

Digital Systems

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IT131
Sequential Circuits – Part II

Lecture 8



Sequential Circuits Part II

Contents

1. Sequential circuits
2. Flip-flops
3. Analysis of clocked sequential circuits
4. Design procedure

Sequential Circuits Part II

Analysis of Clocked Sequential Circuits

- The behavior of a clocked sequential circuit is determined from the inputs, the outputs, and the state of flip-flops
- A clocked sequential circuit can be analyzed by using:
 - (i) The **state equation** obtained by replacing flip-flop input equations in its characteristic equation, or
 - (ii) a **state table**.
- A **state equation** (or **transition equation**) specifies the next state as a function of present state and inputs

Sequential Circuits Part II

- State table – a multiple variable table with the following four sections:
 - Present State – the values of the state variables at time (t) for each allowed state.
 - Input – the input combinations allowed.
 - Next-state – the value of the state at time $(t+1)$ based on the present state and the input
 - Output – the value of the output as a function of the present state and (**Mealy**) the input.

Sequential Circuits Part II

Analysis (Contd.)

- **Procedure:** Write the Boolean expression for each flip-flop input, which is, in fact, the flip-flop output right after the next active clock edge.
- Consider the circuit shown in next slide
- It consists of two D flip-flops A and B, an input x and an output y
- Since the D input of a flip-flop determines the value of the next state, it is possible to write a set of equations for the circuit

Sequential Circuits Part II

Analysis (Contd.)

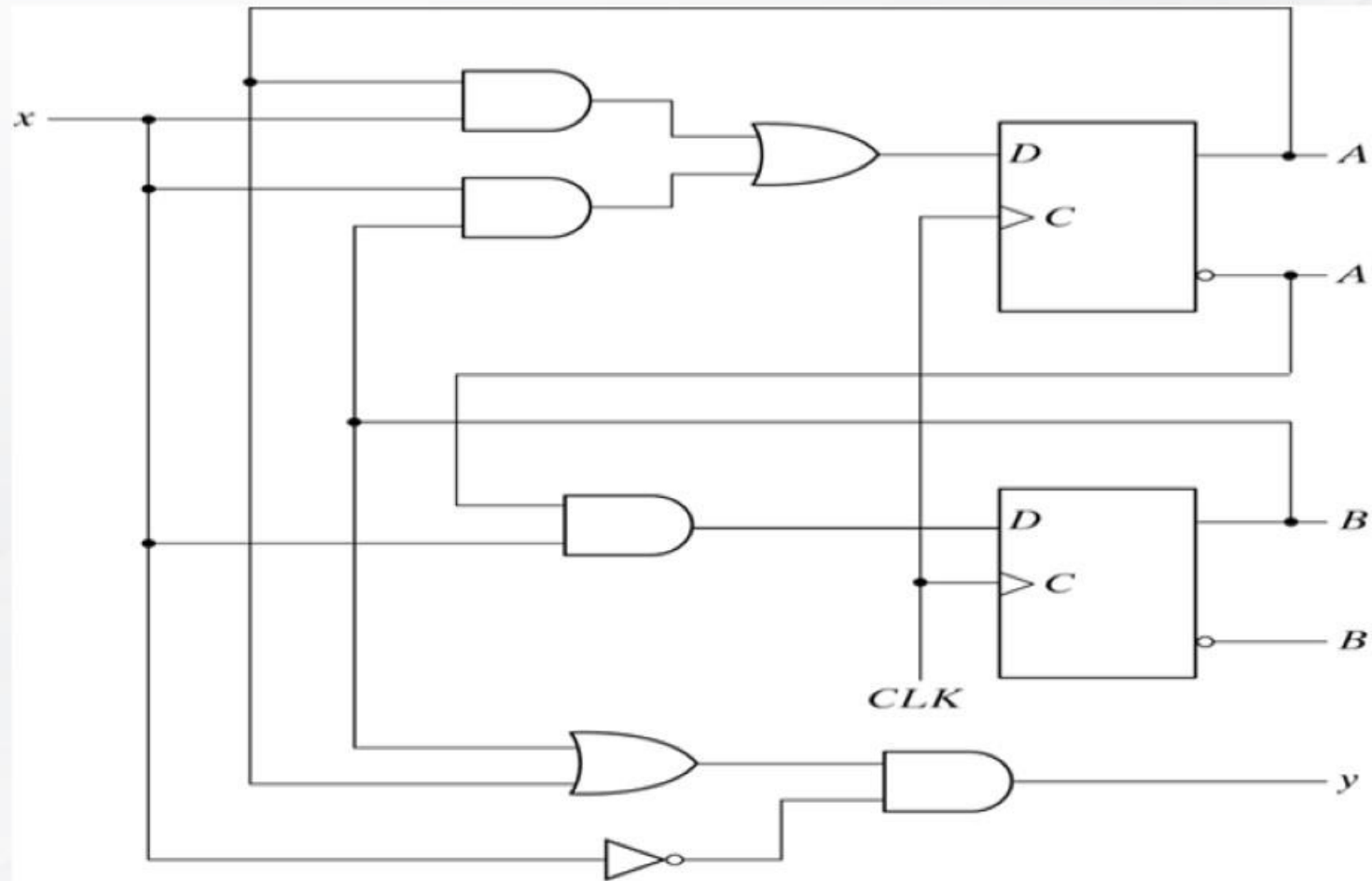


Fig. 5-15 Example of Sequential Circuit

Sequential Circuits Part II

Analysis (Contd.)

➤ **Flip-flop input equations** or excitation equations (for a circuit with D flip-flops, it is the same as the state equation):

$$D_A = Ax + Bx$$

$$D_B = A'x$$

$$y = (A + B)x'$$

➤ As the FF in use is the D FF:

➤ $Q(t+1) = D$

Sequential Circuits Part II

Analysis (Contd.)

- The state equation can be written as:

$$A(t+1) = A(t).x(t) + B(t).x(t)$$

$$B(t+1) = A'(t).x(t)$$

- Or simply:

$$A(t+1) = Ax + Bx$$

$$B(t+1) = A'x$$

- Similarly, the present state of output is:

$$y(t) = [A(t) + B(t)]x'(t),$$

or $y = (A + B)x'$

Sequential Circuits Part II

Analysis (Contd.)

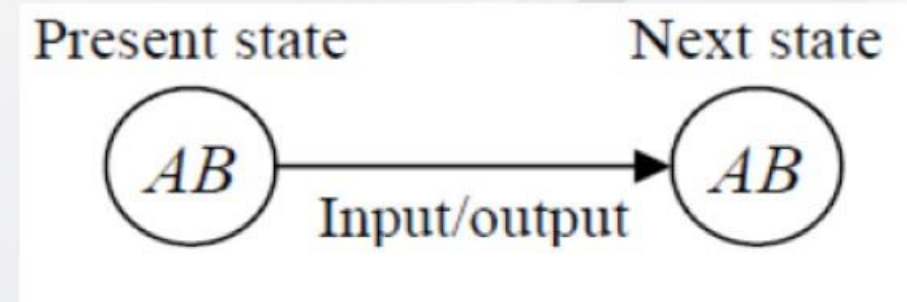
m flip-flops		n inputs	Next state		Output
Present state		Input			
A	B	x	A	B	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

2^{m+n} rows

Sequential Circuits Part II

Analysis (Contd.)

