

Mathematics 3 – MAT132

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Lecture 10 – Mathematical Operations by MATLAB



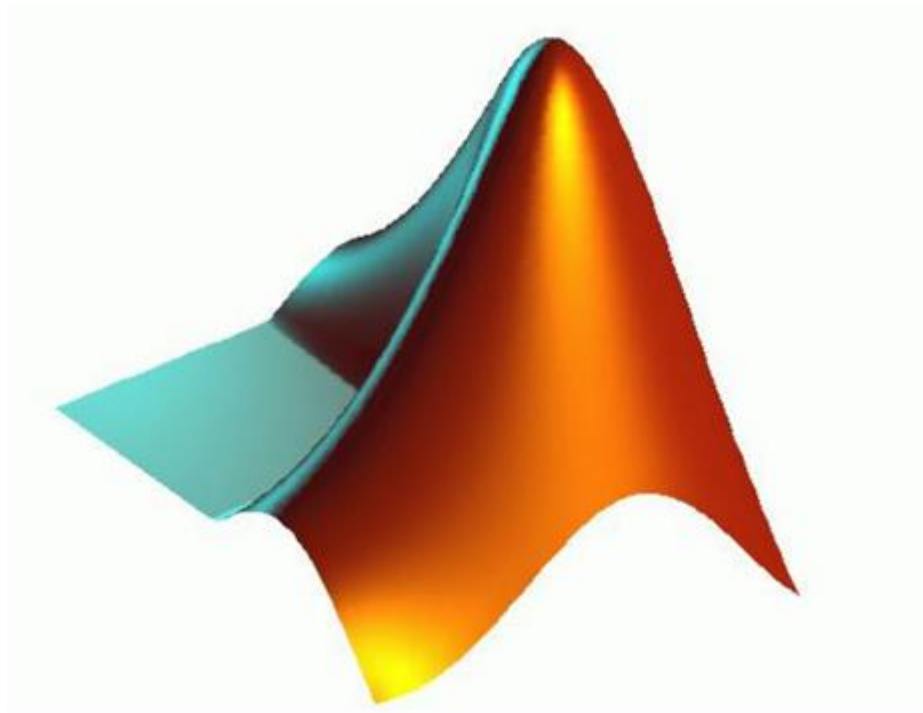
Lecture Objective

- 1- Learn the Importance of MATLAB.
- 2- Explain more about Elementary Functions.
- 3- Give a full illustration for Vectors and Matrices.
- 4- Understand how to conduct Matrix Multiplication.
- 5- Perform many Practical Examples on MATLAB

Lecture Outline

- 1- Importance of MATLAB.
- 2- Elementary Functions.
- 3- Vectors and Matrices.
- 4- Matrix Multiplication.
- 5- Practical Examples on MATLAB

Importance of MATLAB



Importance of MATLAB

- Its basic data element is the [matrix](#). A simple integer is considered an matrix of one row and one column. Several mathematical operations that work on arrays or matrices are built-in to the Mat lab environment. For example, cross-products, dot-products, determinants, inverse matrices.
- Vectorized operations. Adding two arrays together needs only one command, [instead of a for or while loop](#).

Importance of MATLAB

- The graphical output is optimized for interaction. You can plot your data very easily, and then change colors, sizes, scales, etc, by using the graphical interactive tools.
- Mat lab's functionality can be greatly expanded by the addition of toolboxes. These are sets of specific functions that provided more specialized functionality. Ex: Excel link allows data to be written in a format recognized by Excel, Statistics Toolbox allows more specialized statistical manipulation of data (Anova, Basic Fits, etc)

Importance of MATLAB

- Or you can create programs and functions that perform repetitive tasks, just as any other computer language.

Try a simple operation now:
2 + 2 <enter>

Answer is '4' displayed immediately on command window

Importance of MATLAB

One of the most important features of the MATLAB interface is the help. It is very thorough and you can learn almost anything you need from it.

