

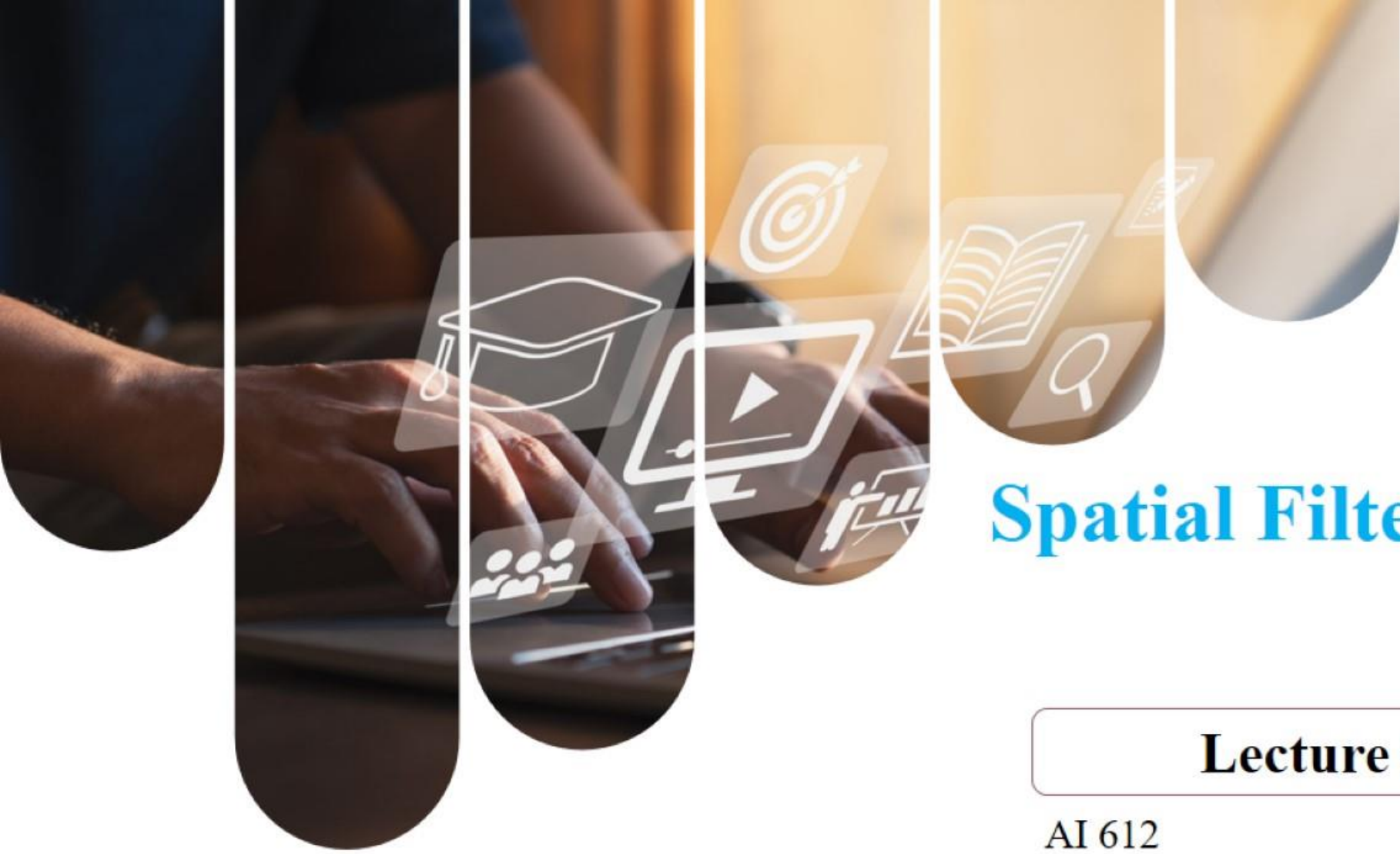
Computer Vision, and Digital Image Processing

Prepared by Dr. Mohamed Mostafa

AI 612

Lecture 5



The background features five vertical panels of varying heights and widths. The first panel on the left is dark blue. The second panel shows hands typing on a laptop. The third panel shows a laptop screen with a play button icon. The fourth panel shows an open book and a magnifying glass icon. The fifth panel is light blue. Overlaid on these panels are several white line-art icons: a graduation cap, a target with an arrow, a play button, a magnifying glass, and a group of people.