- A **state diagram** is a graphical representation of a state table



- represented by a circle, and the transitions between states are indicated by directed lines connecting the circles
- Number of states = **2^{number of Flip Flops}**
- The binary number inside each circle identifies the state of the flip-flops

Sequential Circuits Part II

Analysis (Contd.)

- The directed lines are labeled with two binary numbers separated by a slash
- The state diagram follows from the state table
- The state diagram for the given circuit:

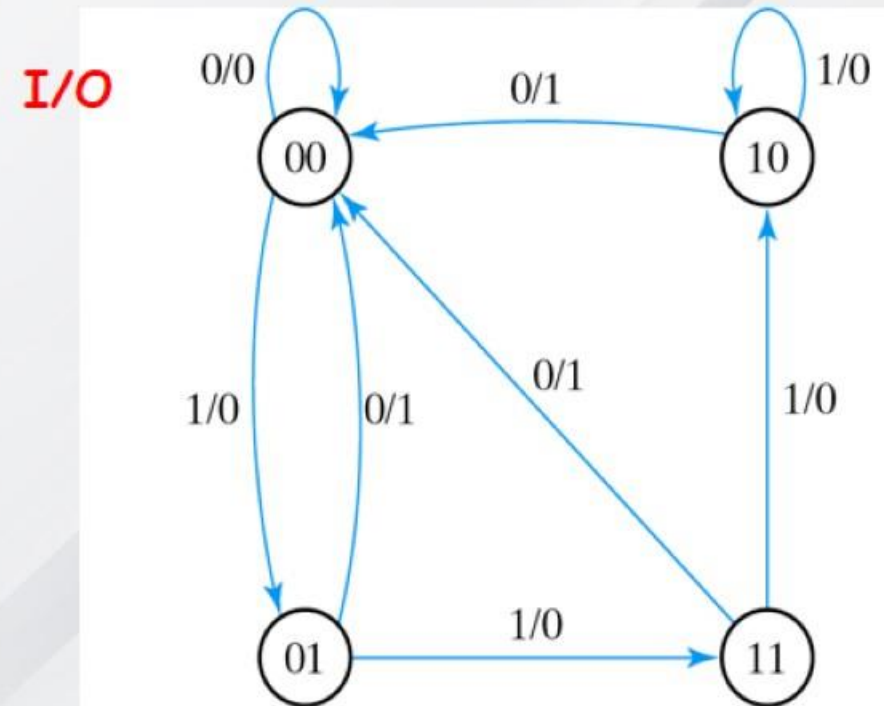


Fig. 5-16 State Diagram of the Circuit of Fig. 5-15

Sequential Circuits Part II

State Diagrams Types

■ NOTE:

- **Mealy model** - the output values are determined by both current state and current inputs.
- **Moore model** - the output values are determined **only** by current state.

Sequential Circuits Part II

Analysis of D FF: summary

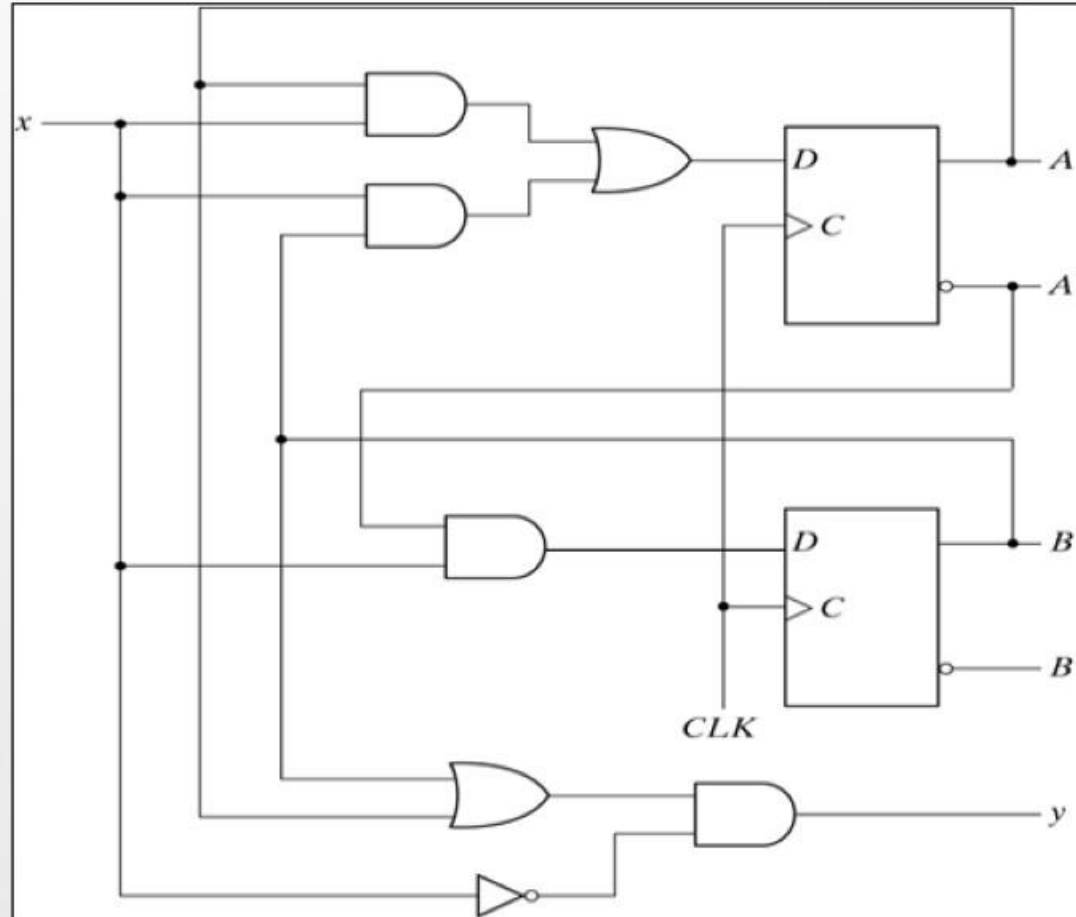


Fig. 5-15 Example of Sequential Circuit

1) FF Input equation:

$$D_A = Ax + Bx$$

$$D_B = A'x$$

2) State equation and output:

Characteristic equation:

$$A(t+1) = D_A, \quad B(t+1) = D_B$$

State equation:

$$A(t+1) = A(t).x(t) + B(t).x(t)$$

$$B(t+1) = A'(t).x(t)$$

Or
$$A(t+1) = Ax + Bx$$

$$B(t+1) = A'x$$

state of output is:

$$y = (A + B)x'$$

3) State table.

4) State diagram (Mealy)

Sequential Circuits Part II

Analysis with JK flip-flops

- The next state values of a sequential circuit that uses flip-flops such as JK can be derived using following procedure:
 - Determine the flip-flop input equations in terms of the present state and inputs
 - Use corresponding flip-flop characteristic equation to determine the next state values
- We consider the sequential circuit with two JK flip-flops A and B, and one input x

Sequential Circuits Part II

Analysis with JK flip-flops (Contd.)

