

Operations Research

Prepared by Dr. Mohamed Eassa

IT 262
Operations Research

Lecture 4



Linear Programming

Simplex Method Part 1

Lecture 4

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Chapter

3

Lecture 3 Topics

- ❖ **Simplex Method**
- ❖ **Maximization Problem**
- ❖ **Optimal Solution**

Simplex Method

Simplex Method Can solve for more than two decision variables.

Simplex Method

The advantage of Simplex Method

1- No graph

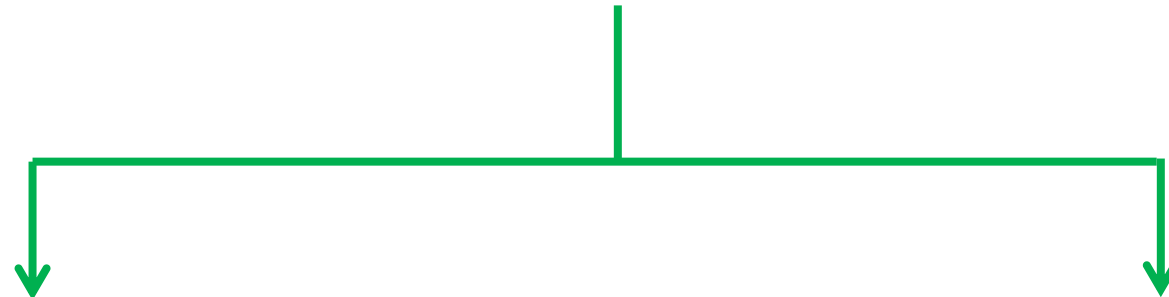
2- can solve for more than 2 variables

Simplex Method

Simplex Method

Maximization Problem

Minimization Problem



Simplex Method

The First : Maximization Problem

Problem 1:

$$\text{Maximize } Z = X_1 + 1.5 X_2$$

Subject to:

$$2X_1 + 4X_2 \leq 12$$

$$3X_1 + 2X_2 \leq 10$$

$$X_1, X_2 \geq 0$$

Simplex Method: *Problem 1*

$$\text{Max } Z = X_1 + 1.5 X_2$$

$$2X_1 + 4X_2 \leq 12$$

$$3X_1 + 2X_2 \leq 10$$

$$X_1, X_2 \geq 0$$

Steps in the Simplex Method :

1- **step 1:** Rewrite the problem in standard form.

$$Z - X_1 - 1.5 X_2 = 0 \quad (\text{obj.})$$

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

$$X_1, X_2 \geq 0$$

Maximize only

Objective function → with R.H.S = 0

Constraints:

(For $\leq$) → equation by $(+s_i)$

(For $\geq$) → equation by $(-s_i)$

Simplex Method: *Problem 1*

2- *step 2: Construct the table and search for the Pivot (Pivot column $\cap$ Pivot Row)*

	X_1	X_2	S_1	S_2	<i>Const.</i>	
R_1	2	4	1	0	12	

$$Z - X_1 - 1.5 X_2 = 0 \quad (obj.)$$

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

Simplex Method: *Problem 1*

2- **step 2:** *Construct the table and search for the Pivot (Pivot column $\cap$ Pivot Row)*

	X_1	X_2	S_1	S_2	<i>Const.</i>	
R_1	2	4	1	0	12	
R_2	3	2	0	1	10	

$$Z - X_1 - 1.5 X_2 = 0 \quad (obj.)$$

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

Simplex Method: *Problem 1*

2- **step 2:** *Construct the table and search for the Pivot (Pivot column $\cap$ Pivot Row)*

For select the pivot column get the lower in objective function

$$Z - X_1 - 1.5 X_2 = 0 \quad (obj.)$$

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

	X_1	X_2	s_1	s_2	Const.	
R_1	2	4	1	0	12	
R_2	3	2	0	1	10	
obj	-1	-1.5	0	0	0	

Simplex Method: *Problem 1*

2- **step 2:** *Construct the table and search for the Pivot (Pivot column $\cap$ Pivot Row)*

