

Computer Organization and Architecture

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Lecture 05



Boolean algebra

Introduction

- ✓ Boolean algebra, named after its creator George Boole, is a fundamental mathematical system with profound applications in various fields, including computer science, digital electronics, and logic design. This algebraic system deals with variables that can take only two values, typically denoted as "0" and "1," representing "false" and "true," or "off" and "on."
- ✓ It is described as an algebra of logic or an algebra of two values i.e True or False.
- ✓ The term logic means a statement having binary decisions i.e True/Yes or False/No.

Introduction

✓ In this algebra, there are fundamental operations such as AND, OR, and NOT, which enable the manipulation of binary variables. These operations are used to create Boolean expressions that can describe logical conditions and control the behavior of digital systems. Boolean algebra also encompasses advanced concepts like De Morgan's laws, truth tables, and Karnaugh maps, which allow for efficient simplification of complex logical expressions.

Applications of boolean algebra

Application of boolean algebra

It is used to perform the logical operations in digital computer.

In digital computer True represent by '1' (high volt) and False represent by '0' (low volt)

Logical operations are performed by logical operators.

The fundamental logical operators are:

1. AND (conjunction)
2. OR (disjunction)
3. NOT (negation/complement)

AND operator

It performs logical multiplication and denoted by (.) dot.

X	Y	X.Y
0	0	0
0	1	0
1	0	0
1	1	1

OR operator

It performs logical addition and denoted by (+) plus.

X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

NOT operator

it performs logical negation and denoted by (-) bar. It operates on single variable.

X	X' (means complement of x)
0	1
1	0

Truth Table

Truth Table

Truth table is a table that contains all possible values of logical variables/statements in a Boolean expression.

No. of possible combination = 2^n , where n =number of variables used in a Boolean expression.

Truth Table

The truth table for $XY + Z$ is as follows:

Dec	X	Y	Z	XY	XY+Z
0	0	0	0	0	0
1	0	0	1	0	1
2	0	1	0	0	0
3	0	1	1	0	1
4	1	0	0	0	0
5	1	0	1	0	1
6	1	1	0	1	1
7	1	1	1	1	1

Implementation of Boolean Algebra

Implementation of Boolean Algebra

**Boolean Algebra applied in computers electronic circuits.
These circuits perform Boolean operations and these are
called logic circuits or logic gates.**

AND Gate

The AND gate is an electronic circuit that gives a high output (1) only if all its inputs are high.

AND gate takes two or more input signals and produce only one output signal.

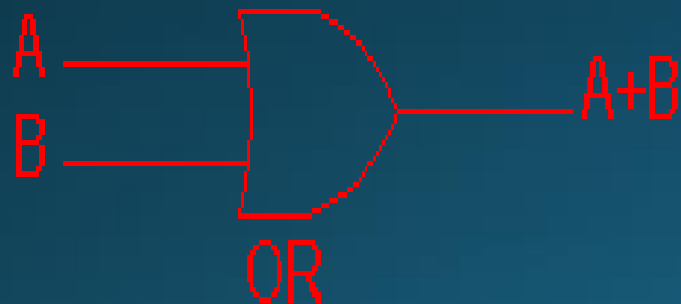


Input A	Input B	Output AB
0	0	0
0	1	0
1	0	0
1	1	1

OR Gate

The OR gate is an electronic circuit that gives a high output (1) if one or more of its inputs are high.

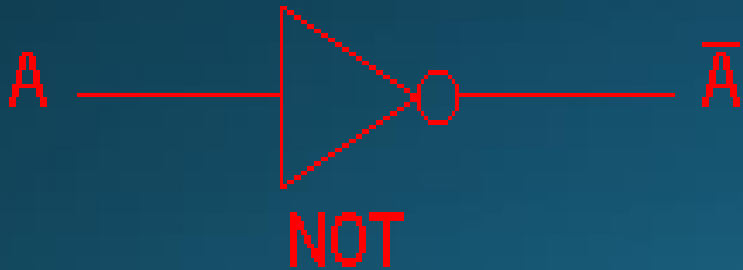
OR gate also takes two or more input signals and produce only one output signal.