Spatial Filtering

Lecture 5

AI 612

Prepared by
Dr. Mohamed Mostafa

Chapter
03

Spatial Filtering

Resources for this
lecture

Chapter 3:

Gonzalez, Rafael C. *Digital image processing*.
Pearson education, India, 2009.



Smoothing filter

Linear Filters

Box Filter

Weighted Filter

Gaussian Filter

Non-linear Filters

Median Filter

Max Filter

Min Filter

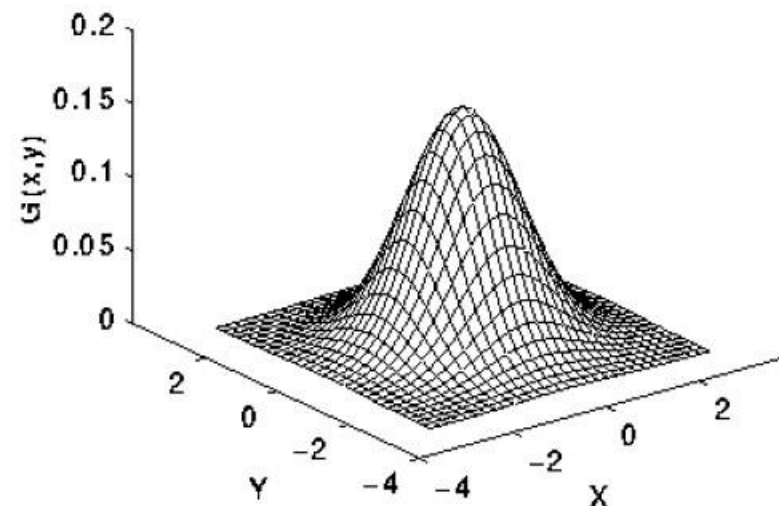
Gaussian Smoothing filter

- The filter is implemented as an Odd sized Symmetric Kernel
- This process smooths the image by reducing high-frequency noise and preserving low-frequency components.
- The Gaussian filter is characterized by a bell-shaped curve, with higher weights assigned to the central pixel and gradually decreasing weights towards the outer pixels.