For select the pivot column get the lower in objective function

$$Z - X_1 - 1.5 X_2 = 0 \quad (obj.)$$

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

	X_1	X_2	s_1	s_2	Const.	
R_1	2	4	1	0	12	
R_2	3	2	0	1	10	
obj	-1	-1.5	0	0	0	

Simplex Method: *Problem 1*

2- **step 2:** Construct the table and search for the Pivot (Pivot column $\cap$ Pivot Row)

To select the Pivot row perform the ratio test and get the low value $Z - X_1 - 1.5 X_2 = 0$ (obj.)

Ratio test = constant / value in pivot column for R_1 & R_2

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

	X_1	X_2	s_1	s_2	Const.	Ratio Test
R_1	2	4	1	0	12	$12/4 = 3$
R_2	3	2	0	1	10	$10/2 = 5$
obj	-1	-1.5	0	0	0	

Simplex Method: *Problem 1*

2- **step 2:** Construct the table and search for the Pivot (Pivot column $\cap$ Pivot Row)

To select the Pivot row perform the ratio test and get the low value $Z - X_1 - 1.5 X_2 = 0$ (obj.)

Ratio test = constant / value in pivot column for R_1 & R_2

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

	X_1	X_2	s_1	s_2	Const.	Ratio Test
R_1	2	4	1	0	12	$12/4 = 3$
R_2	3	2	0	1	10	$10/2 = 5$
obj	-1	-1.5	0	0	0	

Pivot

Simplex Method: *Problem 1*

3- *step 3*: The Pivot must be 1

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
R_1	2	4	1	0	12	$12/4 = 3$
R_2	3	2	0	1	10	$10 / 2 = 5$
obj	-1	-1.5	0	0	0	

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
$(R_1)_{New} = \frac{(R_1)_{old}}{4}$	$1/2$	1	$1/4$	0	3	

$$Z - X_1 - 1.5 X_2 = 0 \quad (obj.)$$

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

Simplex Method: *Problem 1*

4- step 4: use row operation such that in the Pivot column the other element must be 0

	X_1	X_2	S_1	S_2	Const.	Ratio Test
R_1	2	4	1	0	12	$12/4 = 3$
R_2	3	2	0	1	10	$10 / 2 = 5$
obj	-1	-1.5	0	0	0	

$$Z - X_1 - 1.5 X_2 = 0 \quad (obj.)$$

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

Step 4: must be 0 with the new Row

	X_1	X_2	S_1	S_2	Const.	Ratio Test
$(R_1)_{New} = \frac{(R_1)_{old}}{4}$	$1/2$	1	$1/4$	0	3	
$(R_2)_{New} = -2(R_1)_{New} + (R_2)_{old}$	2	0	$-1/2$	1	4	

Simplex Method: *Problem 1*

4- step 4: use row operation such that in the Pivot column the other element must be 0

	X_1	X_2	S_1	S_2	Const.	Ratio Test
R_1	2	4	1	0	12	$12/4 = 3$
R_2	3	2	0	1	10	$10 / 2 = 5$
obj	-1	-1.5	0	0	0	

$$Z - X_1 - 1.5 X_2 = 0 \quad (obj.)$$

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

Step 4: must be 0 with the new Row

	X_1	X_2	S_1	S_2	Const.	Ratio Test
$(R_1)_{New} = \frac{(R_1)_{old}}{4}$	$1/2$	1	$1/4$	0	3	
$(R_2)_{New} = -2(R_1)_{New} + (R_2)_{old}$	2	0	$-1/2$	1	4	
$(obj.)_{New} = 1.5(R_1)_{New} + (obj)_{old}$	$-1/4$	0	$3/8$	0	$9/2$	

Simplex Method: *Problem 1*

5- **step 5:** Repeat step 2 & 3 & 4 until there is no negative numbers in the obj. row.

