

Operations Research

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**IT 262
Operations Research**

Lecture 2



Linear Programming

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Chapter

02

Chapter Topics

- ❖ **Linear Programming**
- ❖ **Model Formulation**
- ❖ **Mathematical Model**

Introduction of Linear Programming (LP)



Linear Programming (LP) is a mathematical model technique designed to optimize the usage of limited resources, such as available materials, labour and machine time.



Linear Programming problems (usually profit seek or to maximize cost) or expressed minimize as an some objective quantity function.

Introduction to Linear Programming

- **The importance of linear programming**
 - Many real-world problems led themselves to use linear programming modeling.
 - Many real-world problems can be approximated by linear models.
- **There are well-known successful application in:**
 - **Manufacturing.**
 - **Marketing.**
 - **Finance (investment)**
 - **Advertising**
 - **Agriculture**

Introduction to Linear Programming

- **The importance of linear programming**
 - ❖ **There are efficient solution techniques that solve linear programming model.**
 - ❖ **The output generated from linear programming packages provides useful “what if” analysis.**

Introduction to Linear Programming

- **A Linear Programming Model seeks to maximize or minimize a linear function, subject to a set of linear constraints.**
- **The linear model consists of the following components:**
 - **A set of decision variables.**
 - **An objective function.**
 - **A set of constraints.**

Linear Programming

The Linear Programming (LP) model includes three basic elements



Decision variables that we seek to determine.



Objective (goal) and an objective function that we aim to optimize.



Constraints that we need to satisfy.

Steps for mathematical formulating of Linear programming (LP) problems:

Step 1:

- Identify the decision variable of the problem

Step 2:

- Formulate the objective function to be optimized (maximized/minimized) as a linear function of the decision variables.

Steps for mathematical formulating of Linear programming (LP) problems:

Step 3:

- Formulate the constraints of the problem such as resource limitations, market conditions, interrelations between variables and others as linear equation or in-equations in terms of the decision variables.

Step 4:

- Add the non-negativity constraint so that negative values of the decisions variables do not have any valid physical interpretation.

mathematical Formulation of a Linear programming (LP) problems:

- 1- Choose decision variables.**
- 2- Choose an objective function.**
- 3- Choose Constrains, Linear inequalities.**
- 4- Choose sign restriction.**

Formulation of linear programming model (LP)

The general form of each model will be:

$$Z = C_1x_1 + C_2x_2 + \dots + C_k x_k$$

Subject to:

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1k}x_k \leq, = \text{ or } \geq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2k}x_k \leq, = \text{ or } \geq b_2$$

$$a_{31}x_1 + a_{32}x_2 + \dots + a_{3k}x_k \leq, = \text{ or } \geq b_3$$

$$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mk}x_k \leq, = \text{ or } \geq b_m$$

Formulation of linear programming model (LP)

where:

C_j is a known (cost or profit) coefficient X_j .

X_j is an unknown variable.

a_{ij} is a known constant.

b_j is a known constant.

$\leq, =$ or $\geq$ for each constraint

Problem 1

A Maximization Model

- A company produces two products that are processed on two assembly lines. Assembly line 1 has 100 available hours, and assembly line 2 has 42 available hours. Each product requires 10 hours of processing time on line 1, while on line 2 product 1 requires 7 hours and product 2 requires 3 hours. The profit for product 1 is \$6 per unit, and the profit for product 2 is \$4 per unit.
- **Formulate a linear programming model for this problem.**

Problem 1

A Maximization Model

Resource Requirements

	Products	Line 1 (hr/unit)	Line 2 (lb/unit)	Profit (\$/unit)
x_1	Product 1	10	7	6
x_2	Product 2	10	2	4

Available Time 100 42

Objective function Maximize

$$Z = 6x_1 + 4x_2$$

subject to:

$$10x_1 + 10x_2 \leq 100$$

$$7x_1 + 2x_2 \leq 42$$

Non-Negativity Constraints: $x_1, x_2 \geq 0$

Problem 2

A Maximization Model

- Product mix problem - Beaver Creek Pottery Company
- How many bowls and mugs should be produced to maximize profits given labor and materials constraints?
- Product resource requirements and unit profit:

	Product	Labor (hr/unit)	Clay (lb/unit)	Profit (\$/unit)
X_1	Bowl	1	4	40
X_2	Mug	2	3	50

Problem 2

A Maximization Model

Resource Availability:

40 hrs of labor per day
120 lbs of clay

Decision Variables:

x_1 = number of bowls to produce per day
 x_2 = number of mugs to produce per day

Objective Function:

Maximize $Z = \$40x_1 + \$50x_2$
Where Z = profit per day

Resource Constraints:

$1x_1 + 2x_2 \leq 40$ hours of labor
 $4x_1 + 3x_2 \leq 120$ pounds of clay

Non-Negativity Constraints:

$x_1 \geq 0; x_2 \geq 0$

Problem 2

A Maximization Model

	Product	Labor (hr/unit)	Clay (lb/unit)	Profit (\$/unit)
x_1	Bowl	1	4	40
x_2	Mug	2	3	50
Available resources		40	120	

Maximize $Z = 40x_1 + 50x_2$

subject to:

$$x_1 + 2x_2 \leq 40$$

$$4x_1 + 3x_2 \leq 120$$

$$x_1, x_2 \geq 0$$

Problem 3

A Maximization Model

A manager wants to know how many units of each product to produce on a daily basis in order to achieve the highest contribution to profit. Production requirement for the products are shown in the following table.

The profit is £8 for each unit of the first product and £6 for each unit of the second product

	Product	Processing time required for the first product (Hours)	Processing time required for the second product (Hours)	Profit (£/unit)
x_1	I	4	2	8
x_2	II	2	4	6

Production capacity
available (Hours)

60

48

Problem 3

A Maximization Model

	Product	Processing time required for the first product (Hours)	Processing time required for the second product (Hours)	Profit (£/unit)
X_1	I	4	2	8
X_2	II	2	4	6
Production capacity available (Hours)		60	48	

- The objective function for the LP is:

$$\text{Maximize } Z = 8X_1 + 6X_2$$

- The two constraints can be expressed as:

$$4X_1 + 2X_2 \leq 60 \quad \text{constraint I}$$

$$2X_1 + 4X_2 \leq 48 \quad \text{constraint II}$$

- The no. negatively restriction is all

$$X_1, X_2 \geq 0$$

Problem 4

The Galaxy Industries Production Problem – A Prototype Example

- Galaxy manufactures two toy doll models:
 - Space Ray.
 - Zapper.
- Resources are limited to
 - 1000 pounds of special plastic.
 - 40 hours of production time per week.

Problem 4

The Galaxy Industries Production Problem – A Prototype Example

- Galaxy manufactures two toy doll models:
 - Space Ray.
 - Zapper.
- Resources are limited to
 - 1000 pounds of special plastic.
 - 40 hours of production time per week.

Marketing requirement

- Total production cannot exceed 700 dozens. $X_1 + X_2 \leq 700$ (Total production)
- Number of dozens of Space Rays cannot exceed number of dozens of Zappers by more than 350. $X_1 - X_2 \leq 350$ (Mix)

Technological input

- Space Rays requires 2 pounds of plastic and 3 minutes of labor per dozen.
- Zappers requires 1 pound of plastic and 4 minutes of labor per dozen.

Problem 4

The Galaxy Industries Production Problem – A

Prototype Example

The current production plan calls for:

- Producing as much as possible of the more profitable product, Space Ray (\$8 profit per dozen).
- Use resources left over to produce Zappers (\$5 profit per dozen), while remaining within the marketing guidelines.

	Product	Number of pound of plastic	Number of minutes	Profit (\$/unit)
x_1	Space Rays	2	3	8
x_2	Zappers	1	4	5

Resources are limited to

1000

2400 m (40 h)

Problem 4

The Galaxy Industries Production Problem – A Prototype Example

Decision's variables:

- X_1 = Weekly production level of Space Rays (in dozens)
- X_2 = Weekly production level of Zappers (in dozens).

