

Business Mathematics

Prepared by Dr. Alfaisal Hasan

MTH 112
Business Mathematics

Lecture NO 5



Business Mathematics

Lecture NO 5

MTH 112

Business Mathematics

Prepared by
Dr. Alfaisal Hasan

www.midocean.ae



Chapter

Topics

- Chapter 1: Mathematical preliminaries
- Chapter 2: The straight line and applications
- Chapter 3: Functions and Graphs
- Chapter 4: Limits and Continuity
- **Chapter 5: Financial Mathematics**
- Chapter 6: Differentiation
- Chapter 7: Methods of Differentiation
- Chapter 8: Function of several variables
- Chapter 9: Integration and applications
- Chapter 10: Linear algebra and applications

Objectives

- 1: Using and Writing Sequence
- 2: Arithmetic Sequence
- 3: Arithmetic Series

Using and Writing Sequence

The numbers in sequences are called terms.

You can think of a **sequence** as a function whose domain is a set of consecutive integers. If a domain is not specified, it is understood that the domain starts with 1.

3 6 9 12 15

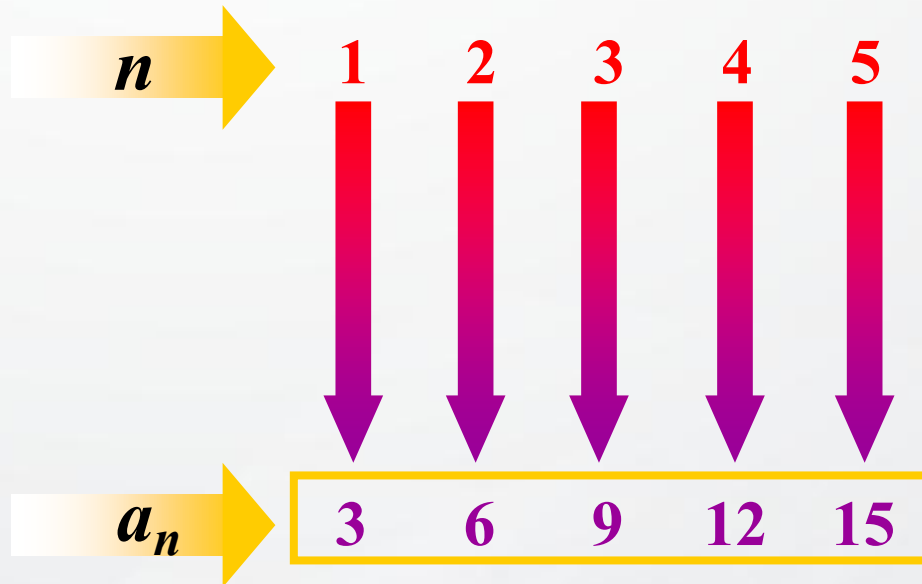
Using and Writing Sequence

Sequence a list of numbers in a specific order.

Term each number in a sequence.

3 6 9 12 15

Using and Writing Sequence



The **domain** gives the **relative position** of each **term**.

The **range** gives the **terms** of the sequence.

This is a **finite sequence** having the rule

$$a_n = 3n,$$

where a_n represents the **n** th term of the sequence.

Using and Writing Sequence

EXAMPLE

Write the first five terms of the sequence $a_n = 2n + 3$.

Using and Writing Sequence

SOLUTION

$$a_1 = 2(1) + 3 = 5$$

1st term

$$a_2 = 2(2) + 3 = 7$$

2nd term

$$a_3 = 2(3) + 3 = 9$$

3rd term

$$a_4 = 2(4) + 3 = 11$$

4th term

$$a_5 = 2(5) + 3 = 13$$

5th term

Using and Writing Sequence

EXAMPLE

Write the first five terms of the sequence $a_n = \frac{n}{n+1}$.