$$\frac{1}{16}$$

1	2	1
2	4	2
1	2	1

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



Gaussian Smoothing filter

1	2	1		
2	4	2	60	70
1	2	1	60	70
	10	20	60	70
	10	20	60	70

$$\frac{1}{16}$$

1	2	1
2	4	2
1	2	1

Gaussian Smoothing filter

	1	2	1	
10	2	4	2	70
10	1	2	1	70
10				70
10				70

$$\frac{1}{16}$$

1	2	1
2	4	2
1	2	1

7.5			

Gaussian Smoothing filter

		1	2	1	
		2	4	2	
10	20	60	70		
10	20	60	70		
10	20	60	70		
10	20	60	70		

$$\frac{1}{16}$$

1	2	1
2	4	2
1	2	1

7.5	19.4		

Gaussian Smoothing filter

10 ¹	20 ²	60 ¹	70
10 ²	20 ⁴	60 ²	70
10 ¹	20 ²	60 ¹	70
10	20	60	70

$$\frac{1}{16}$$

1	2	1
2	4	2
1	2	1

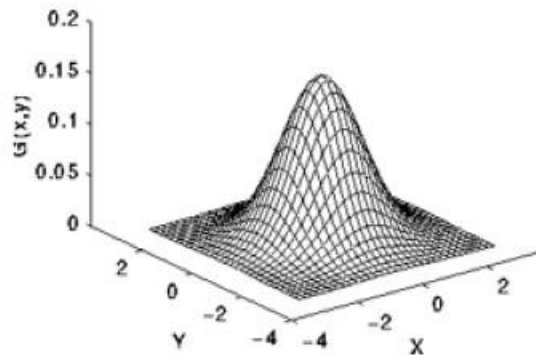
7.5	19.4	39.4	37.5
	27.5		

Gaussian Smoothing filter

10	20 ¹	60 ²	70 ¹
10	20 ²	60 ⁴	70 ²
10	20 ¹	60 ²	70 ¹
10	20	60	70

$$\frac{1}{16}$$

1	2	1
2	4	2
1	2	1



7.5	19.4	39.4	37.5
	27.5	52.5	

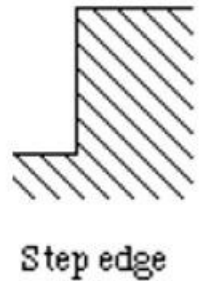
Sharpening Filters

- Sharpening highlights transitions in intensity.
- Image blurring could be accomplished in the spatial domain by pixel averaging (smoothing) in a neighborhood, averaging is analogous to integration.
- Sharpening can be accomplished by spatial differentiation.
- Derivatives of a digital function are defined in terms of differences
- sharpening is often referred to as highpass filtering.
- High frequencies (which are responsible for fine details) are passed, while low frequencies are attenuated or rejected.

Sharpening Filters

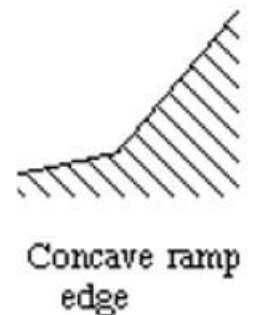
- **Step Discontinuity:**

- A step discontinuity occurs when there is an sudden change in intensity from one region of an image to another.
- It appears as a sharp edge or boundary between two distinct areas of different intensity levels.
- Examples include the edge of an object in an image or the border between two contrasting regions.



- **Ramp Discontinuity:**

- A ramp discontinuity is characterized by a gradual change in intensity across an image.
- It can occur when the intensity changes smoothly across an area, such as a gradient or slope in brightness.