Input A	Input B	Output A+B
0	0	0
0	1	1
1	0	1
1	1	1

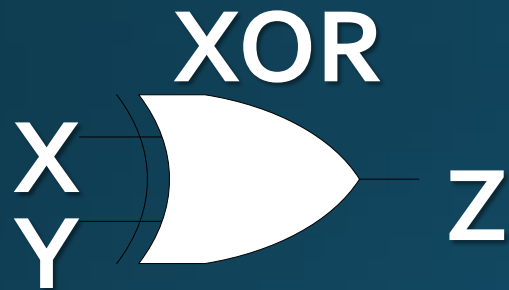
NOT gate

The NOT gate is an electronic circuit that gives a high output (1) if its input is low . NOT gate takes only one input signal and produce only one output signal. The output of NOT gate is complement of its input. It is also called inverter.



Input A	Output A
0	1
1	0

Exclusive-OR Gate



$$Z = X \oplus Y$$

`xor(Z,X,Y)`

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0

Basic Theorem of Boolean Algebra

Basic Theorem of Boolean Algebra

T₁ : Properties of 0

(a) $0 + A = A$

(b) $0 A = 0$

T₂ : Properties of 1

(a) $1 + A = 1$

(b) $1 A = A$

Basic Theorem of Boolean Algebra

T₃ : Commutative Law

$$(a) A + B = B + A$$

$$(b) A B = B A$$

T₄ : Associate Law

$$(a) (A + B) + C = A + (B + C)$$

$$(b) (A B) C = A (B C)$$

