

Mathematics

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Lecture 1

جامعة الجميع الذكية

تسعى لتكون الكيان الأول الذي يتبنى مبدأ المجتمع التعليمي
التعاوني ويقدم العلم مقابل العمل لتنمية وبناء المجتمعات.

MAT 112

Functions and Graphs



جامعة الجميع الذكية
EVERYONE'S SMART UNIVERSITY

- Lecture 1: Functions and Graphs
- Lecture 2: Exponential Functions
- Lecture 3: Limits and Continuity
- Lecture 4: Differentiations
- Lecture 5: Additional Differentiation topics
- Lecture 6: Curve Sketching
- Lecture 7: Integration
- Lecture 8: Methods and Applications of integration
- Lecture 9: Multivariable Calculus
- Lecture 10: Topics by Python

- **Functions**
- **Special Functions**
- **Combinations of functions**
- **Linear Equation**
- **Formatting the linear equation to graph it**
- **Graphs in rectangular coordinates**

1. Functions

- A **function** assigns each input number to one output number.
- The set of all input numbers is the **domain** of the function.
- The set of all output numbers is the **range**.

Equality of Functions

- Two functions f and g are equal ($f = g$):
 1. Domain of f = domain of g ;
 2. $f(x) = g(x)$.

Example 1 – Determining Equality of Functions

Determine which of the following functions are equal.

a. $f(x) = \frac{(x+2)(x-1)}{(x-1)}$

b. $g(x) = x + 2$

c. $h(x) = \begin{cases} x + 2 & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$

d. $k(x) = \begin{cases} x + 2 & \text{if } x \neq 1 \\ 3 & \text{if } x = 1 \end{cases}$



Solution:

When $x = 1$,

$$f(1) \neq g(1),$$

$$f(1) \neq h(1),$$

$$f(1) \neq k(1)$$

By definition, $g(x) = h(x) = k(x)$ for all $x \neq 1$.

Since $g(1) = 3$, $h(1) = 0$ and $k(1) = 3$, we conclude that

$$g = k,$$

$$g \neq h,$$

$$h \neq k$$



Finding Domain and Function Values

Let $g(x) = 3x^2 - x + 5$. Any real number can be used for x , so the domain of g is all real numbers.

- a. Find $g(z)$.
- b. Find $g(r^2)$.
- c. Find $g(x + h)$.

Finding Domain and Function Values

Let $g(x) = 3x^2 - x + 5$. Any real number can be used for x , so the domain of g is all real numbers.

a. Find $g(z)$.

Solution: $g(z) = 3z^2 - z + 5$

b. Find $g(r^2)$.

Solution: $g(r^2) = 3(r^2)^2 - r^2 + 5 = 3r^4 - r^2 + 5$

c. Find $g(x + h)$.

Solution: $g(x + h) = 3(x + h)^2 - (x + h) + 5$
 $= 3x^2 + 6hx + 3h^2 - x - h + 5$



- We begin with *constant function*.

Example 1 – Constant Function

Let $h(x) = 2$. The domain of h is all real numbers.

$$h(10) = 2 \quad h(-387) = 2 \quad h(x + 3) = 2$$

A function of the form $h(x) = c$, where $c = \text{constant}$, is a **constant function**.

Example – Rational Functions

a. $f(x) = \frac{x^2 - 6x}{x + 5}$ is a rational function, since the numerator and denominator are both polynomials.

b. $g(x) = 2x + 3$ is a rational function, since $2x + 3 = \frac{2x + 3}{1}$

Example – Absolute-Value Function

Absolute-value function is defined as $|x|$, e.g.

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

3. Combinations of functions

- We define the operations of function as:

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x).g(x)$$

$$\frac{f}{g}(x) = \frac{f(x)}{g(x)} \text{ for } g(x) \neq 0$$



Example 1 – Combining Functions

If $f(x) = 3x - 1$ and $g(x) = x^2 + 3x$, find

- a. $(f + g)(x)$
- b. $(f - g)(x)$
- c. $(fg)(x)$
- d. $\frac{f}{g}(x)$
- e. $(\frac{1}{2}f)(x)$

Solution:

a. $(f + g)(x) = f(x) + g(x) = (3x - 1) + (x^2 + 3x) = x^2 + 6x - 1$

b. $(f - g)(x) = f(x) - g(x) = (3x - 1) - (x^2 + 3x) = -1 - x^2$

c. $(fg)(x) = f(x)g(x) = (3x - 1)(x^2 + 3x) = 3x^3 + 8x^2 - 3x$

d. $\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{3x - 1}{x^2 + 3x}$

e. $(\frac{1}{2}f)(x) = \frac{1}{2}(f(x)) = \frac{1}{2}(3x - 1) = \frac{3x - 1}{2}$