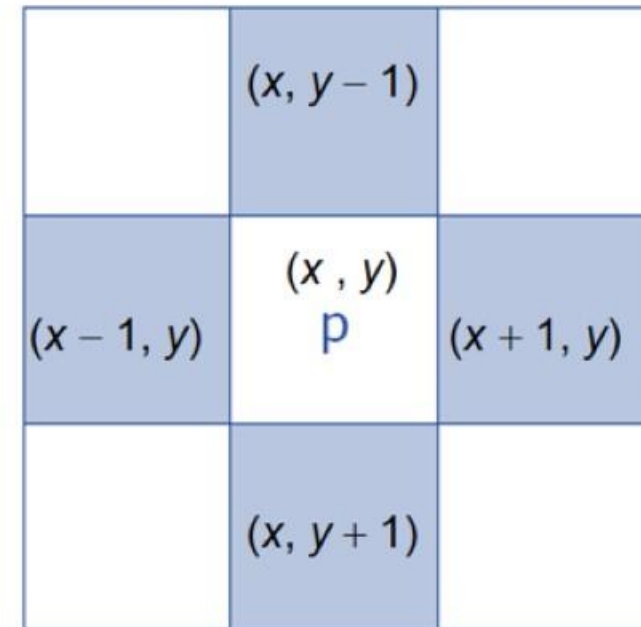
Sharpening Filters

first-order derivative

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

second-order derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$



Sharpening Filters

first-order derivative

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

40	50	70	100	100
----	----	----	-----	-----

10	20	30	0	
----	----	----	---	--

second-order derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

	10	10	-30	
--	----	----	-----	--

Sharpening Filters

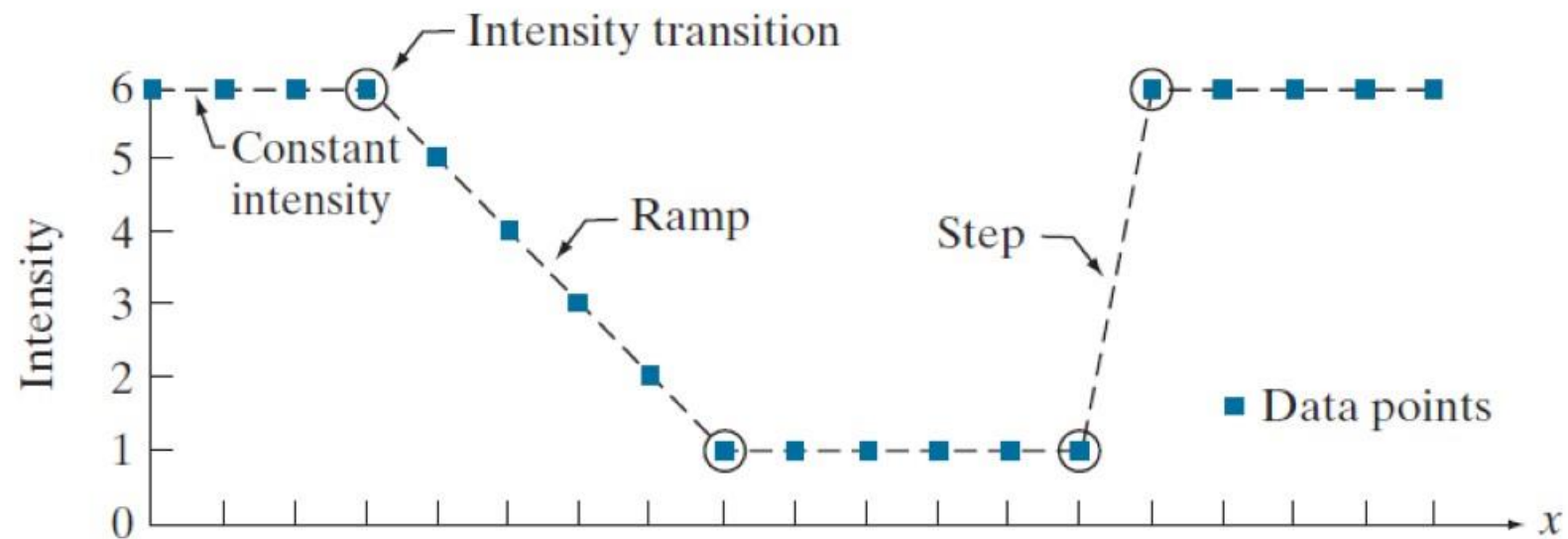
use for **a *first derivative***:

1. Must be zero in areas of constant intensity.
2. Must be nonzero at the onset of an intensity step or ramp.
3. Must be nonzero along intensity ramps.