	X_1	X_2	S_1	S_2	Const.	Ratio Test
R_1	2	4	1	0	12	$12/4 = 3$
R_2	3	2	0	1	10	$10/2 = 5$
Obj.	-1	-1.5	0	0	0	

$$Z - X_1 - 1.5 X_2 = 0 \quad (obj.)$$

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

	X_1	X_2	S_1	S_2	Const.	Ratio Test
$(R_1)_{New}$	$1/2$	1	$1/4$	0	3	
$(R_2)_{New}$	2	0	$-1/2$	1	4	
$(obj.)_{New}$	$-1/4$	0	$3/8$	0	$9/2$	

Simplex Method: *Problem 1*

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
R_1	2	4	1	0	12	$12/4 = 3$
R_2	3	2	0	1	10	$10 / 2 = 5$
obj	-1	-1.5	0	0	0	

$$Z - X_1 - 1.5 X_2 = 0 \quad (obj.)$$

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
R_1	$1/2$	1	$1/4$	0	3	
R_2	2	0	$-1/2$	1	4	
obj	$-1/4$	0	$3/8$	0	$9/2$	

Repeat Step 2: Get the lower value in objective function to get the Pivot column



Simplex Method: *Problem 1*

	X_1	X_2	S_1	S_2	Const.	Ratio Test
R_1	2	4	1	0	12	$12/4 = 3$
R_2	3	2	0	1	10	$10 / 2 = 5$
obj	-1	-1.5	0	0	0	

	X_1	X_2	S_1	S_2	Const.	Ratio Test
R_1	$1/2$	1	$1/4$	0	3	
R_2	2	0	$-1/2$	1	4	
obj	$-1/4$	0	$3/8$	0	$9/2$	

$$Z - X_1 - 1.5 X_2 = 0 \quad (obj.)$$

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

Repeat Step 2: Get the lower value in objective function to get the Pivot column

Simplex Method: *Problem 1*

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
R_1	2	4	1	0	12	$12/4 = 3$
R_2	3	2	0	1	10	$10 / 2 = 5$
obj	-1	-1.5	0	0	0	

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
R_1	$1/2$	1	$1/4$	0	3	$3 / (1/2) = 6$
R_2	2	0	$-1/2$	1	4	$4 / 2 = 2$
obj	$-1/4$	0	$3/8$	0	$9/2$	

$$Z - X_1 - 1.5 X_2 = 0 \quad (obj.)$$

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

Step 2: Calculate the low value in the ratio test to get the Pivot Row

Simplex Method: *Problem 1*

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
R_1	2	4	1	0	12	$12/4 = 3$
R_1	3	2	0	1	10	$10 / 2 = 5$
obj	-1	-1.5	0	0	0	

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
R_1	$1/2$	1	$1/4$	0	3	$3 / (1/2) = 6$
R_2	2	0	$-1/2$	1	4	$4 / 2 = 2$
obj	$-1/4$	0	$3/8$	0	$9/2$	

$$Z - X_1 - 1.5 X_2 = 0 \quad (obj.)$$

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

Step 3: The Pivot must to be 1

Simplex Method: *Problem 1*

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
R_1	2	4	1	0	12	$12/4 = 3$
R_2	3	2	0	1	10	$10 / 2 = 5$
obj	-1	-1.5	0	0	0	

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
R_1	$1/2$	1	$1/4$	0	3	$3 / (1/2) = 6$
R_2	2	0	$-1/2$	1	4	$4 / 2 = 2$
obj	$-1/4$	0	$3/8$	0	$9/2$	

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
$(R_2)_{New} = \frac{(R_2)_{old}}{2}$	1	0	$-1/4$	$1/2$	2	

$$Z - X_1 - 1.5 X_2 = 0 \quad (obj.)$$

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

Step 3: The Pivot must to be 1

Simplex Method: *Problem 1*

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
R_1	2	4	1	0	12	$12/4 = 3$
R_2	3	2	0	1	10	$10 / 2 = 5$
obj	-1	-1.5	0	0	0	

