



Mathematics

Lecture 3

MAT 122

Introduction to Matrix Algebra

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Concept of a matrix.

Order and size of a matrix.

Special Definitions in Matrices.

Special types of matrices.

Extra Practice.

A matrix is an ordered rectangular array of numbers, arranged in rows and columns.

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} = (a_{ij})_{mn}$$

A restaurant offers **4 kinds of pizza** (vegetables, cheese, beef and chicken) with **3 sizes**.

Kind	Size		
	small	Medium	Large
Veg.	15	13	9
Chicken	16	18	12
Beef	13	10	8
cheese	18	20	7

If we want to write it in a form of matrix :

A = Name of the matrix	First column	Second column	Third column	
	15	13	9	→ first row
	16	18	12	→ second row
	13	10	8	→ third row
	18	20	17	→ fourth row
	4 * 3 size of the matrix Size of matrix = m x n , m → row , n → column			

The **size** or **order** of a matrix is described by its number of rows and the number of columns.

*If a matrix, **A**, has **m** rows and **n** columns then **A** is described as an **m** $\times$ **n** matrix.*

A matrix consisting of m horizontal rows and n vertical columns is called an **$m \times n$ matrix** or a **matrix of size $m \times n$** .

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{12} & \cdots & a_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

For the entry a_{ij} , we call i the row subscript and j the column subscript.

Find the size for the following matrices:

A- $\begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$

B- $\begin{bmatrix} 1 & -6 \\ 5 & 1 \\ 9 & 4 \end{bmatrix}$

C- $\begin{bmatrix} 7 \end{bmatrix}$

D- $\begin{bmatrix} 1 & 3 & 7 & -2 & 4 \\ 9 & 11 & 5 & 6 & 8 \\ 6 & -2 & -1 & 1 & 1 \end{bmatrix}$

Find the size for the following matrices:

a. The matrix $\begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$ has size 1×3 .

b. The matrix $\begin{bmatrix} 1 & -6 \\ 5 & 1 \\ 9 & 4 \end{bmatrix}$ has size 3×2 .

Find the size for the following matrices:

C- The matrix $\begin{bmatrix} 7 \end{bmatrix}$ has size 1×1 .

d. The matrix $\begin{bmatrix} 1 & 3 & 7 & -2 & 4 \\ 9 & 11 & 5 & 6 & 8 \\ 6 & -2 & -1 & 1 & 1 \end{bmatrix}$ has size 3×5

- Elements of a matrix
- Row Matrix
- Column Matrix
- Matrices of the same order
- Equal Matrices

The numbers in a matrix are called its **elements**. The element in the i^{th} row and j^{th} column of a matrix is generally denoted by a^{ij} . A matrix with m rows and n columns is written $(a_{ij})_{mn}$ or (a_{ij}) .

A matrix with just one row is called a row matrix (or row vector).

$$A = [a_1 a_2 \dots, a_n] = \{a_j\}_{(1 \times n)}$$

A matrix with just one column is called a **column matrix**.

$$A = \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_m \end{bmatrix} = \{a_i\}_{(m \times 1)}$$

Two matrices which have the Same number of rows and columns are said to be matrices of the **same order**.

Two matrices A and B are said to be equal if, and only if, each element a_{ij} of A is equal to the corresponding element b_{ij} of B .

Two matrices $A = (a_{ij})$ and $B = (b_{ij})$ are said to be equal if, and only if, each element a_{ij} of A is equal to the corresponding element b_{ij} of B .

In symbolic form this reads:

$$A=B \Leftrightarrow a_{ij} = b_{ij} \text{ for all } i \text{ and } j$$

From this it follows that equal matrices are of the same order but matrices of the same order are not necessarily equal.

- Null Matrix
- Identity Matrix
- Square Matrix
- Transpose Matrix

Any matrix all of whose elements are zero is called a null or zero matrix

Any matrix, all of whose elements are zero, is called a **null** or **zero** matrix and is denoted by O .

$$[0], \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Square matrix with ones on the diagonal and zeros elsewhere.

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Same number of rows and columns

$$B = \begin{bmatrix} 5 & 4 & 7 \\ 3 & 6 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

Rows become columns and columns become rows

$$A^T = \begin{bmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & a_{m2} \\ \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n} & \dots & a_{mn} \end{bmatrix}$$

The transpose of an $m \times n$ matrix A ,
denoted A^T , is the $n \times m$ matrix.

Let's Practice

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Find A^T .

Transpose of matrix A

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$\text{If } A = \begin{bmatrix} 2 & -2 & 5 \\ -3 & 4 & 1 \\ -7 & 2 & 5 \end{bmatrix}$$

Find A^T

$$A^T = \begin{bmatrix} 2 & -3 & -7 \\ -2 & 4 & 2 \\ 5 & 2 & 5 \end{bmatrix}$$

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Find A^T

Matrix A is 2×3 , so A^T is 3×2

→ First row of A becomes the first column of A^T

→ Second row of A becomes the second column of A^T

$$\therefore A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

Examples:

If $A = \begin{pmatrix} -2 & 0 & \frac{1}{2} \\ 1 & 6 & -4 \end{pmatrix}$

Find a) The size of the matrix

a) a_{22} , a_{12} and a_{23}

Solution

a) $\text{size} = 2 \times 3$

b) $a_{22} = 6$, $a^{12} = 0$ $a_{23} = -4$

If $B = \begin{pmatrix} 4 & 5 \\ -1 & 8 \end{pmatrix}$

Find a) the size

a) b_{11} & b_{22}

Solution

a) the size = 2×2

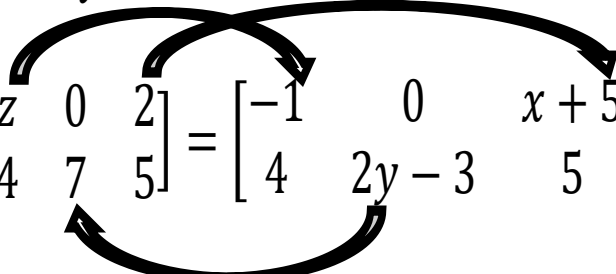
b) $b_n = 4$

$b_{22} = 8$

Example

Find x , y and z

If $\begin{bmatrix} z & 0 & 2 \\ 4 & 7 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & x+5 \\ 4 & 2y-3 & 5 \end{bmatrix}$

A diagram with three curved arrows indicating the correspondence between elements of the two matrices. One arrow points from the element 'z' in the first row, first column of the left matrix to the element '-1' in the first row, first column of the right matrix. A second arrow points from the element '0' in the first row, second column of the left matrix to the element '0' in the first row, second column of the right matrix. A third arrow points from the element '2' in the first row, third column of the left matrix to the element 'x+5' in the first row, third column of the right matrix.

$$Z = -1$$

$$X + 5 = 2 \rightarrow x = 2 - 5 = -3$$

$$2y - 3 = 7 \rightarrow 2y = 7 + 3 \rightarrow 2y = 10$$

$$\rightarrow y = \frac{10}{2} = 5$$

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