Using and Writing Sequence

SOLUTION

$$a_1 = \frac{(1)}{(1)+1} = \frac{1}{2}$$

1st term

$$a_2 = \frac{(2)}{(2)+1} = \frac{2}{3}$$

2nd term

$$a_3 = \frac{(3)}{(3)+1} = \frac{3}{4}$$

3rd term

$$a_4 = \frac{(4)}{(4)+1} = \frac{4}{5}$$

4th term

$$a_5 = \frac{(5)}{(5)+1} = \frac{5}{6}$$

5th term

Using and Writing Sequence

EXAMPLE

Write the first five terms of the sequence $a_n = n! - 2$.

Using and Writing Sequence

SOLUTION

$$a_1 = (1)! - 2 = -1$$

1st term

$$a_2 = (2)! - 2 = 0$$

2nd term

$$a_3 = (3)! - 2 = 4$$

3rd term

$$a_4 = (4)! - 2 = 22$$

4th term

$$a_5 = (5)! - 2 = 118$$

5th term

Using and Writing Sequence

EXAMPLE

Write the first five terms of the sequence $f(n) = (-2)^{n-1}$.

Using and Writing Sequence

SOLUTION

$$f(1) = (-2)^{1-1} = 1$$

1st term

$$f(2) = (-2)^{2-1} = -2$$

2nd term

$$f(3) = (-2)^{3-1} = 4$$

3rd term

$$f(4) = (-2)^{4-1} = -8$$

4th term

$$f(5) = (-2)^{5-1} = 16$$

5th term

Arithmetic Sequence

- **Arithmetic Sequence:** *sequence whose consecutive terms have a common difference.*
- *Example:* 3, 5, 7, 9, 11, 13, ...
- The terms have a common difference of 2.
- The common difference is the number d .
- To find the common difference you use $a_{n+1} - a_n$
- *Example: Is the sequence arithmetic?*
-45, -30, -15, 0, 15, 30
- **Yes, the common difference is 15**

Arithmetic Sequence

Find the next four terms of $-9, -2, 5, \dots$

Arithmetic Sequence

$$-2 - (-9) = 7 \text{ and } 5 - (-2) = 7$$

7 is referred to as the common difference (d)

Common Difference (d) – what we ADD to get next term

Next four terms.....12, 19, 26, 33

Arithmetic Sequence

Find the next four terms of 0, 7, 14, ...

Arithmetic Sequence, $d = 7$

21, 28, 35, 42

Arithmetic Sequence

Find the next four terms of $-3x, -2x, -x, \dots$

Arithmetic Sequence, $d = x$

$0, x, 2x, 3x$

Arithmetic Sequence

Find the next four terms of $5k, -k, -7k, \dots$

Arithmetic Sequence, $d = -6k$

$-13k, -19k, -25k, -31k$

Arithmetic Sequence

Find the next four terms of 2, 8, 14, ...

Arithmetic Sequence, $d = 6$

20, 26, 32, 38

Arithmetic Sequence

Find the next four terms of 1, 11, 21, ...

Arithmetic Sequence, $d = 10$

31, 41, 51, 61

Arithmetic Sequence

Find the next four terms of 100, 80, 60, ...

Arithmetic Sequence, $d = -20$

40, 20, 0, -20

Arithmetic Sequence

How do you find any term in this sequence?

To find any term in an arithmetic sequence, use the formula

$$a_n = a_1 + (n - 1)d$$

where d is the common difference.

Arithmetic Sequence

Vocabulary of Sequences (Universal)

$a_1 \rightarrow$ First term

$a_n \rightarrow$ nth term

$n \rightarrow$ number of terms

$S_n \rightarrow$ sum of n terms

$d \rightarrow$ common difference

nth term of arithmetic sequence $\rightarrow a_n = a_1 + (n - 1)d$

sum of n terms of arithmetic sequence $\rightarrow S_n = \frac{n}{2}(a_1 + a_n)$

Arithmetic Sequence

EXAMPLE

Find the 14th term of the arithmetic sequence

4, 7, 10, 13,

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n - 1)d$$

$$\begin{aligned} a_{14} &= 4 + (14 - 1) \\ &= 4 + (13)3 \\ &= 4 + 39 \\ &= 43 \end{aligned}$$

Arithmetic Sequence

EXAMPLE

Find the 12th term of the arithmetic sequence

2, 6, 10, 14,.....