Objective Function:

- Weekly profit, to be maximized

Problem 4

The Galaxy Industries Production Problem – A Prototype Example

Max $z = 8X_1 + 5X_2$ (Weekly profit)
subject to

$$2X_1 + 1X_2 \leq 1000 \quad (\text{Plastic})$$

$$3X_1 + 4X_2 \leq 2400 \quad (\text{Production Time})$$

$$X_1 + X_2 \leq 700 \quad (\text{Total production})$$

$$X_1 - X_2 \leq 350 \quad (\text{Mix})$$

$$X_1, X_2 \geq 0 \quad (\text{Nonnegativity})$$

	Product	Number of pound of plastic	Number of minutes	Profit (\$/unit)
X_1	Space Rays	2	3	8
X_2	Zappers	1	4	5
Resources are limited to		1000	2400 m (40 h)	

Problem 4

$$\text{Max } z = 8X_1 + 5X_2 \quad (\text{Weekly profit})$$

current production plan consists of:

Space Rays = 450 dozen

Zapper = 100 dozen

Profit = \$4100 per week

$$8(450) + 5(100)$$


Problem 5

Windsor Glass Company is planning to launch two new products

The Windsor Glass Company is planning to launch two new products. Product 1 is an 8-foot glass door with aluminum framing and Product 2 a 4x6 foot double-hung wood-framed window

Aluminum frames are made in Plant 1, wood frames are made in Plant 2, and Plant 3 produces the glass and assembles the products. Product 1 requires some of the production capacity in Plants 1 and 3, but none in Plant 2. Product 2 needs only Plants 2 and 3. The marketing division has concluded that the company could sell as much of either product as could be processed by these plants. The management of the company wants to determine what mixture of both products would be the most profitable. The following table provides the information available.

Plant	Production time per batch, hours		Production time available per week, hours
	Product		
	1	2	
1	1	0	4
2	0	2	12
3	3	2	18
Profit per batch	\$3,000	\$5,000	

Problem 5

Windsor Glass Company is planning to launch
two new products

Plant	Production time per batch, hours		Production time available per week, hours
	Product		
	1	2	
1	1	0	4
2	0	2	12
3	3	2	18
Profit per batch	\$3,000	\$5,000	

	Product	Plant 1	Plant 2	Plant 3	Profit (\$/unit)
X_1	Product 1	1	0	3	3000
X_2	Product 2	0	2	2	5000

Production time
available per
week, hours

4

12

18

Problem 5

Windsor Glass Company is planning to launch
two new products

Formulation as a linear programming problem

Decision variables:

x_1 = Number of batches of product 1 produced per week

x_2 = Number of batches of product 2 produced per week Objective function:

Maximize $z = 3000 X_1 + 5000 X_2$ (in thousands of dollars)

Subject to:

$x_1 \leq 4$ (Production Available in Plant 1)

$2x_2 \leq 12$ (Production Available in Plant 2)

$3x_1 + 2x_2 \leq 18$ (Production Available in Plant 3)

$x_1, x_2 \geq 0$

	Product	Plant 1	Plant 2	Plant 3	Profit (\$/unit)
x_1	Product 1	1	0	3	3000
x_2	Product 2	0	2	2	5000
	Production time available per week, hours	4	12	18	

Problem 6

Reddy Mikks Problem

- The Reddy Mikks company owns a small paint factory that produces both interior and exterior house paints for wholesale distribution. Two basic raw materials, A and B, are used to manufacture the paints.
- The maximum availability of A is 6 tons a day; that of B is 8 tons a day. The daily requirements of the raw materials per ton of interior and exterior paints are summarized in the following table.

	Tons of Raw Material per Ton of Paint		Maximum Availability (tons)
	Exterior (X_E)	Interior (X_I)	
Raw Material A	1	2	6
Raw Material B	2	1	8

- A market survey has established that the daily demand for the interior paint cannot exceed that of exterior paint by more than 1 ton. The survey also showed that the maximum demand for the interior paint is limited to 2 tons daily.

$$X_I - X_E \leq 1$$

$$X_I \leq 2$$

- The wholesale price per ton is \$3000 for exterior paint and \$2000 per interior paint. How much interior and exterior paint should the company produce daily to maximize gross income?