Basic Theorem of Boolean Algebra

T5 : Distributive Law

$$(a) A (B + C) = A B + A C$$

$$(b) A + (B C) = (A + B) (A + C)$$

$$(c) A + A'B = A + B$$

T6 : Indempotence (Identity) Law

$$(a) A + A = A$$

$$(b) A A = A$$

T7 : Absorption (Redundance) Law

$$(a) A + A B = A$$

$$(b) A (A + B) = A$$

Basic Theorem of Boolean Algebra

T8 : Complementary Law

$$(a) X + X' = 1$$

$$(b) X \cdot X' = 0$$

T9 : Involution

$$(a) x'' = x$$

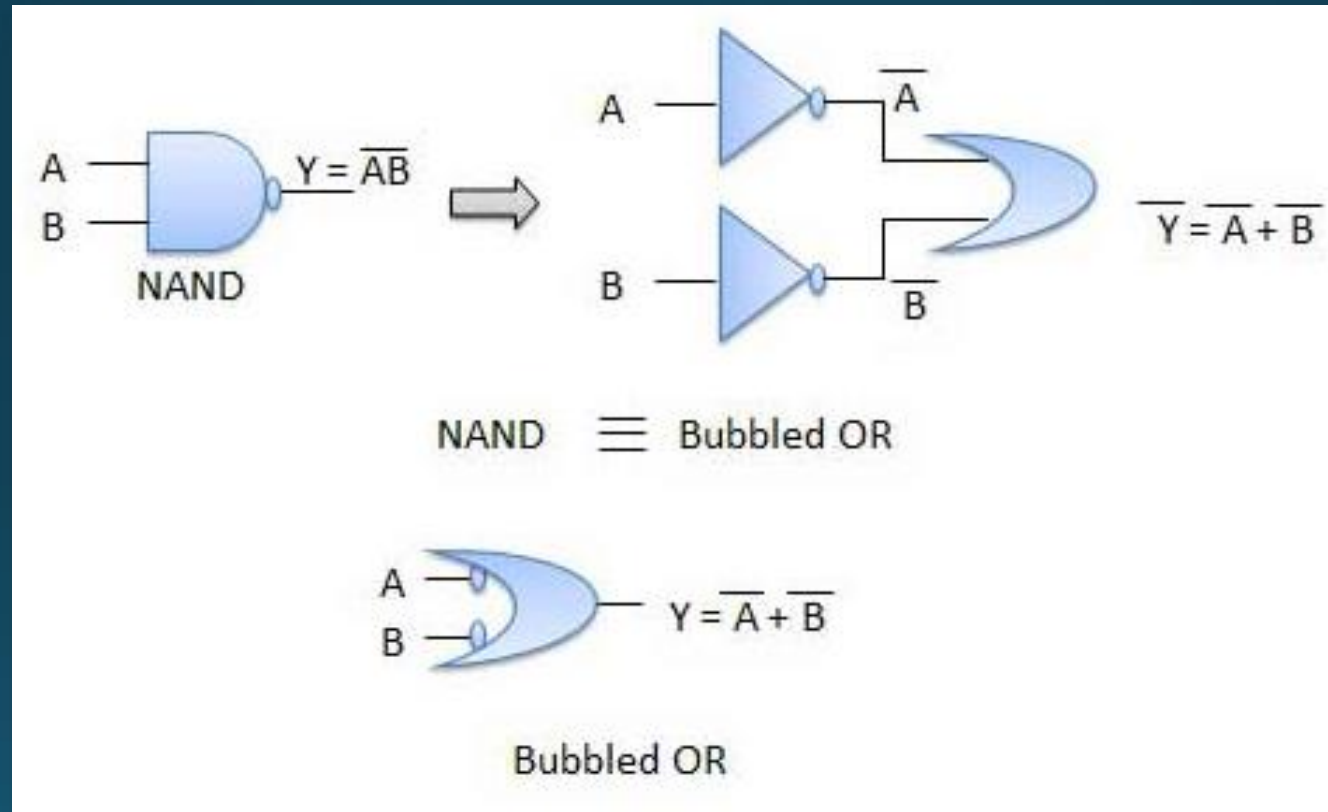
De Morgan's Theorem

De Morgan's Theorem 1

$$\overline{A.B} = \overline{A} + \overline{B}$$

NAND = Bubbled OR

De Morgan's Theorem 1



De Morgan's Theorem 1

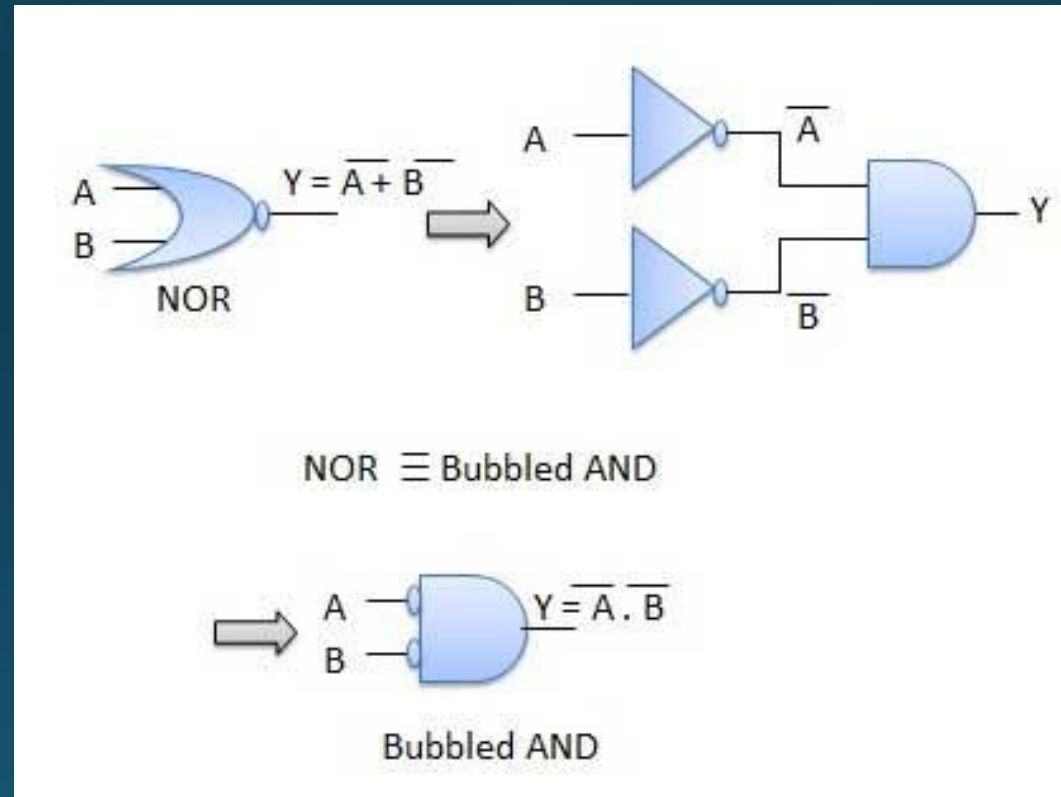
A	B	$\overline{AB}$	$\overline{A}$	$\overline{B}$	$\overline{A} + \overline{B}$
0	0	1	1	1	1
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	0	0	0