Arithmetic Sequence

2, 6, 10, 14,.....

$$a_n = a_1 + (n-1)d$$

$$a_1 = 2, d = 6 - 2 = 4, n = 12$$

$$a_{12} = 2 + (12-1)4$$

$$a_{12} = 2 + 44$$

$$a_{12} = 46$$

Arithmetic Sequence

EXAMPLE

Find a_{16} if $a_1 = 1.5$ and $d = 0.5$

Arithmetic Sequence

Find a_{16} if $a_1 = 1.5$ and $d = 0.5$

1.5 $a_1 \rightarrow$ First term

a_{16} $a_n \rightarrow$ nth term

16 $n \rightarrow$ number of terms

NA $S_n \rightarrow$ sum of n terms

0.5 $d \rightarrow$ common difference

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n - 1)d$$

$$a_{16} = 1.5 + (16 - 1)(0.5)$$

$$a_{16} = 1.5 + (15)(0.5)$$

$$a_{16} = 1.5 + 7.5$$

$$a_{16} = 9$$

Arithmetic Sequence

EXAMPLE

Find a_{20} if $a_1 = 2.0$ and $d = 3.0$

Arithmetic Sequence

Find a_{20} if $a_1 = 2.0$ and $d = 3.0$

$$a_n = a_1 + (n-1)d$$

$$a_1 = 2, \quad d = 3, \quad n = 20$$

$$a_{20} = 2 + (20-1)3$$

$$a_{20} = 2 + 57$$

$$a_{20} = 59$$

Arithmetic Sequence

EXAMPLE

The table shows typical costs for a construction company to rent a crane for one, two, three, or four months. Assuming that the arithmetic sequence continues, how much would it cost to rent the crane for twelve months?

Months	Cost (\$)
1	75,000
2	90,000
3	105,000
4	120,000

Arithmetic Sequence

SOLUTION

Months	Cost (\$)
1	75,000
2	90,000
3	105,000
4	120,000

75,000 $a_1 \rightarrow$ First term

a_{12} $a_n \rightarrow$ nth term

12 $n \rightarrow$ number of terms

NA $S_n \rightarrow$ sum of n terms

15,000 $d \rightarrow$ common difference

Arithmetic Sequence

$$a_n = a_1 + (n - 1)d$$

$$a_{12} = 75,000 + (12 - 1)(15,000)$$

$$a_{12} = 75,000 + (11)(15,000)$$

$$a_{12} = 75,000 + 165,000$$

$$a_{12} = \$240,000$$

Arithmetic Sequence

EXAMPLE

In the arithmetic sequence 4,7,10,13,..., which term has a value of 301?

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n - 1)d$$

$$301 = 4 + (n - 1)3$$

$$301 = 4 + 3n - 3$$

$$301 = 1 + 3n$$

$$300 = 3n$$

$$100 = n$$

Arithmetic Sequence

EXAMPLE

Find n if $a_n = 633$, $a_1 = 9$, and $d = 24$

Arithmetic Sequence

Find n if $a_n = 633$, $a_1 = 9$, and $d = 24$

9 $a_1 \rightarrow$ First term

633 $a_n \rightarrow$ nth term

n $n \rightarrow$ number of terms

NA $S_n \rightarrow$ sum of n terms

24 $d \rightarrow$ common difference

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n-1)d$$

$$633 = 9 + (n-1)(24)$$

$$633 = 9 + 24n - 24$$

$$633 = 24n - 15$$

$$648 = 24n$$

$$n = 27$$

Arithmetic Sequence

EXAMPLE

Given an arithmetic sequence with

$a_{15} = 38$ and $d = -3$, find a_1 .