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
R_1	$1/2$	1	$1/4$	0	3	$3 / (1/2) = 6$
R_2	2	0	$-1/2$	1	4	$4 / 2 = 2$
obj	$-1/4$	0	$3/8$	0	$9/2$	

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
$(R_1)_{New} = -1/2 (R_2)_{New} + (R_1)_{old}$	0	1	$3/8$	$-1/4$	2	
$(R_2)_{New} = \frac{(R_2)_{old}}{2}$	1	0	$-1/4$	$1/2$	2	

$$Z - X_1 - 1.5 X_2 = 0 \quad (obj.)$$

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

Step 4: use row operation such that in the Pivot column the other element must be 0

Simplex Method: *Problem 1*

	X_1	X_2	S_1	S_2	Const.	Ratio Test
R_1	2	4	1	0	12	$12/4 = 3$
R_2	3	2	0	1	10	$10/2 = 5$
obj	-1	-1.5	0	0	0	

	X_1	X_2	S_1	S_2	Const.	Ratio Test
R_1	$1/2$	1	$1/4$	0	3	$3 / (1/2) = 6$
R_2	2	0	$-1/2$	1	4	$4 / 2 = 2$
obj	$-1/4$	0	$3/8$	0	$9/2$	

	X_1	X_2	S_1	S_2	Const.	Ratio Test
$(R_1)_{New} = -1/2 (R_2)_{New} + (R_1)_{old}$	0	1	$3/8$	$-1/4$	2	
$(R_2)_{New} = \frac{(R_2)_{old}}{2}$	1	0	$-1/4$	$1/2$	2	
$(obj)_{New} = 1/4 (R_2)_{New} + (obj)_{old}$	0	0	$5/6$	$1/8$	5	

$$Z - X_1 - 1.5 X_2 = 0 \quad (obj.)$$

$$2X_1 + 4X_2 + s_1 = 12 \quad (R_1)$$

$$3X_1 + 2X_2 + s_2 = 10 \quad (R_2)$$

Step 4: use row operation such that in the Pivot column the other element must be 0

there is no negative numbers in the obj. row

Simplex Method: *Problem 1*

	X_1	X_2	S_1	S_2	Const.	Ratio Test
R_1	2	4	1	0	12	$12/4 = 3$
R_1	3	2	0	1	10	$10/2 = 5$
obj	-1	-1.5	0	0	0	

	X_1	X_2	S_1	S_2	Const.	Ratio Test
R_1	$1/2$	1	$1/4$	0	3	$3 / (1/2) = 6$
R_2	2	0	$-1/2$	1	4	$4 / 2 = 2$
obj	$-1/4$	0	$3/8$	0	$9/2$	

	X_1	X_2	S_1	S_2	Const.	Ratio Test
$(R_1)_{New} = -1/2(R_2)_{New} + (R_1)_{old}$	0	1	$3/8$	$-1/4$	2	
$(R_2)_{New} = \frac{(R_2)_{old}}{2}$	1	0	$-1/4$	$1/2$	2	
$(obj)_{New} = 1/4(R_2)_{New} + (obj)_{old}$	0	0	$5/6$	$1/8$	5	

$$\text{Max } Z = X_1 + 1.5 X_2$$

$$2X_1 + 4X_2 \leq 12$$

$$3X_1 + 2X_2 \leq 10$$

$$X_1, X_2 \geq 0$$

$$X_2 = 2$$

Simplex Method

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
R_1	2	4	1	0	12	$12/4 = 3$
R_1	3	2	0	1	10	$10 / 2 = 5$
obj	-1	-1.5	0	0	0	

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
R_1	$1/2$	1	$1/4$	0	3	$3 / (1/2) = 6$
R_2	2	0	$-1/2$	1	4	$4 / 2 = 2$
obj	$-1/4$	0	$3/8$	0	$9/2$	