Similarly, any definition of **a *second derivative***

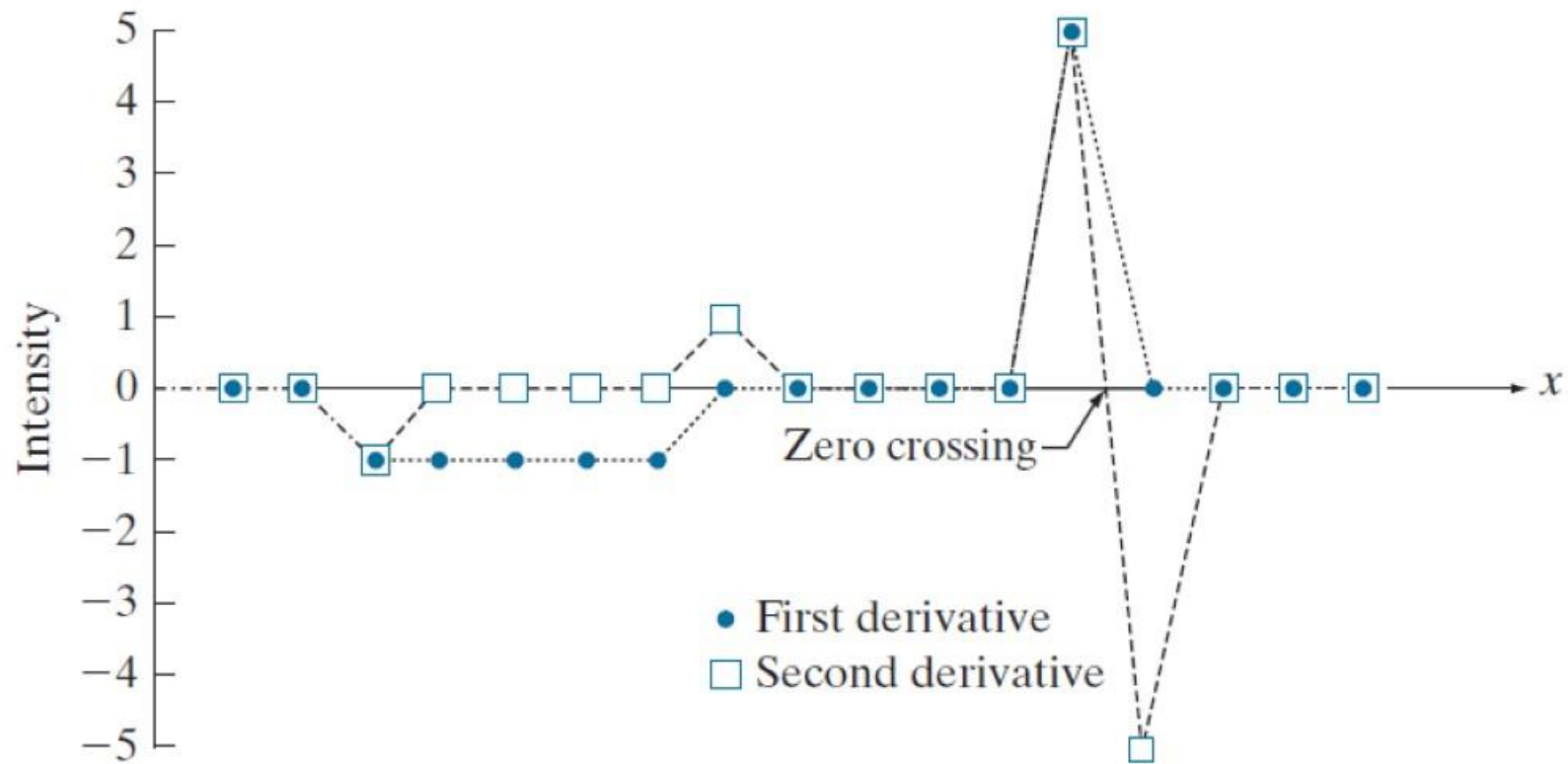
1. Must be zero in areas of constant intensity.
2. Must be nonzero at the onset *and* end of an intensity step or ramp.
3. Must be zero along intensity ramps.

Sharpening Filters



Values of scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6	x
1st derivative	0	0	-1	-1	-1	-1	-1	0	0	0	0	0	0	5	0	0	0	0	0	
2nd derivative	0	0	-1	0	0	0	0	1	0	0	0	0	0	5	-5	0	0	0	0	

Sharpening Filters



Sharpening Filters

The 2nd derivative is more useful for image enhancement than the 1st derivative

- Stronger response to fine detail, because it provides information about the rate of change of the image intensity.
- We can detect edges and other significant features more effectively compared to the first derivative.