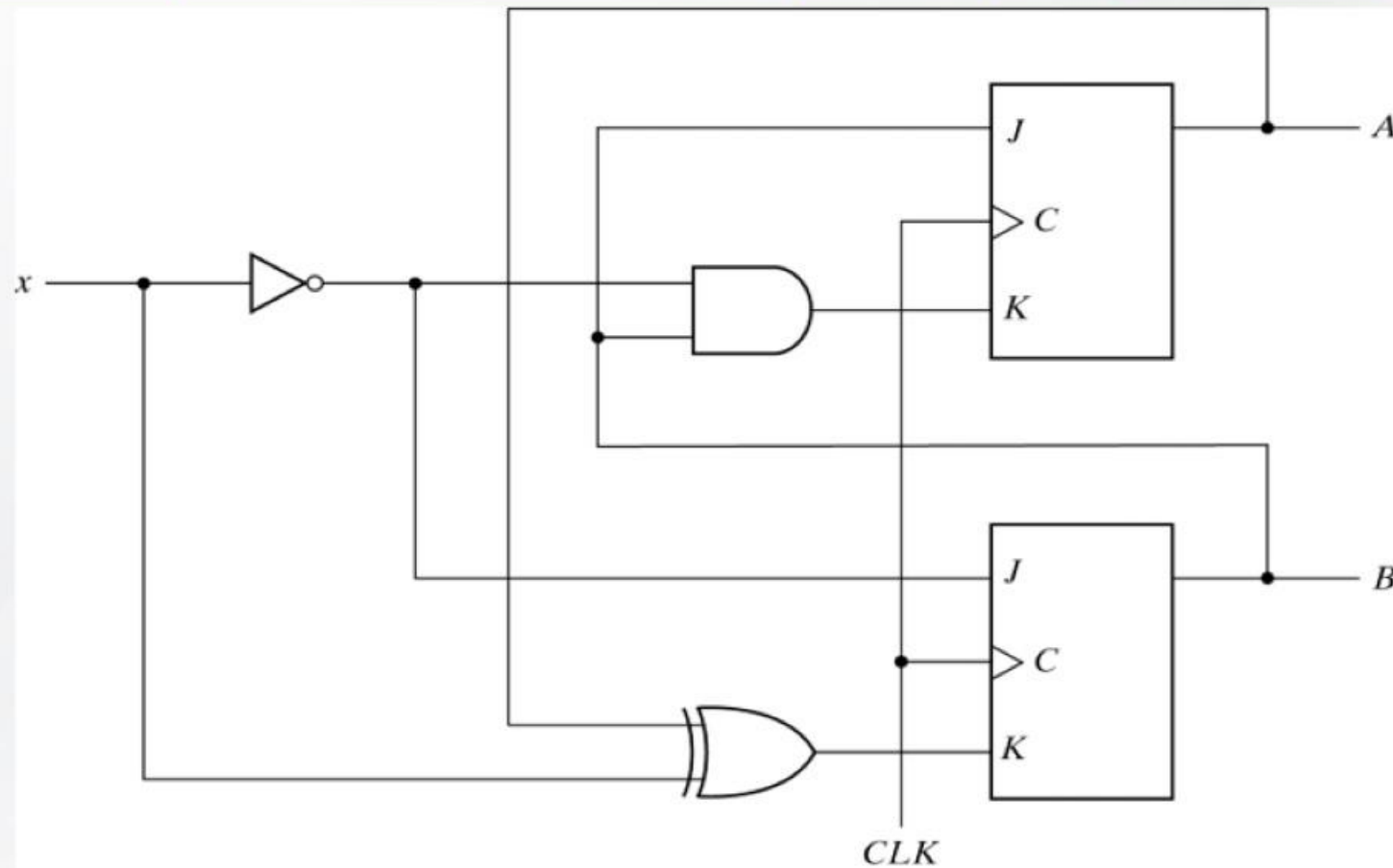


Fig. 5-18 Sequential Circuit with JK Flip-Flop

Sequential Circuits Part II

Analysis with JK flip-flops (Contd.)

- The circuit has no outputs and, so, the state table does not need an output column
- The outputs of the flip-flops may be considered as the outputs in this case
- The circuit can be specified by the flip-flop input equations

$$J_A = B$$

$$K_A = Bx'$$

$$J_B = x'$$

$$K_B = A'x + Ax' = A \oplus x$$

Sequential Circuits Part II

Analysis with JK flip-flops (Contd.)

- find the next state from the state equations by substituting the input equations into the flip-flop characteristic equations.
- Characteristic equation for the *JK* flip-flop is:

$$A(t+1) = J_A A' + K'_A A$$

$$B(t+1) = J_B B' + K'_B B$$

Sequential Circuits Part II

Analysis with JK flip-flops (Contd.)

➤ Substituting the values of J_A and K_A , and J_B and K_B , from the input equations, we obtain state equations for A and B as:

$$A(t+1) = BA' + (Bx')'A = A'B + AB' + Ax$$

$$B(t+1) = x'B' + (A \oplus x)'B = B'x' + ABx + A'Bx'$$

Sequential Circuits Part II

Analysis with JK flip-flops (Contd.)

Present state		Input	Next State	
<i>A</i>	<i>B</i>	<i>x</i>	<i>A</i>	<i>B</i>
0	0	0	0	1
0	0	1	0	0
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	1	1

$$A(t+1) = A'B + AB' + Ax$$

$$B(t+1) = B'x' + ABx + A'Bx'$$

Sequential Circuits Part II

Analysis with JK flip-flops (Contd.)

➤ The state diagram of the sequential circuit is shown right-side. Since the circuit has no outputs, the directed lines out of circles are marked with one binary number only to designate the value of input x .

