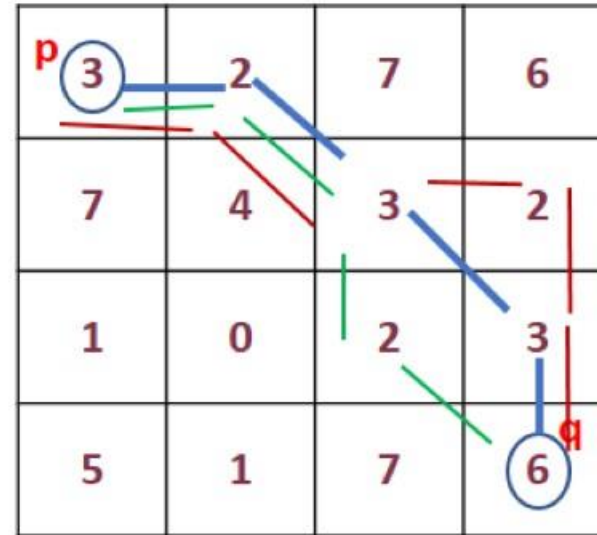
Basic Relations between Pixels

Distance Measures

- *D8-path between q and p*

1. let's consider the $V = \{2, 3, 6\}$

- **Answer:** *There is more than one path between p and q ,
So we choose the shorts*



Basic Relations between Pixels

Distance Measures

- m-distance depends on the path values*

1. let's consider the $V = \{2, 3\}$

p 3	2	7	6
7	4	3	2
1	0	2	3
5	1	7	6 ^q

- Answer:** *There is no path between p and q since 6 is not in the V set*

Basic Relations between Pixels

Practice

- let's consider the $V = \{2, 3\}$ consider the following find the relationship between p and q
- 4-adjacency a 4-path
- 8-adjacency a 8-path
- m-adjacency a m-path
- path length ?
- Number of pixels involved

3	1	1	3 (q)
1	3	2	4
3	2	1	4
(p) 2	0	0	1

Basic Relations between Pixels

Practice

- let's consider the $V = \{2, 3\}$ consider the following:
 - 4-adjacency a 4-path
 - 8-adjacency a 8-path
 - m-adjacency a m-path
 - path length ?
 - Number of pixels involved
- **Answer:** *There is no path between p and q*

3	1	1	3 (q)
1	3	2	4
3	2	1	4
(p) 2	0	0	1

Basic Relations between Pixels

Practice

- let's consider the $V = \{2, 3\}$ consider the following:
 - 4-adjacency a 4-path
 - **8-adjacency a 8-path**
 - m-adjacency a m-path
 - path length ?
 - Number of pixels involved

3	1	1	3 (q)
1	3	2	4
3	2	1	4
(p) 2	0	0	1

- **Answer:** *There are many 8-path between p and q*

Basic Relations between Pixels

Practice

- let's consider the $V = \{2, 3\}$ consider the following:
 - 4-adjacency a 4-path
 - 8-adjacency a 8-path
 - **m-adjacency a m-path**
 - path length ?
 - Number of pixels involved

3	1	1	3 (q)
1	3	2	4
3	2	1	4
(p) 2	0	0	1

- **Answer:** *There is only m-path between p and q*



Thank You