Arithmetic Sequence

Given an arithmetic sequence with

$$a_{15} = 38 \text{ and } d = -3, \text{ find } a_1.$$

a_1 $a_1 \rightarrow$ First term

38 $a_n \rightarrow$ nth term

15 $n \rightarrow$ number of terms

NA $S_n \rightarrow$ sum of n terms

-3 $d \rightarrow$ common difference

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n - 1)d$$

$$38 = a_1 + (15 - 1)(-3)$$

$$38 = a_1 + (14)(-3)$$

$$38 = a_1 - 42$$

$$a_1 = 80$$

Arithmetic Sequence

EXAMPLE

Find d if $a_1 = -6$ and $a_{29} = 20$

Arithmetic Sequence

Find d if $a_1 = -6$ and $a_{29} = 20$

-6 $a_1 \rightarrow$ First term

20 $a_n \rightarrow$ nth term

29 $n \rightarrow$ number of terms

NA $S_n \rightarrow$ sum of n terms

d $d \rightarrow$ common difference

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n - 1)d$$

$$20 = -6 + (29 - 1)(d)$$

$$20 = -6 + (28)(d)$$

$$26 = 28d$$

$$d = \frac{26}{28}$$

Arithmetic Sequence

EXAMPLE

Write an equation for the n^{th} term of the arithmetic sequence
 $8, 17, 26, 35, \dots$

Arithmetic Sequence

Write an equation for the n^{th} term of the arithmetic sequence
8, 17, 26, 35, ...

8 $a_1 \rightarrow$ First term

9 $d \rightarrow$ common difference

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n - 1)d$$

$$a_n = 8 + (n - 1)(9)$$

$$a_n = 8 + 9n - 9$$

$$a_n = 9n - 1$$

Arithmetic Sequence

Arithmetic Mean: The terms between any two nonconsecutive terms of an arithmetic sequence.

Ex. 19, 30, 41, 52, 63, 74, 85, 96

41, 52, 63 are the Arithmetic Mean between 30 and 74

Arithmetic Sequence

EXAMPLE

Find two arithmetic means between -4 and 5

$-4, \underline{\quad}, \underline{\quad}, 5$

Arithmetic Sequence

Find two arithmetic means between -4 and 5

$-4, \underline{\quad}, \underline{\quad}, 5$

- -4 $a_1 \rightarrow$ First term
- 5 $a_n \rightarrow$ nth term
- 4 $n \rightarrow$ number of terms
- NA $S_n \rightarrow$ sum of n terms
- d $d \rightarrow$ common difference

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n - 1)d$$

$$5 = -4 + (4 - 1)(d)$$

$$5 = -4 + (3)(d)$$

$$9 = (3)(d)$$

$$d = 3$$

The two arithmetic means are -1 and 2 , since $-4, -1, 2, 5$ forms an arithmetic sequence

Arithmetic Sequence

EXAMPLE

Find three arithmetic means between 21 and 45

21, _____, _____, _____, 45

Arithmetic Sequence

Find three arithmetic means between 21 and 45

21, _____, _____, _____, 45

21 $a_1 \rightarrow$ First term

45 $a_n \rightarrow$ nth term

5 $n \rightarrow$ number of terms

NA $S_n \rightarrow$ sum of n terms

d $d \rightarrow$ common difference

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n - 1)d$$

$$45 = 21 + (5 - 1)(d)$$

$$45 = 21 + (4)(d)$$

$$24 = (4)(d)$$

$$d = 6$$

The three arithmetic means are 27, 33, and 39
since 21, 27, 33, 39, 45 forms an arithmetic sequence

Arithmetic Series

Arithmetic Series: *The sum of terms in an arithmetic sequence.*

Arithmetic Sequence

VS

Arithmetic Series

3, 5, 7, 9, 11, 13

$3 + 5 + 7 + 9 + 11 + 13$

Arithmetic Series

Vocabulary of Sequences (Universal)

$a_1 \rightarrow$ First term

$a_n \rightarrow$ nth term

$n \rightarrow$ number of terms

$S_n \rightarrow$ sum of n terms

$d \rightarrow$ common difference

nth term of arithmetic sequence $\rightarrow a_n = a_1 + (n - 1)d$

Sum of n terms of a **FINITE** arithmetic sequence $\rightarrow S_n = \frac{n}{2}(a_1 + a_n)$

Arithmetic Series

EXAMPLE

Find the sum of the first 63 terms of the arithmetic sequence $-19, -13, -7, \dots$

Arithmetic Series

Find the sum of the first 63 terms of the arithmetic sequence -19, -13, -7,...