Help <enter>



Importance of MATLAB

The best way for you to get started with MATLAB is to learn how to handle matrices !!!!!

You can enter matrices into MATLAB in several different ways:

1. Enter an explicit list of elements.
2. Load matrices from external data files.
3. Generate matrices using built-in functions.
4. Create matrices with your own functions in M-files.

2- Elementary Functions

Some commonly used functions, where variables x and y can be numbers, vectors, or matrices are listed in the following Table:

Table : Elementary functions

$\cos(x)$	Cosine	$\text{abs}(x)$	Absolute value
$\sin(x)$	Sine	$\text{sign}(x)$	Signum function
$\tan(x)$	Tangent	$\max(x)$	Maximum value
$\text{acos}(x)$	Arc cosine	$\min(x)$	Minimum value
$\text{asin}(x)$	Arc sine	$\text{ceil}(x)$	Round towards $+\infty$
$\text{atan}(x)$	Arc tangent	$\text{floor}(x)$	Round towards $-\infty$
$\exp(x)$	Exponential	$\text{round}(x)$	Round to nearest integer
$\text{sqrt}(x)$	Square root	$\text{rem}(x)$	Remainder after division
$\log(x)$	Natural logarithm	$\text{angle}(x)$	Phase angle
$\log_{10}(x)$	Common logarithm	$\text{conj}(x)$	Complex conjugate

Creating MATLAB variables

Operations	Operators	Examples
Addition	+	>> 5 + 3
Subtraction	-	>> 5 - 3
Multiplication	*	>> 5 * 3
Right Division	/	>> 5/3
Left Division	\	>> 5\3 = 3/5
Exponentiation	^	>> 5^3 (means $5^3 = 125$)

Precedence Order	Operators
1	Parentheses (). For nested parentheses, the innermost
2	are executed first.
3	Exponentiation, ^
4	Multiplication, *; Division, /, \
	Addition, +; Subtraction, -

Elementary Functions

In addition to the elementary functions, MATLAB includes a number of predefined *constant values*. A list of the most common values is given in the following Table :

Table : Predefined constant values

pi	The π number, $\pi = 3.14159 \dots$
i, j	The imaginary unit i , $\sqrt{-1}$
Inf	The infinity, ∞
NaN	Not a number

3- Matrices on MATLAB

Matrices on MATLAB

Matrices and Vectors

Input matrix

$$A = [1 \ 2 \ 5; 3 \ 9 \ 0]$$

Use semicolon ";" to suppress output

Use ... for line continuation

Matrices and Vectors

- ❑ Vector is a special case of matrix
- ❑ Scalar is considered 1x1 matrix
 $g = 9.81$ (no $[\dots]$ needed)
- ❑ $X = []$ creates a null matrix

Matrices and Vectors

1. Separate the elements of a row with blanks or commas.
2. Use a semicolon, ; , to indicate the end of each row.
3. Surround the entire list of elements with square brackets, []

A = [16 3 2 13; 5 10 11 8; 9 6 7 12; 4 15 14 1]

This is what MATLAB displays after you hit <enter>

A =

MAGIC SQUARE

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1



Magic Square

Let's prove it is a magic square. Let's get the sum
of all columns by typing 'sum(A)'

The answer (ans) is:

ans =

34 34 34 34

This result is a row vector. Each column in A adds
up to 34. That's magic!

Transpose the matrix

How about the row sums? MATLAB has a preference for working with the columns of a matrix, so the easiest way to get the row sums is to **transpose the matrix**, compute the column sums of the transpose, and then transpose the result. The transpose operation is denoted by an **apostrophe or single quote, '**. It flips a matrix about its main diagonal and it turns a row vector into a column vector.