Composition

- Composite of f with g is defined by

$$(f \circ g)(x) = f(g(x))$$



Example – Composition

If $F(p) = p^2 + 4p - 3$, $G(p) = 2p + 1$, and $H(p) = |p|$, find

- a. $F(G(p))$
- b. $F(G(H(p)))$
- c. $G(F(1))$

Solution:

- a. $F(G(p)) = F(2p + 1) = (2p + 1)^2 + 4(2p + 1) - 3 = 4p^2 + 12p + 2 = (F \circ G)(p)$
- b. $F(G(H(p))) = (F \circ (G \circ H))(p) = ((F \circ G) \circ H)(p) = (F \circ G)(H(p)) = (F \circ G)(|p|) = 4|p|^2 + 12|p| + 2 = 4p^2 + 12|p| + 2$
- c. $G(F(1)) = G(1^2 + 4 \times 1 - 3) = G(2) = 2 \times 2 + 1 = 5$



Definition:

A linear equation in the variable (X) is an equation that is equivalent to one that be written in the form.

$$ax + b = 0$$

Where a and b are constant and $a \neq 0$

→linear equation is also called a first-degree equation, since the highest power of the variable that occurs in Equation (1) is the first.

The general form of a linear equation is:

$$Ax + By = C$$

Example: find A, B & C

$$(1) \ x + 4y = 6 \quad \Rightarrow \ A = 1 \quad B = 4 \\ C = 6$$

$$(2) \ y = -\frac{1}{2}x - \frac{3}{4} \quad \Rightarrow \ \frac{1}{2}x + y = -\frac{3}{4} \\ A = \frac{1}{2} \\ B = 1 \\ C = -\frac{3}{4}$$

How to Graph a linear equation:

To graph a linear equation you need to substitute (X) by zero to get (Y) & substitute (Y) by zero to get (X), to get two points.

(1) $y = x - 4$

Step1:

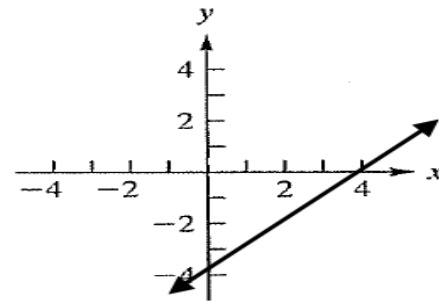
let $x = 0$ so the point is $(0, -4)$

$$Y = 0 - 4 \therefore y = -4$$

$$\text{Let } y = 0 \Rightarrow 0 = x - 4 \therefore x = 4$$

So the point is $(4, 0)$

Step 2:



Lecture 01

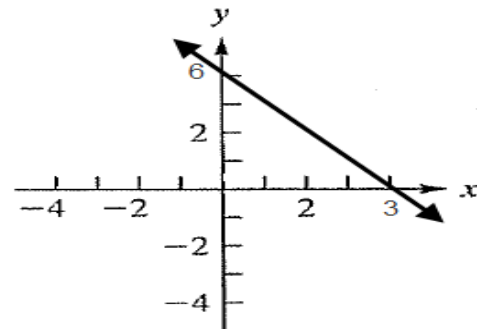
Example

$$(2) y = -2x + 6$$

Step1: let $x = 0$ $(0, 6)$

Let $y = 0$ $(3, 0)$

Step2:



Lecture 01

Example

(3) $y = 3x$

Step1:

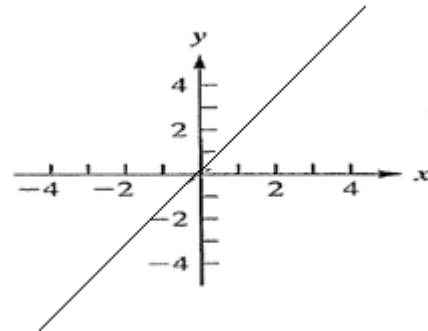
let $x = 0$

$(0, 0)$

Let $x = 2$

$(2, 6)$

Step2:



How to get the slope and the equation:

Rules:

$$y = \textcircled{m}x + \textcircled{b}$$

Diagram illustrating the components of the linear equation $y = mx + b$:

- An arrow points from the circled m to the word "slope".
- An arrow points from the circled b to the text "y - intercept".