Sharpening Filters- The Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f(x, y) = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

Sharpening Filters- The Laplacian

20	50	30	0
20	40	60	70
30	20	60	70
67	60	75	80

$$\nabla^2 f(x, y) = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y) = -10$$

Sharpening Filters- The Laplacian

0	1	0	1	1	1	0	-1	0	-1	-1	-1
1	-4	1	1	-8	1	-1	4	-1	-1	8	-1
0	1	0	1	1	1	0	-1	0	-1	-1	-1

$$\nabla^2 f(x, y) = f(x + 1, y) + f(x - 1, y) + f(x, y + 1) + f(x, y - 1) - 4f(x, y)$$

Sharpening Filters- The Laplacian

0	1	0
1	-4 -ve	1
0	1	0

1	1	1
1	-8 -ve	1
1	1	1

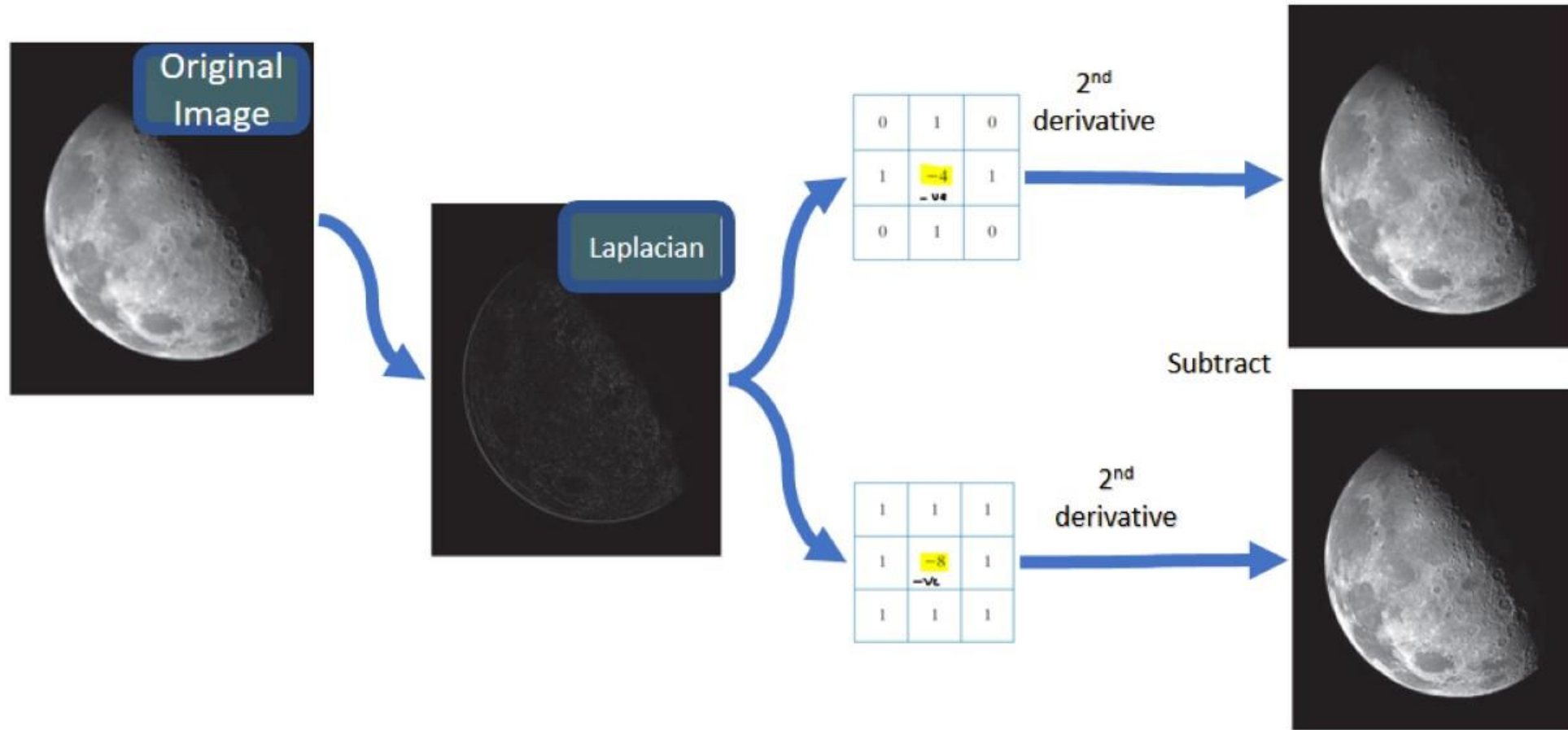
0	-1	0
-1	4 +ve	-1
0	-1	0

-1	-1	-1
-1	8 +ve	-1
-1	-1	-1

$$g(x, y) = f(x, y) - c[\nabla^2 f(x, y)]$$

$$g(x, y) = f(x, y) + c[\nabla^2 f(x, y)]$$

Sharpening Filters- The Laplacian



Sharpening Filters- The Composite Laplacian

Step 1

$$\nabla^2 f(x, y) = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

Step 2

$$g(x, y) = f(x, y) + c[\nabla^2 f(x, y)]$$