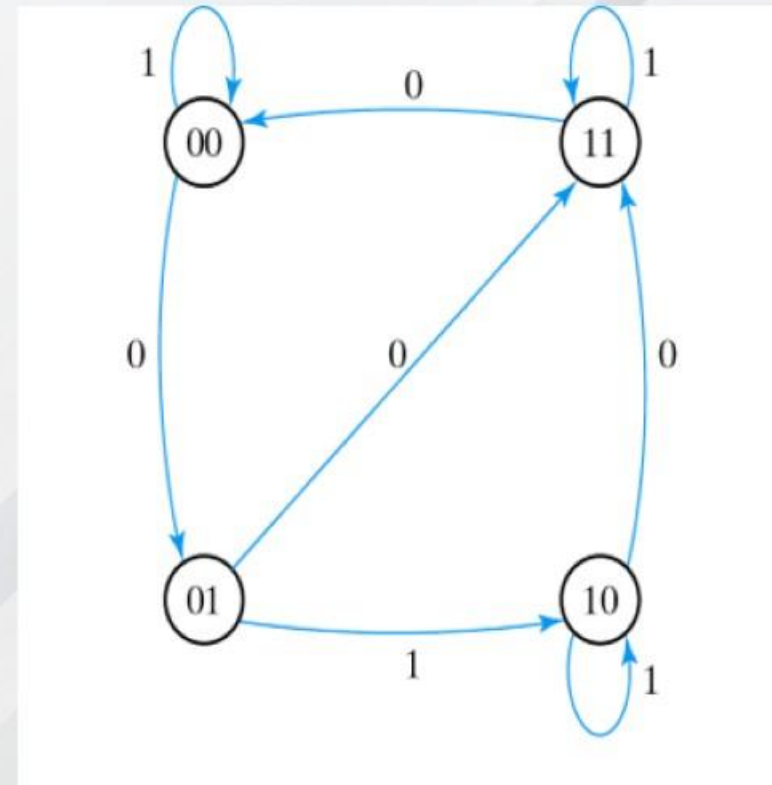
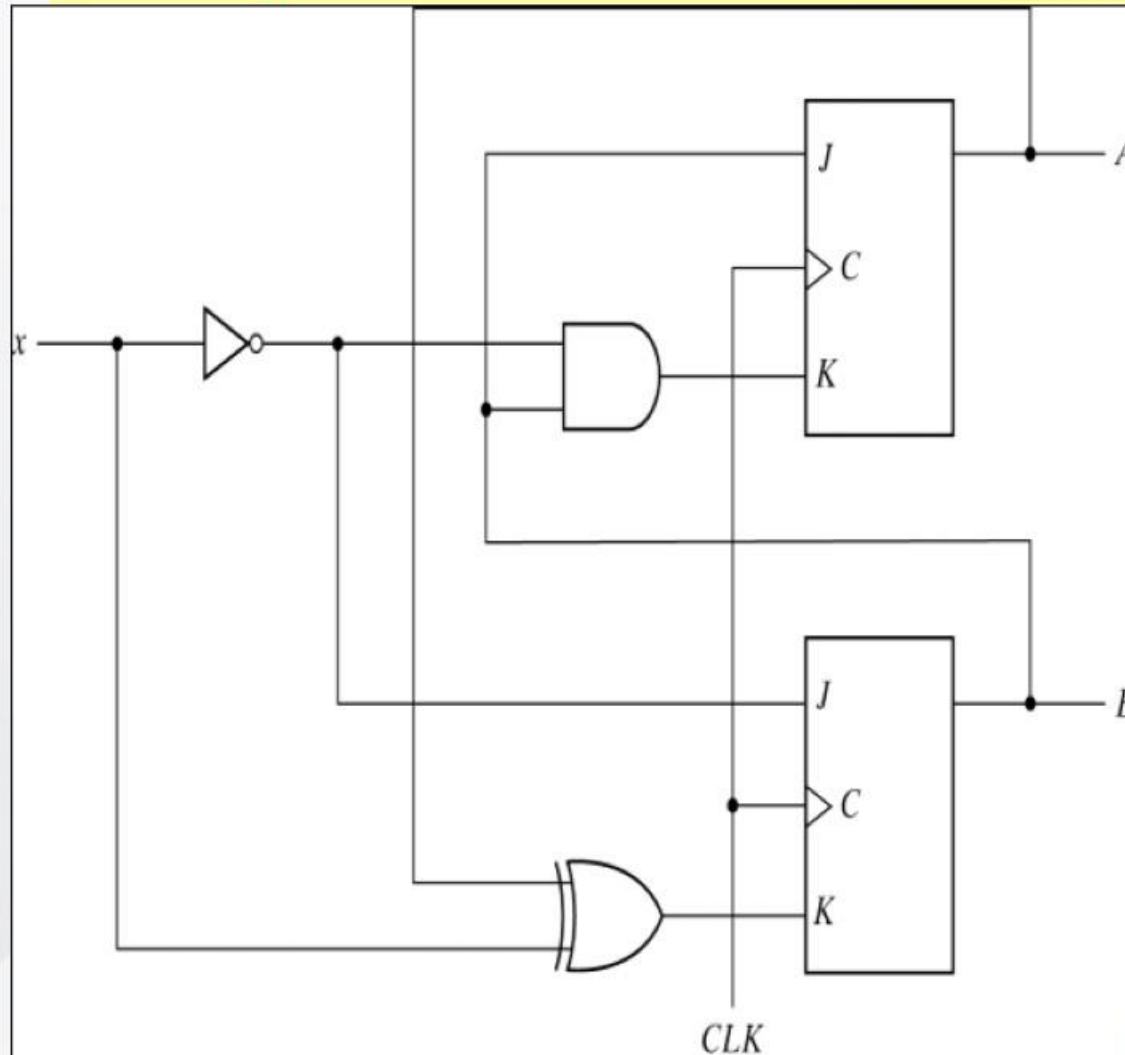


Fig. 5-19 State Diagram of the Circuit of Fig. 5-18

Sequential Circuits Part II

Analysis of JK FF: summary



1) FF Input equation:

$$J_A = B \quad K_A = Bx'$$

$$J_B = x' \quad K_B = A \oplus x$$

2) State equation and output:

Characteristic equation:

$$A(t+1) = J_A A' + K'_A A$$

$$B(t+1) = J_B B' + K'_B B$$

State equation:

$$A(t+1) = BA' + (Bx')'A$$

$$= A'B + AB' + Ax$$

$$B(t+1) = x'B' + (A \oplus x)'B$$

$$= B'x' + ABx + A'Bx'$$

3) State table.

4) State diagram.

Sequential Circuits Part II

Analysis with T flip-flops

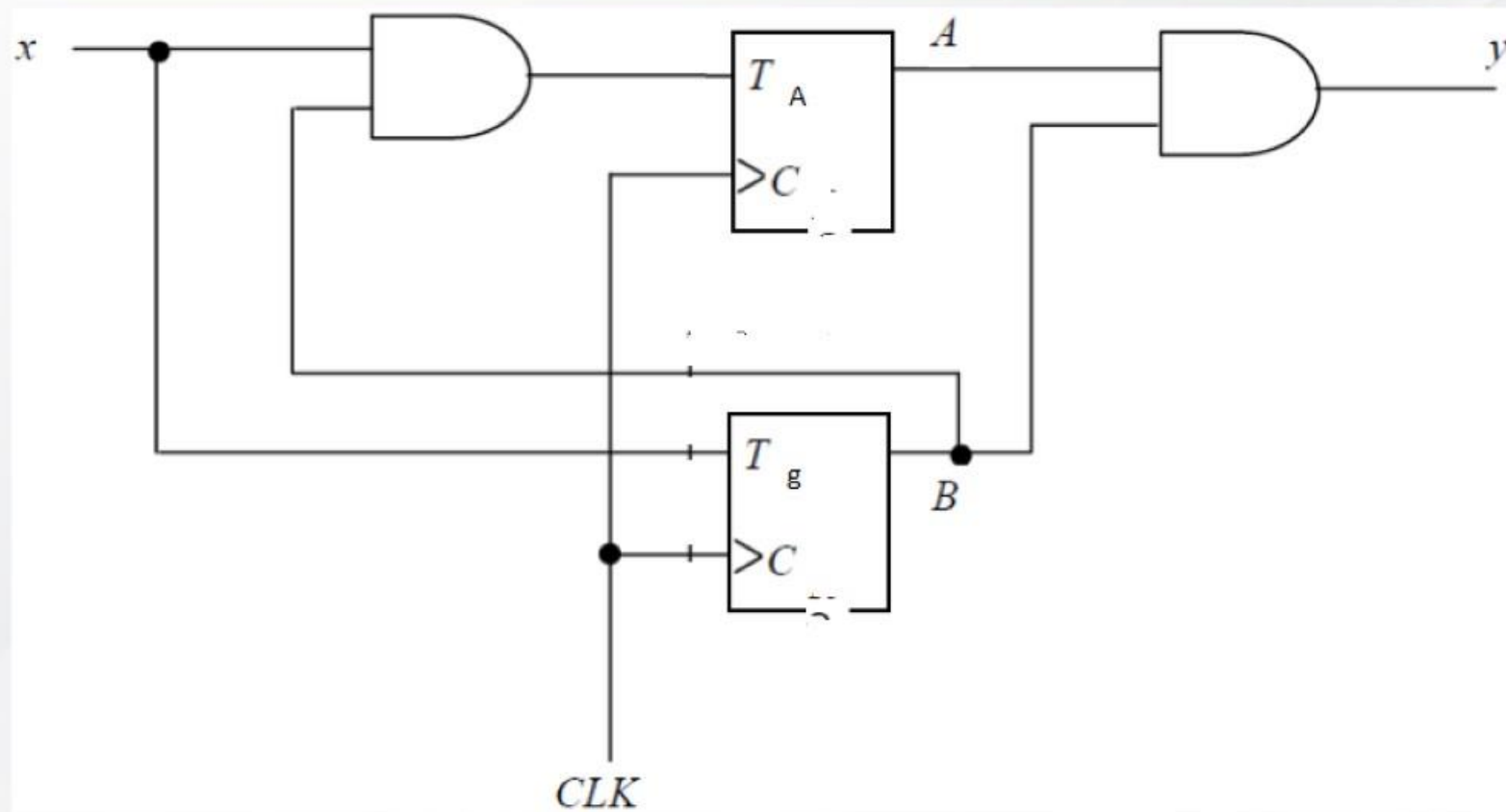
- Similar to the analysis with *JK* flip-flops, the state table can be obtained by using characteristic table or the characteristic equation

