

The background features a series of vertical panels with rounded ends. The leftmost panel shows a person's hands typing on a laptop. The subsequent panels contain various white line-art icons on a semi-transparent background: a graduation cap, a play button, a target with an arrow, an open book, a magnifying glass, and a group of people. The overall color palette is warm, with shades of orange and brown.

Mathematics

Lecture 9

MAT 122
Vector Space

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- Definition of vector space
- Difference between vector space and vector
- Is zero a vector space?
- What are Equal Vectors?
- Vector Space Axioms
- Conditions for Vector Addition
- Conditions for Scalar Multiplication

Definition of Vector Space

A **vector space** or a linear space is a group of objects called vectors, added collectively and multiplied (“scaled”) by numbers, called scalars. Scalars are usually considered to be real numbers.

A space comprised of vectors, collectively with the associative and commutative law of addition of vectors and also the associative and distributive process of multiplication of vectors by scalars is called vector space. A vector space consists of a set of V (elements of V are called vectors), a field F (elements of F are scalars).

Difference between vector space and vector

A vector is a part of a vector space whereas vector space is a group of objects which is multiplied by scalars and combined by the vector space axioms.

Is zero a vector space?

The trivial vector space, represented by $\{0\}$, is an example of vector space which contains zero vector or null vector. In this case, the addition and scalar multiplication are trivial.

What are Equal Vectors?

The vectors which have the same magnitude and the same direction are called equal vectors. When two vectors are equal, the addressed line segments are parallel. Also, their vector columns are identical.

Vector Space Axioms

All the axioms should be universally quantified. For vector addition and scalar multiplication, it should obey some of the axioms. Here eight axiom rules are given.

Conditions for Vector Addition

An operation vector addition $+$ must satisfy the following conditions:

- Commutative Law.
- Associative Law.
- Additive Identity.
- Additive inverse.

Closure : If x and y are any vectors in the vector space V , then $x + y$ belongs to V

1- Commutative Law : For all vectors x and y in V , then $x + y = y + x$

Closure : If x and y are any vectors in the vector space V , then $x + y$ belongs to V

2- Associative Law : For all vectors x , y and z in V , then $x + (y + z) = (x + y) + z$

Closure : If x and y are any vectors in the vector space V , then $x + y$ belongs to V

3- Additive Identity : For any vector x in V , the vector space contains the additive identity element and it is denoted by ' 0 ' such that $0 + x = x$ and $x + 0 = x$

Closure : If x and y are any vectors in the vector space V , then $x + y$ belongs to V

4- Additive inverse : For each vector x in V , there is an additive inverse $-x$ to get a solution in V .

Conditions for Scalar Multiplication

An operation scalar multiplication is defined between a scalar and a vector and it should satisfy the following condition:

- Associative Law
- Distributive law.
- Distributive law
- Unitary Law

Closure: If x is any vector and c is any real number in the vector space V , then $x \cdot c$ belongs to V

Associative Law:

For all real numbers c and d , and the vector x in V , then $c \cdot (d \cdot v) = (c \cdot d) \cdot v$

Closure: If x is any vector and c is any real number in the vector space V , then $x \cdot c$ belongs to V

Distributive law:

For all real numbers c and d , and the vector x in V , $(c + d) \cdot v = c \cdot v + c \cdot d$

Closure: If x is any vector and c is any real number in the vector space V , then $x \cdot c$ belongs to V

Distributive law:

For all real numbers c and the vectors x and y in V , $c \cdot (x + y) = c \cdot x + c \cdot y$

Closure: If x is any vector and c is any real number in the vector space V , then $x \cdot c$ belongs to V

Distributive law:

For all real numbers c and the vectors x and y in V , $c \cdot (x + y) = c \cdot x + c \cdot y$

Closure: If x is any vector and c is any real number in the vector space V , then $x \cdot c$ belongs to V

Unitary Law :

For all vectors x in V , then $1 \cdot v = v \cdot 1 = v$

Let's Practice

True or False:

Associative law condition for Vector Addition is considered as For all vectors x ,

y and z in V , then $x + (y + z) = (x + y) + z$

Answer: TRUE

True or False:

Associative law condition for Scalar Multiplication is considered as For all

vectors x, y and z in V , then $x + (y + z) = (x + y) + z$

Answer: False

Justification :

Associative Law for all real numbers c and d , and the vector x in V , then $c \cdot (d \cdot v) = (c \cdot d) \cdot v$

True or False:

Distributive law condition for Scalar Multiplication is considered as For all

vectors x, y and z in V , then $x + (y + z) = (x + y) + z$

Answer: False

Justification :

Distributive law for Scalar Multiplication is stated as: For all real numbers c and d , and the vector x in V , $(c + d).v = c.v + c.d$

True or False:

Unitary Law is a condition for Vector Addition

For all vectors x in V , then $1.v = v.1 = v$

Answer: False

Justification:

Unitary Law is a condition for scalar multiplication

For all vectors x in V , then $1.v = v.1 = v$

True or False:

A vector Space is a part of a vector

Answer: False

Justification:

A vector is a part of a vector space whereas vector space is a group of objects which is multiplied by scalars and combined by the vector space axioms.

True or False:

Zero is not a vector space

Answer: False

Justification :

Zero is a vector space. The trivial vector space, represented by $\{0\}$, is an example of vector space which contains zero vector or null vector. In this case, the addition and scalar multiplication are trivial.

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