$$\begin{aligned} g(x, y) &= f(x, y) - [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) + 4f(x, y)] \\ &= 5f(x, y) - [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] \end{aligned}$$

Sharpening Filters- The Composite Laplacian

0	-1	0
-1	5	-1
0	-1	0

-1	-1	-1
-1	9	-1
-1	-1	-1

Sharpening Filters- The Laplacian vs The Composite

	0	-1	0		
	-1	4	-1	60	70
	0	-1	0	60	70
		10	20	60	70
		10	20	60	70

$$\nabla^2 f(x, y)$$

$$g(x, y) = f(x, y) + c [\nabla^2 f(x, y)]$$

Sharpening Filters- The Laplacian vs The Composite

	0	-1	0		
-1	10 ⁴	20 ⁻¹	60	70	
0	10 ⁻¹	20 ⁰	60	70	
	10	20	60	70	
	10	20	60	70	

10			

$$\nabla^2 f(x, y)$$

20			

$$g(x, y) = f(x, y) + c [\nabla^2 f(x, y)]$$

Sharpening Filters- The Laplacian vs The Composite

	0	-1	0	
	-1	4	-1	
10	20	60	70	
10	20	60	70	
10	20	60	70	
10	20	60	70	

10	-10		

$$\nabla^2 f(x, y)$$

20	10		

$$g(x, y) = f(x, y) + c [\nabla^2 f(x, y)]$$

Sharpening Filters- The Laplacian vs The Composite

			0	-1	0
			-1	4	-1
10	20	60	70		
		0	-1	0	
10	20	60	70		
10	20	60	70		
10	20	60	70		