$$Q(t+1) = T \oplus Q = T'Q + TQ'$$

- Consider the sequential circuit given in the next slide
- It has two flip-flops A and B, one input x, and one output y

Sequential Circuits Part II

Analysis with T flip-flops (Contd.)



Sequential Circuits Part II

Analysis with T flip-flops (Contd.)

- It can be described algebraically by two input equations and one output equation

$$T_A = Bx, \quad T_B = x, \quad y = AB$$

- The characteristic equations are

$$A(t+1) = T_A \oplus A, \quad B(t+1) = T_B \oplus B$$

- Substituting the input equations with characteristic equations we get

$$A(t+1) = (Bx)'A + (Bx)A' = AB' + Ax' + A'Bx$$

$$B(t+1) = x \oplus B$$

Sequential Circuits Part II

Analysis with T flip-flops (Contd.)

➤ The state table is as follows:

Present state		Input	Next State		Output
<i>A</i>	<i>B</i>	<i>x</i>	<i>A</i>	<i>B</i>	<i>y</i>
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	1	0
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	1

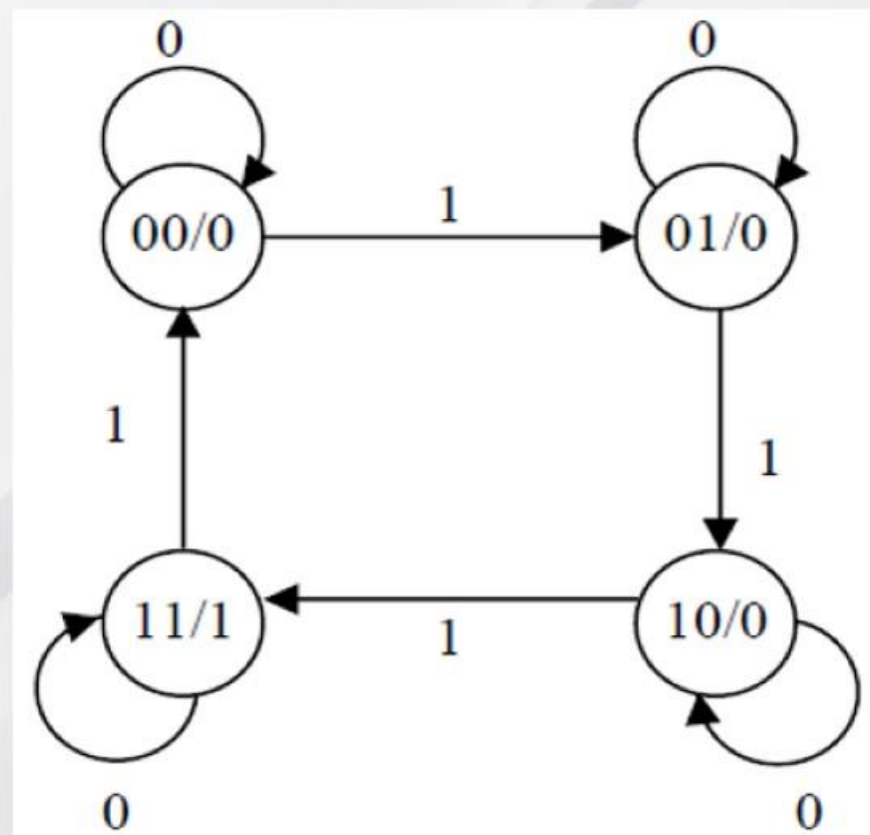
$$A(t+1) = AB' + Ax' + A'Bx$$

$$B(t+1) = x \oplus B$$

Sequential Circuits Part II

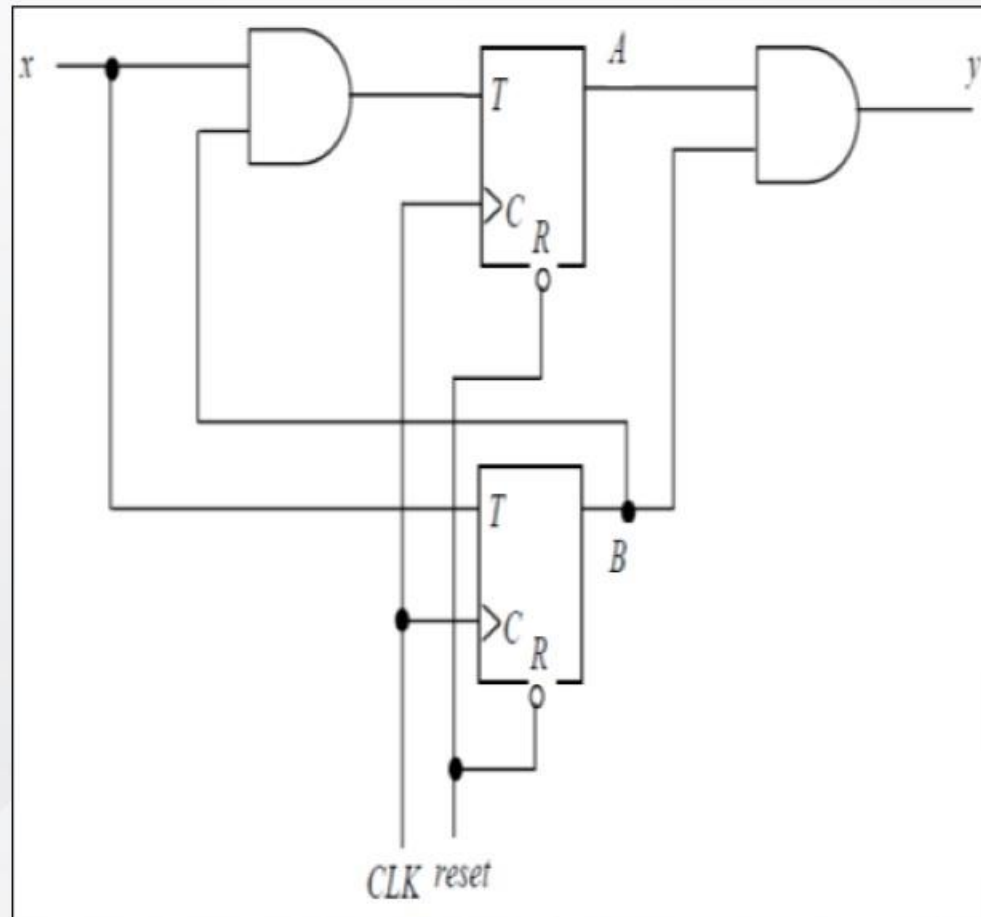
Analysis with T flip-flops (Contd.)