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
$(R_1)_{New} = -1/2 (R_2)_{New} + (R_1)_{old}$	0	1	$3/8$	$-1/4$	2	
$(R_2)_{New} = \frac{(R_2)_{old}}{2}$	1	0	$-1/4$	$1/2$	2	
$(obj)_{New} = 1/4 (R_2)_{New} + (obj)_{old}$	0	0	$5/6$	$1/8$	5	

$$\text{Max } Z = X_1 + 1.5 X_2$$

$$2X_1 + 4X_2 \leq 12$$

$$3X_1 + 2X_2 \leq 10$$

$$X_1, X_2 \geq 0$$

$$X_1 = 2$$

Simplex Method: *Problem 1*

	X_1	X_2	S_1	S_2	Const.	Ratio Test
R_1	2	4	1	0	12	$12/4 = 3$
R_1	3	2	0	1	10	$10 / 2 = 5$
obj	-1	-1.5	0	0	0	

	X_1	X_2	S_1	S_2	Const.	Ratio Test
R_1	$1/2$	1	$1/4$	0	3	$3 / (1/2) = 6$
R_2	2	0	$-1/2$	1	4	$4 / 2 = 2$
obj	$-1/4$	0	$3/8$	0	$9/2$	

	X_1	X_2	S_1	S_2	Const.	Ratio Test
$(R_1)_{New} = -1/2 (R_2)_{New} + (R_1)_{old}$	0	1	$3/8$	$-1/4$	2	
$(R_2)_{New} = \frac{(R_2)_{old}}{2}$	1	0	$-1/4$	$1/2$	2	
$(obj)_{New} = 1/4 (R_2)_{New} + (obj)_{old}$	0	0	$5/6$	$1/8$	5	

$$\text{Max } Z = X_1 + 1.5 X_2$$

$$2X_1 + 4X_2 \leq 12$$

$$3X_1 + 2X_2 \leq 10$$

$$X_1, X_2 \geq 0$$

$$X_1 = 2, X_2 = 2$$

The Solution:

$$Z = 5$$

$$X_1 = 2, X_2 = 2$$

$$Z = 5$$

For check you can calculate to make sure the result is true

$$\text{Max } Z = X_1 + 1.5 X_2 = 2 + 1.5 (2) = 5$$

The Result is true

Simplex Method

The First : Maximization Problem

Problem 2:

$$\text{Maximize } Z = 40X_1 + 50X_2$$

Subject to:

$$X_1 + 2X_2 \leq 40$$

$$4X_1 + 3X_2 \leq 120$$

$$X_1, X_2 \geq 0$$

Simplex Method: *Problem 2*

$$\begin{aligned}\text{Max } Z &= 40X_1 + 50X_2 \\ X_1 + 2X_2 &\leq 40 \\ 4X_1 + 3X_2 &\leq 120 \\ X_1, X_2 &\geq 0\end{aligned}$$

Steps in the Simplex Method :

1- **step 1:** Rewrite the problem in standard form.

$$Z - 40X_1 - 50X_2 = 0 \quad (\text{obj.})$$

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

$$X_1, X_2 \geq 0$$

Maximize only

Objective function → with R.H.S = 0

Constraints:

(For $\leq$) → equation by $(+s_i)$

(For $\geq$) → equation by $(-s_i)$

Simplex Method: *Problem 2*

2- *step 2: Construct the table and search for the Pivot (Pivot column $\cap$ Pivot Row)*

	X_1	X_2	S_1	S_2	<i>Const.</i>	
R_1	1	2	1	0	40	

$$Z - 40X_1 - 50 X_2 = 0 \quad (obj.)$$

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

Simplex Method: *Problem 2*

2- *step 2: Construct the table and search for the Pivot (Pivot column $\cap$ Pivot Row)*