De Morgan's Theorem 2

$$\overline{A + B} = \overline{A} . \overline{B}$$

NOR = Bubbled AND

De Morgan's Theorem 2



De Morgan's Theorem 2

A	B	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	0

Problem 1

- Simplify the following expression using Boolean Algebra:

(a) $\overline{ABC} + \overline{A}BC + ABC + \overline{A}\overline{B}C + A\overline{B}C$

Solution:

$$= \overline{A} + \overline{B} + \overline{C} + BC(\overline{A} + A) + A(\overline{B} + \overline{C}) + A\overline{B}C$$

$$= \overline{A} + \overline{B} + \overline{C} + BC + A\overline{B} + A\overline{C} + A\overline{B}C$$

$$= \overline{A} + (\overline{B} + A\overline{B} + A\overline{B}C) + (\overline{C} + A\overline{C}) + BC$$

$$= \overline{A} + \overline{B}(1 + A + AC) + \overline{C}(1 + A) + BC$$

$$= \overline{A} + \overline{B} + (\overline{C} + BC)$$

$$= \overline{A} + \overline{B} + \overline{C} + B$$

$$= \overline{A} + 1 + \overline{C} = 1$$

Another solution :

Replacing ABC by X

$$\overline{X} + \overline{A}BC + X + \overline{A}\overline{B}C + A\overline{B}C$$

$$= 1 + \overline{A}BC + \overline{A}\overline{B}C + A\overline{B}C = 1$$

Problem 1

- Simplify the following expression using Boolean Algebra:

$$(b) (B + \bar{C})(\bar{B} + C) + \overline{A + B + C}$$

Solution:

$$= BC + \bar{B}\bar{C} + A\bar{B}C$$

$$= BC + \bar{B}(\bar{C} + AC)$$

$$= BC + \bar{B}(\bar{C} + A)$$

$$= BC + \bar{B}\bar{C} + A\bar{B}$$

$$= \overline{B \oplus C} + A\bar{B}$$

$$= B \odot C + A\bar{B}$$

Problem 1

- Simplify the following expression using Boolean Algebra:

$$(c) \overline{(C + D)} + \bar{A}C\bar{D} + A\bar{B}\bar{C} + \bar{A}\bar{B}CD + AC\bar{D}$$

Solution:

$$\begin{aligned} &= \bar{C}\bar{D} + \bar{A}C\bar{D} + AC\bar{D} + A(\bar{B} + \bar{C}) + \bar{A}\bar{B}CD \\ &= \bar{C}\bar{D} + C\bar{D}(\bar{A} + A) + A\bar{C} + A\bar{B} + \bar{A}\bar{B}CD \\ &= \bar{D}(\bar{C} + C) + A\bar{C} + \bar{B}(A + \bar{A}CD) \\ &= \bar{D} + A\bar{C} + \bar{B}(A + CD) \\ &= \bar{D} + \bar{B}CD + A\bar{C} + A\bar{B} \\ &= \bar{D} + \bar{B}C + A\bar{C} + A\bar{B} \\ &= \bar{D} + \bar{B}C + A(\bar{C} + \bar{B}) \end{aligned}$$

Problem 2

- Simplify the following expression using DeMorgan's theorems:

$$(a) \overline{A(\overline{B + \overline{C}D})}$$

Solution:

$$\begin{aligned} &= \overline{A} + \overline{(\overline{B + \overline{C}D})} \\ &= \overline{A} + \overline{(\overline{B + \overline{C}}) + \overline{D}} \\ &= \overline{A} + B + \overline{C} + \overline{D} \end{aligned}$$