Problem 6

Reddy Mikks Problem

	Tons of Raw Material per Ton of Paint		Maximum Availability (tons)
	Exterior (X_E)	Interior (X_I)	
Raw Material A	1	2	6
Raw Material B	2	1	8

	Product	Raw Material A	Raw Material B	Profit (\$/unit)
X_E	Tons of Raw Material per Ton of Paint Exterior (X_E)	1	2	3000
X_I	Tons of Raw Material per Ton of Paint Interior (X_I)	2	1	2000
	Maximum Availability (tons)	6	8	

Problem 6 Reddy Mikks Problem

Define:

X_E = Tons of exterior paint to be produced

X_I = Tons of interior paint to be produced

Thus, the LP formulation of the Reddy-Mikks Company is as follows:

$$\text{Maximize } z = 3000X_E + 2000X_I$$

Subject to:

$$X_E + 2X_I \leq 6 \quad (1) \text{ (availability of raw material A)}$$

$$2X_E + X_I \leq 8 \quad (2) \text{ (availability of raw material B)}$$

$$X_I - X_E \leq 1 \quad (3) \text{ (Restriction in production)}$$

$$X_I \leq 2 \quad (4) \text{ (Demand Restriction)}$$

$$X_E, X_I \geq 0$$

	Product	Raw Material A	Raw Material B	Profit (\$/unit)
X_E	Tons of Raw Material per Ton of Paint Exterior (X_E)	1	2	3000
X_I	Tons of Raw Material per Ton of Paint Interior (X_I)	2	1	2000
	Maximum Availability (tons)	6	8	

Problem 7

Dorian Auto Problem

- Dorian Auto manufactures luxury cars and trucks.
- The company believes that its most likely customers are high-income women and men.
- To reach these groups, Dorian Auto has embarked on an ambitious TV advertising campaign and will purchase 1-minute commercial spots on two type of programs: comedy shows and football games.

Problem 7

Dorian Auto Problem

- Each comedy commercial is seen by 7 million high income women and 2 million high-income men and costs \$50,000.
- Each football game is seen by 2 million high-income women and 12 million high-income men and costs \$100,000.
- Dorian Auto would like for commercials to be seen by at least 28 million high-income women and 24 million high-income men.
- Use LP to determine how Dorian Auto can meet its advertising requirements at minimum cost.

Problem 7

Dorian Auto Problem

	number of 1-minute	Women	Men	Cost (\$/unit)
x_1	comedy ads	7	2	50
x_2	football ads	2	12	100
Target		28	24	

Problem 7

Dorian Auto Problem

- Dorian must decide how many comedy and football ads should be purchased, so the decision variables are

x_1 = number of 1-minute comedy ads

x_2 = number of 1-minute football ads

- Dorian wants to minimize total advertising cost.

- Dorian's objective functions is

$$\min z = 50 x_1 + 100x_2$$

- Dorian faces the following the constraints

Commercials must reach at least 28 million high-income women.

$$(7x_1 + 2x_2 \geq 28)$$

Commercials must reach at least 24 million high-income men.

$$(2x_1 + 12x_2 \geq 24)$$

- The sign restrictions are necessary, so $x_1, x_2 \geq 0$.

	number of 1-minute	Women	Men	Cost (\$/unit)
x_1	comedy ads	7	2	50
x_2	football ads	2	12	100
	Target	28	24	

Problem 7

Dorian Auto Problem

Solution:

$$\text{Min } Z = 50 x_1 + 100 x_2$$

Subject to

$$7 x_1 + 2 x_2 \geq 28$$

$$2 x_1 + 12 x_2 \geq 24$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

Linear Programming Overview

- Course Content:

- Mathematical modeling to optimize resources (materials, labor, machine time).
- Focus: Maximize or minimize an objective function (profit, cost).

- Core Elements:

- Decision Variables: Values to optimize.
- Objective Function: The goal (e.g., maximize profit, minimize cost).
- Constraints: Conditions that must be met.

- Steps in Formulating LP Problems:

1. Define objective and decision variables.
2. Write objective function.
3. Define constraints and their coefficients



Thank You