10	-10	30	150

$$\nabla^2 f(x, y)$$

20	10	90	220

$$g(x, y) = f(x, y) + c [\nabla^2 f(x, y)]$$

Sharpening Filters- The Laplacian vs The Composite

10	20 ⁰	60 ⁻¹	70 ⁰
10	20 ⁻¹	60 ⁴	70 ⁻¹
10	20 ⁰	60 ⁻¹	70 ⁰
10	20	60	70

10	-10	90	150
		30	

$$\nabla^2 f(x, y)$$

20	10	150	220
		90	

$$g(x, y) = f(x, y) + c [\nabla^2 f(x, y)]$$

Sharpening Filters- The Laplacian vs The Composite

10	20 ⁰	60 ⁻¹	70 ⁰
10	20 ⁻¹	60 ⁴	70 ⁻¹
10	20 ⁰	60 ⁻¹	70 ⁰
10	20	60	70

10	-10	90	150
		30	

$$\nabla^2 f(x, y)$$

20	10	150	220
		90	

$$g(x, y) = f(x, y) + c [\nabla^2 f(x, y)]$$

Sharpening Filters- The Laplacian vs The Composite

	0	-1	0		
	-1	5	-1	60	70
	0	-1	0	60	70
		10	20	60	70
		10	20	60	70

The Composite

Sharpening Filters- The Laplacian vs The Composite

	0	-1	0		
	-1	5	-1	60	70
	0	-1	0	60	70
		10	20	60	70
		10	20	60	70

20			

The Composite

Sharpening Filters- The Laplacian vs The Composite

	0	-1	0	
	-1	5	-1	
10	20	60	70	
10	20	60	70	
10	20	60	70	
10	20	60	70	

20	10		

The Composite

Sharpening Filters- The Laplacian vs The Composite

		0	-1	0
		-1	5	-1
10	20	60	70	
	0	-1	0	
10	20	60	70	
10	20	60	70	
10	20	60	70	

20	10	150	

The Composite

Sharpening Filters- The Laplacian vs The Composite

			0	-1	0
			-1	5	-1
10	20	60	70		
		0	-1	0	
10	20	60	70		
10	20	60	70		
10	20	60	70		

20	10	150	220

The Composite

Sharpening Filters- The Laplacian vs The Composite

0	10 ⁻¹	20 ⁰	60	70
-1	10 ⁵	20 ⁻¹	60	70
0	10 ⁻¹	20 ⁰	60	70
	10	20	60	70

20	10	150	220
10	-10		

The Composite

Sharpening Filters- The Laplacian vs The Composite

10 ⁰	20 ⁻¹	60 ⁰	70
10 ⁻¹	20 ⁵	60 ⁻¹	70
10 ⁰	20 ⁻¹	60 ⁰	70
10	20	60	70

20	10	150	220
10	-10		

The Composite

Sharpening Filters- The Laplacian vs The Composite

10	20 ⁰	60 ⁻¹	70 ⁰
10	20 ⁻¹	60 ⁵	70 ⁻¹
10	20 ⁰	60 ⁻¹	70 ⁰
10	20	60	70

20	10	150	220
10	-10	90	

The Composite

Sharpening Filters- The Laplacian vs The Composite

10	20	60 ⁰	70 ⁻¹	0
10	20	60 ⁻¹	70 ⁵	-1
10	20	60 ⁰	70 ⁻¹	0
10	20	60	70	

20	10	150	220
10	-10	90	150

The Composite

Sharpening Filters- Unsharp Masking

- A process used many years in the publishing industry to sharpen images
- Sharpened image: Subtracting a blurred version of an image from the image itself

Sharpening Filters- Unsharp Masking

1. Blur the original image.
2. Subtract the blurred image from the original (the resulting difference is called the *mask*.)

$$g_{\text{mask}}(x, y) = f(x, y) - \bar{f}(x, y)$$

Original Image Blurred image

3. Add the mask to the original.

$$g(x, y) = f(x, y) + k g_{\text{mask}}(x, y)$$

When $k = 1 \rightarrow$ unsharp

When $k > 1, \rightarrow$ highboost filtering

Sharpening Filters- Unsharp Masking

Original Image



Blurred image



Mask



Unsharped image



Highboost

Sharpening Filters- Unsharp Masking

Example: Apply Unsharp filter on the following blurred edge

f					f'				$mask = f - f'$			
50	60	90	100	Std. avg 1/9 1/9 1/9 1/9 1/9 1/9 1/9 1/9 1/9	53	67	83	97	-3	-7	7	3
50	60	90	100		53	67	83	97	-3	-7	7	3
50	60	90	100		53	67	83	97	-3	-7	7	3
50	60	90	100		53	67	83	97	-3	-7	7	3

$$g_{Unsharp} = f + mask$$

47	53	97	103
47	53	97	103
47	53	97	103
47	53	97	103

As we note, the difference between the resulted edge (53,97) is larger than the original edge (60,90), but if this sharpening is not enough we will try to enlarge the value of the mask using high-boost filter by multiplying with value > 1

$$g_{highboost} = f + 3 * mask$$

41	39	111	109
41	39	111	109
41	39	111	109
41	39	111	109

Source: Dr. Hoda
Karajah lectures

Exercise

- Show the result of applying the following operators to the image.
Consider padding the edges with 0s.
 - Box filter 3x3
 - The Laplacian operator

0	1	0
1	-4	1
0	1	0

10	20	110	110
10	20	110	110
10	20	110	110
10	20	110	110



Thank You