	X_1	X_2	S_1	S_2	<i>Const.</i>	
R_1	1	2	1	0	40	
R_2	4	3	0	1	120	

$$Z - 40X_1 - 50 X_2 = 0 \quad (obj.)$$

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

Simplex Method: *Problem 2*

2- **step 2:** *Construct the table and search for the Pivot (Pivot column $\cap$ Pivot Row)*

For select the pivot column get the lower in objective function $Z - 40X_1 - 50 X_2 = 0$ (*obj.*)

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

	X_1	X_2	s_1	s_2	Const.	
R_1	1	2	1	0	40	
R_2	4	3	0	1	120	
<i>obj</i>	-40	-50	0	0	0	

Simplex Method: *Problem 2*

2- **step 2:** *Construct the table and search for the Pivot (Pivot column $\cap$ Pivot Row)*

For select the pivot column get the lower in objective function

$$Z - 40X_1 - 50 X_2 = 0 \quad (obj.)$$

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

	X_1	X_2	s_1	s_2	Const.	
R_1	1	2	1	0	40	
R_2	4	3	0	1	120	
obj	-40	-50	0	0	0	

Simplex Method: *Problem 2*

2- **step 2:** Construct the table and search for the Pivot (Pivot column $\cap$ Pivot Row)

To select the Pivot row perform the ratio test and get the low value $Z - 40X_1 - 50X_2 = 0$ (obj.)

Ratio test = constant / value in pivot column for R_1 & R_2

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

	X_1	X_2	s_1	s_2	Const.	Ratio Test
R_1	1	2	1	0	40	$40/2 = 20$
R_2	4	3	0	1	120	$120/3 = 40$
obj	-40	-50	0	0	0	

Simplex Method: *Problem 2*

2- **step 2:** Construct the table and search for the Pivot (Pivot column $\cap$ Pivot Row)

To select the Pivot row perform the ratio test and get the low value

Ratio test = constant / value in pivot column for R_1 & R_2

	X_1	X_2	S_1	S_2	Const.	Ratio Test
R_1	1	2	1	0	40	$40/2 = 20$
R_2	4	3	0	1	120	$120 / 3 = 40$
obj	-40	-50	0	0	0	

$$Z - 40X_1 - 50 X_2 = 0 \quad (obj.)$$

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

Pivot

Simplex Method: *Problem 2*

3- *step 3*: The Pivot must be 1

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
R_1	1	2	1	0	40	$40/2 = 20$
R_2	4	3	0	1	120	$120 / 3 = 40$
obj	-40	-50	0	0	0	

$$Z - 40X_1 - 50 X_2 = 0 \quad (obj.)$$

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
$(R_1)_{New} = \frac{(R_1)_{old}}{2}$	$1/2$	1	$1/2$	0	20	

Simplex Method: *Problem 2*

4- step 4: use row operation such that in the Pivot column the other element must be 0

	X_1	X_2	S_1	S_2	Const	Ratio Test
R_1	1	2	1	0	40	$40/2 = 20$
R_2	4	3	0	1	120	$120/3 = 40$
obj	-40	-50	0	0	0	

$$Z - 40X_1 - 50X_2 = 0 \quad (obj.)$$

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

	X_1	X_2	S_1	S_2	Const.	Ratio Test
$(R_1)_{New} = \frac{(R_1)_{old}}{2}$	$1/2$	1	$1/2$	0	20	
$(R_2)_{New} = -3(R_1)_{New} + (R_2)_{old}$	$5/2$	0	$-3/2$	1	60	
$(obj.)_{New} = 50(R_1)_{New} + (obj)_{old}$	-15	0	25	0	1000	

Step 4: must be 0 with the new Row

Simplex Method: *Problem 2*

5- **step 5:** Repeat step 2 & 3 & 4 until there is no negative numbers in the obj. row.