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \text{to get the slope from two points.}$$

$$y - y_1 = m(x - x_1) \Rightarrow \text{to get the equation.}$$

Example:

Find the equation of a line passing through:

a) $(0, 4)$, $(2, 2)$

b) $(2, 6)$, $(1, 4)$

Solution:

(a) Step1:

find $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{2 - 4}{2 - 0} = \frac{-2}{2} = -1$$

Step2:

Substitute in $y - y_1 = m(x - x_1)$

$$y - 4 = -1(x - 0)$$

$$y - 4 = -x$$

$$\therefore y = -x + 4$$



(b) Step1:

find $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$M = \frac{4 - 6}{1 - 2} = \frac{-2}{-1} = 2$$

Step2:

substitute in the equation:

$$y - y_1 = m (x - x_1)$$

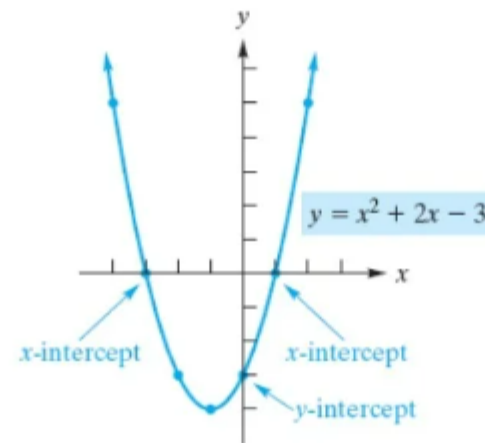
$$y - 6 = 2 (x - 2)$$

$$y - 6 = 2x - 4$$

$$\therefore y = 2x + 2$$



- The **rectangular coordinate system** provides a geometric way to graph equations in two variables.
- An **x-intercept** is a point where the graph intersects the x-axis. **Y-intercept** is vice versa.



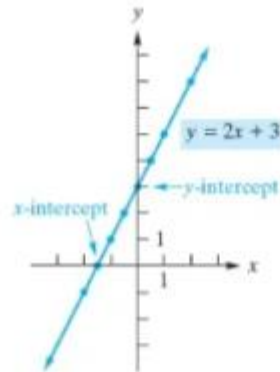
Example 1 – Intercepts and Graph

Find the x - and y -intercepts of the graph of $y = 2x + 3$, and sketch the graph.

Solution:

When $y = 0$, we have $0 = 2x + 3$ so that $x = -\frac{3}{2}$

When $x = 0$, $y = 2(0) + 3 = 3$



x	0	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	1	-1	2	-2
y	3	0	4	2	5	1	7	-1



Example – Intercepts and Graph

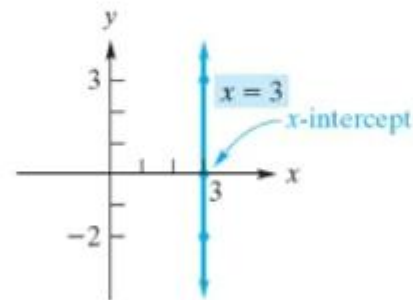
Determine the intercepts of the graph of $x = 3$, and sketch the graph.

Lecture 01

Example

Solution:

There is no y -intercept, because x cannot be 0.



x	3	3	3
y	0	3	-2



Example – Graph of a Case-Defined Function

Graph the case-defined function

$$f(x) = \begin{cases} x & \text{if } 0 \leq x < 3 \\ x - 1 & \text{if } 3 \leq x \leq 5 \\ 4 & \text{if } 5 < x \leq 7 \end{cases}$$

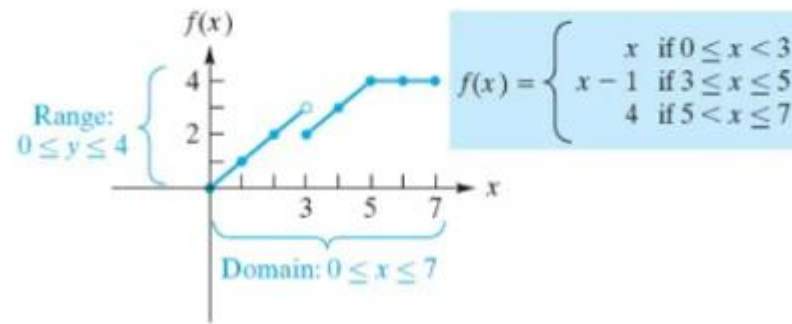


Lecture 01

Example

Solution:

x	0	1	2	3	4	5	6	7
$f(x)$	0	1	2	2	3	4	4	4



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