- The state diagram is in right-side
- The output is only a function of current state, not the input, and that is why the output is written inside state circles



Sequential Circuits Part II

Analysis of T FF: summary



1) FF Input equation:

$$T_A = Bx, \quad T_B = x,$$

2) State equation and output:

Characteristic equation:

$$A(t+1) = T'_A A + T_A A'$$

$$B(t+1) = T'_B B + T_B B'$$

State equation:

$$\begin{aligned} A(t+1) &= (Bx)'A + (Bx)A' \\ &= AB' + Ax' + A'Bx \end{aligned}$$

$$\begin{aligned} B(t+1) &= x'B + xB' \\ &= x \oplus B \end{aligned}$$

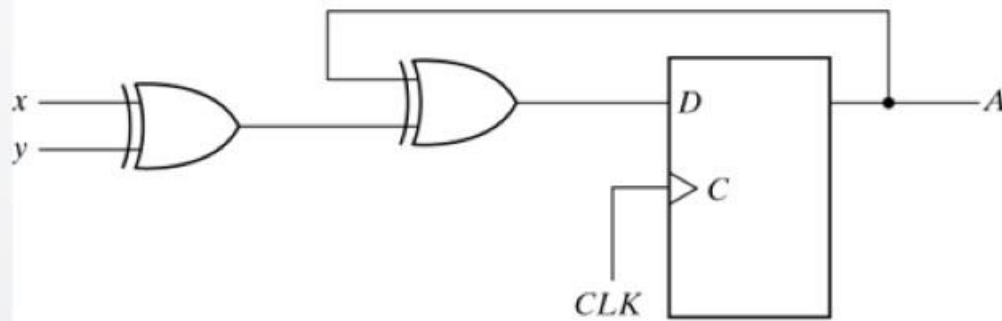
State of the output:

$$y = AB$$

3) State table.

4) State diagram (Moore)

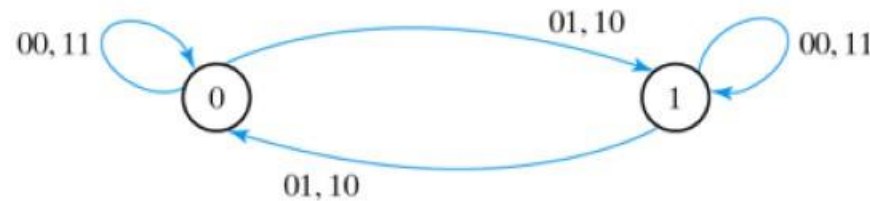
Sequential Circuits Part II



(a) Circuit diagram

Present state	Inputs		Next state
A	x	y	A
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

(b) State table



(c) State diagram

Fig. 5-17 Sequential Circuit with D Flip-Flop

A sequential circuit with one D flip-flops . two inputs x and y , and one output A is specified by the following logic diagram of the circuit.

- List the state table for the sequential circuit.
- Draw the corresponding state diagram.

$$D_A = A \oplus x \oplus y$$

$$A(t+1) = A \oplus x \oplus y$$

Sequential Circuits Part II

A sequential circuit with two D flip-flops A and B, two inputs x and y, and one output z is specified by the following next-state and output equations:

$$A(t+1) = x'y + xA$$

$$B(t+1) = x'B + xA$$

$$Z = B$$

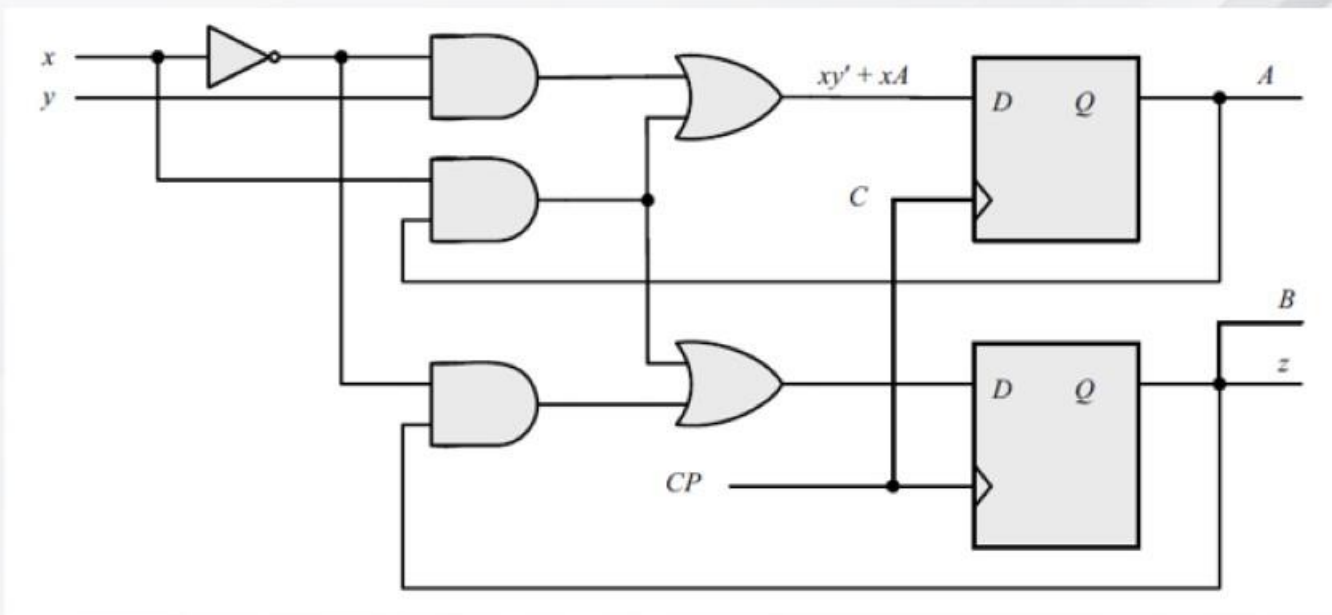
- 1- Draw the logic diagram of the circuit.
- 2- List the state table for the sequential circuit.
- 3- Draw the corresponding state diagram.