	X_1	X_2	S_1	S_2	$Const$	$Ratio\ Test$
R_1	1	2	1	0	40	$40/2 = 20$
R_2	4	3	0	1	120	$120 / 3 = 40$
obj	-40	-50	0	0	0	

$$Z - 40X_1 - 50 X_2 = 0 \quad (obj.)$$

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
$(R_1)_{New}$	$1/2$	1	$1/2$	0	20	
$(R_2)_{New}$	$5/2$	0	$-3/2$	1	60	
$(obj.)_{New}$	-15	0	25	0	1000	

Simplex Method: *Problem 2*

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
R_1	1	2	1	0	40	$40/2 = 20$
R_2	4	3	0	1	120	$120 / 3 = 40$
obj	-40	-50	0	0	0	

$$Z - 40X_1 - 50 X_2 = 0 \quad (obj.)$$

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
$(R_1)_{New}$	$1/2$	1	$1/2$	0	20	
$(R_2)_{New}$	$5/2$	0	$-3/2$	1	60	
$(obj.)_{New}$	-15	0	25	0	1000	

Repeat Step 2: Get the lower value in objective function to get the Pivot column



Simplex Method: *Problem 2*

	X_1	X_2	S_1	S_2	Const.	Ratio Test
R_1	1	2	1	0	40	$40/2 = 20$
R_2	4	3	0	1	120	$120 / 3 = 40$
obj	-40	-50	0	0	0	

$$Z - 40X_1 - 50 X_2 = 0 \quad (obj.)$$

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

	X_1	X_2	S_1	S_2	Const.	Ratio Test
(R_1)	$1/2$	1	$1/2$	0	20	
(R_2)	$5/2$	0	$-3/2$	1	60	
(obj.)	-15	0	25	0	1000	

Repeat Step 2: Get the lower value in objective function to get the Pivot column



Simplex Method: *Problem 2*

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
R_1	1	2	1	0	40	$40/2 = 20$
R_2	4	3	0	1	120	$120 / 3 = 40$
obj	-40	-50	0	0	0	

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
(R_1)	$1/2$	1	$1/2$	0	20	$20 / (1/2) = 40$
(R_2)	$5/2$	0	$-3/2$	1	60	$60 / (5/2) = 24$
$(obj.)$	-15	0	25	0	1000	

$$Z - 40X_1 - 50 X_2 = 0 \quad (obj.)$$

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

Step 2: Calculate the low value in the ratio test to get the Pivot Row

Simplex Method: *Problem 2*

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
R_1	1	2	1	0	40	$40/2 = 20$
R_2	4	3	0	1	120	$120 / 3 = 40$
obj	-40	-50	0	0	0	

	X_1	X_2	S_1	S_2	$Const.$	$Ratio\ Test$
(R_1)	$1/2$	1	$1/2$	0	20	$20 / (1/2) = 40$
(R_2)	$5/2$	0	$-3/2$	1	60	$60 / (5/2) = 24$
$(obj.)$	-15	0	25	0	1000	

$$Z - 40X_1 - 50 X_2 = 0 \quad (obj.)$$

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

Step 3: The Pivot must to be 1

Simplex Method: *Problem 2*

	X_1	X_2	S_1	S_2	Const.	Ratio Test
R_1	1	2	1	0	40	$40/2 = 20$
R_2	4	3	0	1	120	$120 / 3 = 40$
obj	-40	-50	0	0	0	

	X_1	X_2	S_1	S_2	Const.	Ratio Test
(R_1)	$1/2$	1	$1/2$	0	20	$20 / (1/2) = 40$
(R_2)	$5/2$	0	$-3/2$	1	60	$60 / (5/2) = 24$
(obj.)	-15	0	25	0	1000	

	X_1	X_2	S_1	S_2	Const.	Ratio Test
$(R_2)_{New} = \frac{(R_2)_{old}}{(5/2)}$	1	0	$-3/5$	$2/5$	24	

$$Z - 40X_1 - 50X_2 = 0 \quad (obj.)$$

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

Step 3: The Pivot must to be 1

Simplex Method: *Problem 2*

	X_1	X_2	S_1	S_2	Const.	Ratio Test
R_1	1	2	1	0	40	$40/2 = 20$
R_2	4	3	0	1	120	$120 / 3 = 40$
obj	-40	-50	0	0	0	