Problem 2

- Simplify the following expression using DeMorgan's theorems:

$$(b) \overline{(M + \bar{N})(\bar{M} + N)}$$

Solution:

$$\begin{aligned} &= \overline{(M + \bar{N})} + \overline{(\bar{M} + N)} \\ &= \bar{M}N + M\bar{N} \\ &= M \oplus N \end{aligned}$$

Problem 2

- Simplify the following expression using DeMorgan's theorems:

$$(c) \overline{\overline{\overline{A}BCD}}$$

Solution:

$$= \overline{\overline{\overline{A}BC}} + \overline{D}$$

$$= \overline{ABC} + \overline{D}$$

$$= (\overline{A} + \overline{B})C + \overline{D}$$

$$= \overline{A}C + \overline{B}C + \overline{D}$$

Problem 3

- Simplify the following Boolean expression:

$$F(A, B, C) = A'B'C + ABC + A'BC + A'BC'$$

Solution:

To simplify the expression, we can use Boolean algebra rules. We'll use the distributive law to factor out common terms:

$$\begin{aligned} F(A, B, C) &= A'B'C + ABC + A'BC + A'BC' \\ &= A'C(B' + B) + BC(A + A') \\ &= A'C + BC \end{aligned}$$

So, the simplified expression is $F(A, B, C) = A'C + BC$.

Problem 5

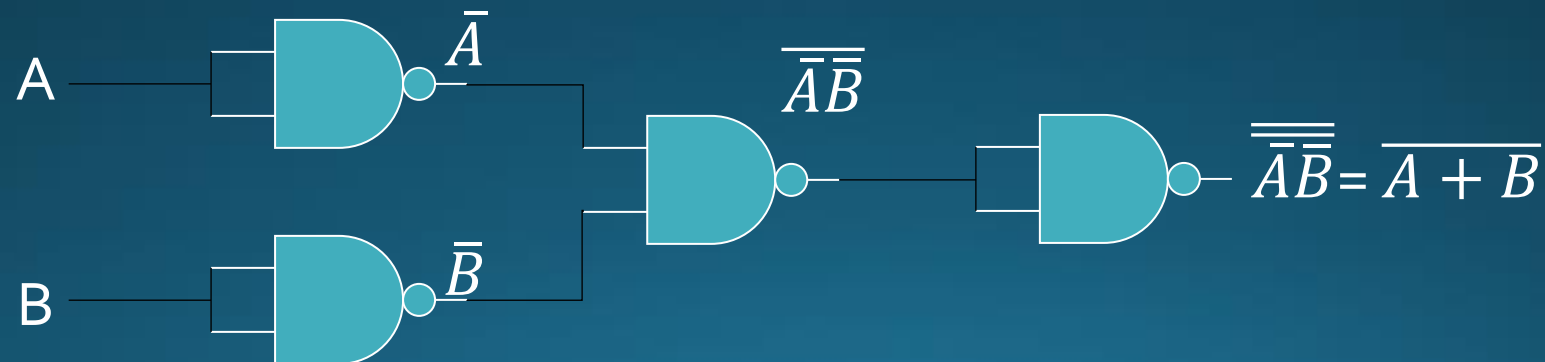
- Show how a two-input NOR gate can be constructed from two-input NAND gates.

Solution:

$$\overline{A + B}$$

$$= \overline{A} \overline{B}$$

$$= \overline{\overline{\overline{A} \overline{B}}}$$



Problem 6

- Change the following expression to both Sum-Of-Product (SOP) and Product-of-Sum (POS):

$$\overline{\overline{ABCD}}$$

Solution:

To Convert to SOP simplify all terms

$$= \bar{A} + \overline{\overline{BCD}}$$

$$= \bar{A} + \bar{B} + CD$$

To Convert to POS leave at least one overall Bar $\overline{XYZ}$ or use a double bar if there is no Bars then simplify under this bar to have SOP to use DeMorgan's Theorem that converts SOP to POS

$$= \overline{AB(\bar{C} + \bar{D})}$$

$$= \overline{AB\bar{C} + AB\bar{D}}$$

$$= (\bar{A} + \bar{B} + C)(\bar{A} + \bar{B} + D)$$



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