Sequential Circuits Part II

$$A(t+1) = x'y + xA$$

$$B(t+1) = x'B + xA$$

$$Z = B$$



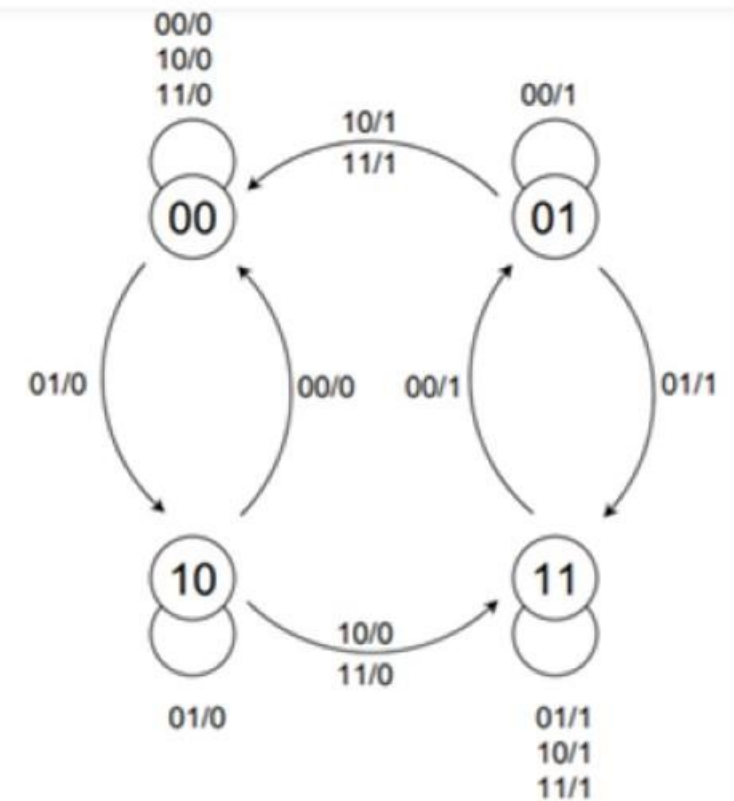
Sequential Circuits Part II

$$A(t+1) = x'y + xA$$

$$B(t+1) = x'B + xA$$

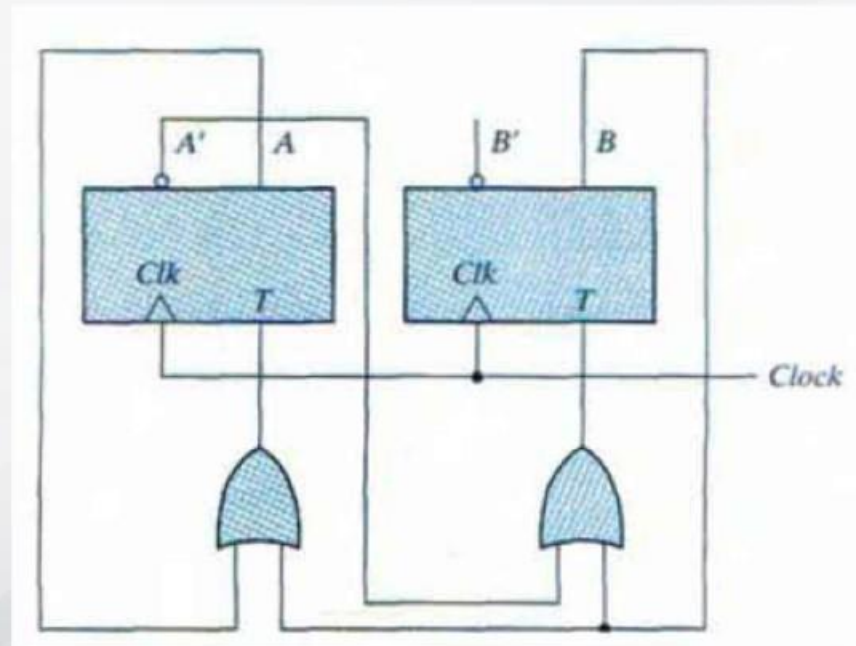
$$Z = B$$

Present State	Inputs	Next State	Output
AB	xy	AB	z
00	00	00	0
00	01	10	0
00	10	00	0
00	11	00	0
01	00	01	1
01	01	11	1
01	10	00	1
01	11	00	1
10	00	00	0
10	01	10	0
10	10	11	0
10	11	11	0
11	00	01	1
11	01	11	1
11	10	11	1
11	11	11	1



Sequential Circuits Part II

Derive the state table and the state diagram of the sequential circuit shown in the following Figure.



Sequential Circuits Part II

<i>Present state</i>		<i>Next state</i>	
<i>A</i>	<i>B</i>	<i>A</i>	<i>B</i>
0	0	0	1
0	1	1	0
1	0	0	0
1	1	0	0

$$T_A = (A + B)$$

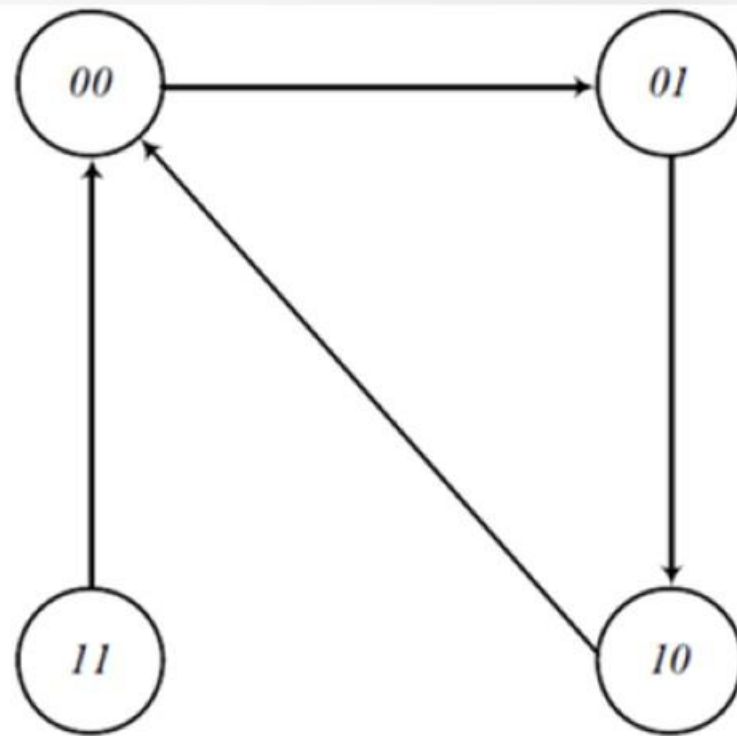
$$T_B = (A' + B)$$

$$A(t+1) = T_A \oplus A$$

$$= (A + B) \oplus A$$

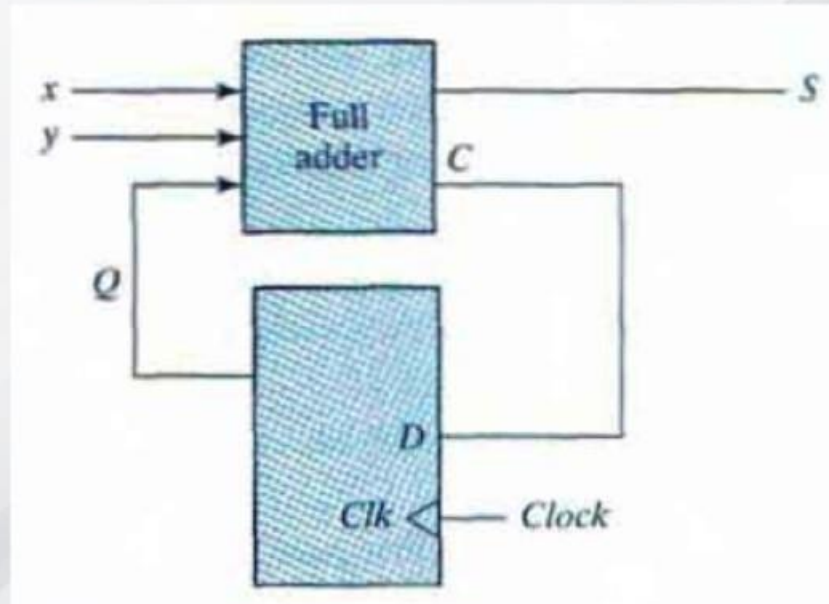
$$B(t+1) = T_B \oplus B$$

$$= (A' + B) \oplus B$$

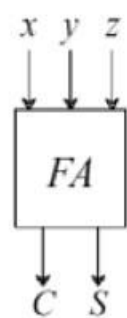


Sequential Circuits Part II

A sequential circuit has one flip-flop Q , two inputs x and y , and one output S . It consists of a full adder circuit connected to a D flip-flop, as shown in the following Figure. Derive the state table and state diagram of the sequential circuit.