-19 $a_1 \rightarrow$ First term

?? $a_n \rightarrow$ nth term

63 $n \rightarrow$ number of terms

S_n $S_n \rightarrow$ sum of n terms

6 $d \rightarrow$ common difference

Arithmetic Series

$$a_n = a_1 + (n - 1)d$$

$$a_{63} = -19 + (63 - 1)(6)$$

$$a_{63} = 353$$

Arithmetic Series

Find the sum of the first 63 terms of the arithmetic sequence -19, -13, -7,...

-19 $a_1 \rightarrow$ First term

353 ?? $a_n \rightarrow$ nth term

63 $n \rightarrow$ number of terms

S_n $S_n \rightarrow$ sum of n terms

6 $d \rightarrow$ common difference

Arithmetic Series

SOLUTION

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{63} = \frac{63}{2}(-19 + 353)$$

$$S_{63} = 10521$$

Arithmetic Series

EXAMPLE

Find the sum of the first 55 terms of the arithmetic sequence 1, 5, 9,...

$$a_1 = 1, d = 4, n = 55$$

$$a_n = a_1 + (n-1)d$$

$$a_1 = 1, d = 4, n = 55$$

$$a_{55} = 1 + (55-1)4$$

$$a_{55} = 1 + 216$$

$$a_{55} = 217$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$a_1 = 1, a_{55} = 217, n = 55$$

$$S_{55} = \frac{55}{2}(1 + 217)$$

$$S_{55} = 5995$$

Arithmetic Series

EXAMPLE

Find the first 3 terms for an arithmetic series in which $a_1 = 9$, $a_n = 105$, $S_n = 741$.

Arithmetic Series

Find the first 3 terms for an arithmetic series in which $a_1 = 9$, $a_n = 105$, $S_n = 741$.

9 $a_1 \rightarrow$ First term

105 $a_n \rightarrow$ nth term

?? $n \rightarrow$ number of terms

741 $S_n \rightarrow$ sum of n terms

?? $d \rightarrow$ common difference

Arithmetic Series

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$741 = \frac{n}{2}(9 + 105)$$

$$741 = \frac{n}{2}(114)$$

$$741 = 57n$$

$$n = 13$$

Arithmetic Series

Find the first 3 terms for an arithmetic series in which $a_1 = 9$, $a_n = 105$, $S_n = 741$.

9 $a_1 \rightarrow$ First term

105 $a_n \rightarrow$ nth term

13 ?? $n \rightarrow$ number of terms

741 $S_n \rightarrow$ sum of n terms

?? $d \rightarrow$ common difference

Arithmetic Series

$$a_n = a_1 + (n - 1)d$$

$$105 = 9 + (13 - 1)d$$

$$105 = 9 + (12)d$$

$$96 = 12d$$

$$8 = d$$

Arithmetic Series

Find the first 3 terms for an arithmetic series in which $a_1 = 9$, $a_n = 105$, $S_n = 741$.

9 $a_1 \rightarrow$ First term

105 $a_n \rightarrow$ nth term

13 ?? $n \rightarrow$ number of terms

741 $S_n \rightarrow$ sum of n terms

8 ?? $d \rightarrow$ common difference

Arithmetic Series

Find the first 3 terms for an arithmetic series in which $a_1 = 9$, $a_n = 105$, $S_n = 741$.

9 $a_1 \rightarrow$ First term

105 $a_n \rightarrow$ nth term

13 $n \rightarrow$ number of terms

741 $S_n \rightarrow$ sum of n terms

8 $d \rightarrow$ common difference

9, 17, 25

Un Solved Examples

Write the first five terms of the sequence $a_n = 3n + 1$.

Un Solved Examples

Given that $2, 6, 10, 14, \dots$ Is an arithmetic sequence?

Find the next 3 terms

Find the common difference d

Un Solved Examples

For an arithmetic sequence given that $a_1 = 9$, $a_n = 105$

Find the sum of first 15 terms

Un Solved Examples

For an arithmetic sequence given that $a_1 = -19$, $n=63$ and $d=6$

Find a_n



Thank You