	X_1	X_2	S_1	S_2	Const.	Ratio Test
(R_1)	$1/2$	1	$1/2$	0	20	$20 / (1/2) = 40$
(R_2)	$5/2$	0	$-3/2$	1	60	$60 / (5/2) = 24$
(obj.)	-15	0	25	0	1000	

	X_1	X_2	S_1	S_2	Const.	Ratio Test
$(R_1)_{New} = -1/2 (R_2)_{New} + (R_1)_{old}$	0	1	$4/5$	$-1/5$	8	
$(R_2)_{New} = \frac{(R_2)_{old}}{(5/2)}$	1	0	$-3/5$	$2/5$	24	
$(obj)_{New} = 15(R_2)_{New} + (obj)_{old}$	0	0	16	6	1360	

$$Z - 40X_1 - 50 X_2 = 0 \quad (obj.)$$

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

there is no negative numbers in the obj. row

Step 4: use row operation such that in the Pivot column the other element must be 0

Simplex Method: *Problem 2*

	X_1	X_2	S_1	S_2	Const.	Ratio Test
R_1	1	2	1	0	40	$40/2 = 20$
R_2	4	3	0	1	120	$120/3 = 40$
obj	-40	-50	0	0	0	

$$Z - 40X_1 - 50X_2 = 0 \quad (obj.)$$

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

	X_1	X_2	S_1	S_2	Const.	Ratio Test
(R_1)	$1/2$	1	$1/2$	0	20	$20 / (1/2) = 40$
(R_2)	$5/2$	0	$-3/2$	1	60	$60 / (5/2) = 24$
(obj.)	-15	0	25	0	1000	

there is no negative numbers in the obj. row

	X_1	X_2	S_1	S_2	Const.	Ratio Test
$(R_1)_{New} = -1/2 (R_2)_{New} + (R_1)_{old}$	0	1	$4/5$	$-1/5$	8	
$(R_2)_{New} = \frac{(R_2)_{old}}{(5/2)}$	1	0	$-3/5$	$2/5$	24	
$(obj)_{New} = 15(R_2)_{New} + (obj)_{old}$	0	0	16	6	1360	

Step 4: use row operation such that in the Pivot column the other element must be 0

Simplex Method: *Problem 2*

	X_1	X_2	S_1	S_2	Const.	Ratio Test
R_1	1	2	1	0	40	$40/2 = 20$
R_2	4	3	0	1	120	$120/3 = 40$
obj	-40	-50	0	0	0	

	X_1	X_2	S_1	S_2	Const.	Ratio Test
(R_1)	$1/2$	1	$1/2$	0	20	$20 / (1/2) = 40$
(R_2)	$5/2$	0	$-3/2$	1	60	$60 / (5/2) = 24$
(obj.)	-15	0	25	0	1000	

	X_1	X_2	S_1	S_2	Const.	Ratio Test
$(R_1)_{New} = -1/2 (R_2)_{New} + (R_1)_{old}$	0	1	$4/5$	$-1/5$	8	
$(R_2)_{New} = \frac{(R_2)_{old}}{(5/2)}$	1	0	$-3/5$	$2/5$	24	
$(obj)_{New} = 15(R_2)_{New} + (obj)_{old}$	0	0	16	6	1360	

$$Z - 40X_1 - 50X_2 = 0 \quad (obj.)$$

$$X_1 + 2X_2 + s_1 = 40 \quad (R_1)$$

$$4X_1 + 3X_2 + s_2 = 120 \quad (R_2)$$

$$X_2 = 8$$

$$X_1 = 24$$

The Solution:

$$Z = 1360$$

$$X_1 = 24, X_2 = 8$$

$$Z = 1360$$

For check you can calculate to make sure the result is true

$$\text{Max } Z = 40X_1 + 50X_2 = 40(24) + 50(8) = 1360$$

The Result is true



Thank You