Sequential Circuits Part II



x	y	z	S	C
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = z \oplus (x \oplus y)$$

$$C = z(x \oplus y) + xy$$



Output Equation

$$S = Q \oplus (x \oplus y)$$

Input Equation:

$$D = C$$

$$C = Q(x \oplus y) + xy$$

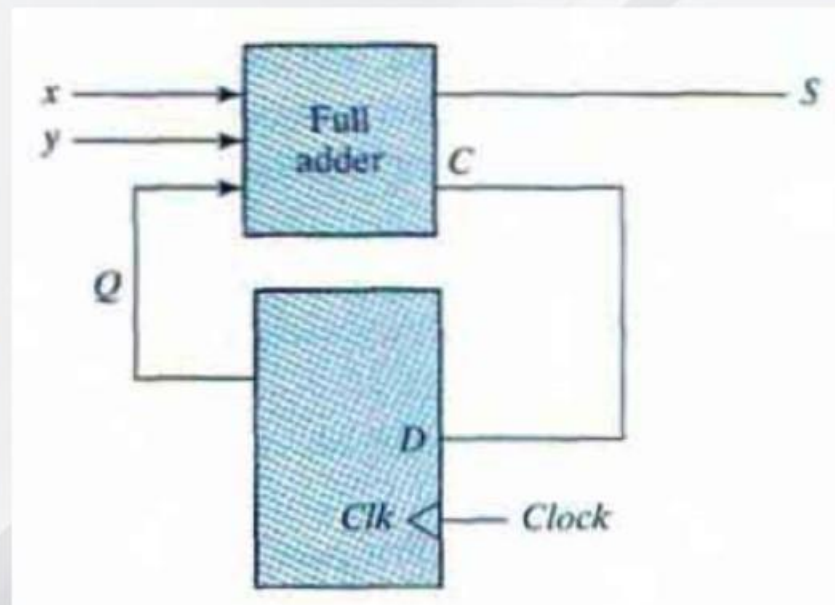
$$D = Q(x \oplus y) + xy$$

Since $Q(t+1) = D$

State Equation

$$Q(t+1) = C$$

$$= Q(x \oplus y) + xy$$



Sequential Circuits Part II

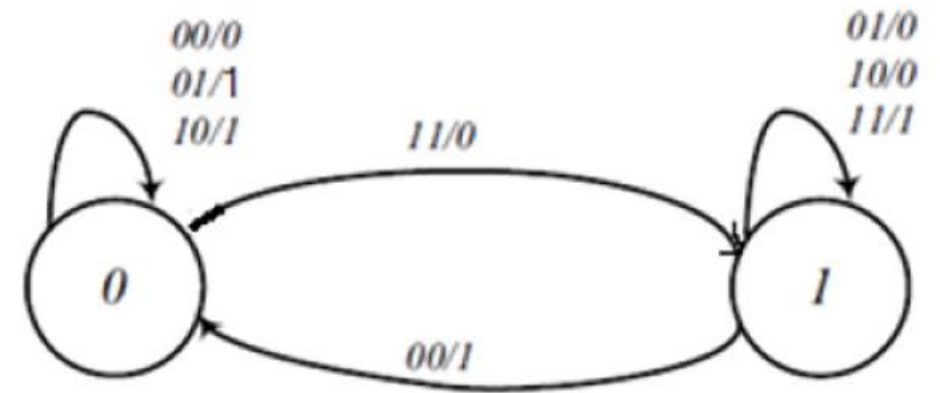
Present state	Inputs		Next state	Output
Q	x	y	Q	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

State Equation:

$$Q(t+1) = C$$

Output Equation:

$$S = Q \oplus (x \oplus y)$$



Sequential Circuits Part II

- A sequential circuit has two JK flip-flops A and B and one input x. The circuit is described by the following flip-flop input equation.

$$\begin{aligned} J_A &= x & K_A &= B' \\ J_B &= x & K_B &= A \end{aligned}$$

1. Derive the state equations $A(t+1)$ and $B(t+1)$ by Substituting the input equations for the J and K variables,
2. Draw the state diagram of the circuit.

Sequential Circuits Part II

State Equations:

$$A(t+1) = J_A A' + K'_A A = xA' + BA$$

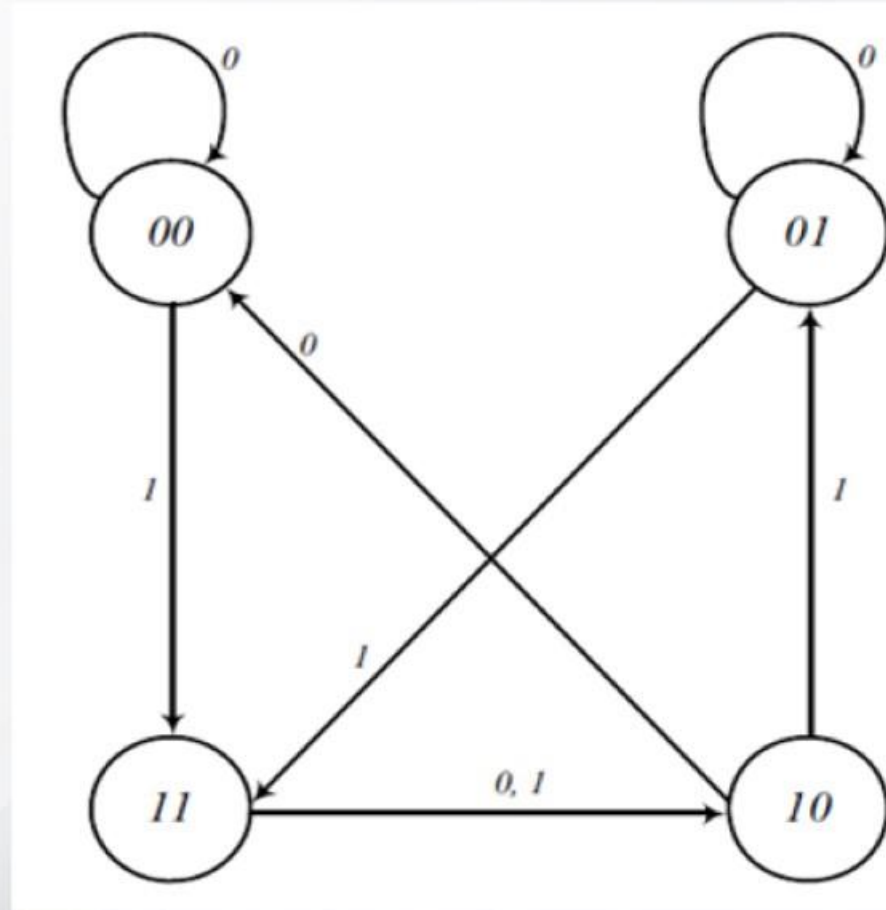
$$B(t+1) = J_B B' + K'_B B = xB' + A'B$$

State Table:

Present State			Next State	
A	B	x	A	B
0	0	0	0	0
0	0	1	1	1
0	1	0	0	1
0	1	1	1	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	0
1	1	1	1	0

Sequential Circuits Part II

State Diagram





Thank You