A'

ans =

16	5	9	4
3	10	6	15
2	11	7	14
13	8	12	1

Row Sums

Now: **sum(A)'** produces a column vector containing the row sums

ans =

34

34

34

34

Extra Practice – Example 1

❖ Example

As the value of the expression $y = e^{-a} \sin(x) + 10\sqrt{y}$, for $a = 5$, $x = 2$, and $y = 8$ is computed by

Extra Practice – Example 1 solution

As the value of the expression $y = e^{-a} \sin(x) + 10\sqrt{y}$, for $a = 5$, $x = 2$, and $y = 8$ is computed by

```
>> a = 5; x = 2; y = 8;  
>> y = exp(-a)*sin(x)+10*sqrt(y)  
y =  
28.2904
```

Extra Practice – Example 2

❖ Example

To calculate $\sin(\pi/4)$ and e^{10} , we enter the following commands in MATLAB,

Extra Practice – Example 2 solution

To calculate $\sin(\pi/4)$ and e^{10} , we enter the following commands in MATLAB,

```
>> sin(pi/4)
ans =
    0.7071

>> exp(10)
ans =
    2.2026e+004
```

MATLAB Interface

OPERATION	MATRIX	ARRAY
Addition	+	+
Subtraction	-	-
Multiplication	*	.*
Division	/	./
Left division	\	.\
Exponentiation	^	.^

Table : Summary of matrix and array operations

Extra Practice – Example 3

produces another matrix **C** of the same size with elements $c_{ij} = a_{ij}b_{ij}$. For example, using the same 3×3 matrices,

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \\ 70 & 80 & 90 \end{bmatrix}$$

Extra Practice – Example 3 solution

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \\ 70 & 80 & 90 \end{bmatrix}$$

we have,

```
>> C = A.*B
```

C

300	360	420
660	810	960
1020	1260	1500

Extra Practice – Example 4

$$\begin{array}{c} A \\ \begin{bmatrix} 3 & 8 & 0 \\ 1 & 2 & 5 \end{bmatrix} \end{array} * \begin{array}{c} B \\ \begin{bmatrix} 1 & 2 & 3 & 1 \\ 4 & 5 & 1 & 2 \\ 0 & 2 & 3 & 0 \end{bmatrix} \end{array} = \begin{array}{c} C \\ \begin{bmatrix} 35 & 46 & 17 & 19 \\ 9 & 22 & 20 & 5 \end{bmatrix} \end{array}$$

In MATLAB, the * operator will perform this matrix multiplication:

Extra Practice – Example 4 Solution

$$\begin{matrix} & A & & B & & C \\ \begin{bmatrix} 3 & 8 & 0 \\ 1 & 2 & 5 \end{bmatrix} & * & \begin{bmatrix} 1 & 2 & 3 & 1 \\ 4 & 5 & 1 & 2 \\ 0 & 2 & 3 & 0 \end{bmatrix} & = & \begin{bmatrix} 35 & 46 & 17 & 19 \\ 9 & 22 & 20 & 5 \end{bmatrix} \end{matrix}$$

In MATLAB, the * operator will perform this matrix multiplication:

```
>> A = [3 8 0; 1 2 5];  
>> B = [1 2 3 1; 4 5 1 2; 0 2 3 0];  
>> C = A*B  
C =  
    35    46    17    19  
     9    22    20     5
```

Extra Practice – Example 5

For example, consider the following system of linear equations

$$\begin{cases} x + 2y + 3z &= 1 \\ 4x + 5y + 6z &= 1 \\ 7x + 8y &= 1 \end{cases}$$

Extra Practice – Example 5 Solution

$$\begin{cases} x + 2y + 3z &= 1 \\ 4x + 5y + 6z &= 1 \\ 7x + 8y &= 1 \end{cases}$$

The coefficient matrix A is

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \text{and the vector } b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

With matrix notation, a system of simultaneous linear equations is written

Extra Practice – Example 5 Solution

1. The first one is to use the matrix inverse, `inv`.

```
>> A = [1 2 3; 4 5 6; 7 8 0];  
>> b = [1; 1; 1];  
>> x = inv(A)*b  
x      =  
      -1.0000  
       1.0000  
      -0.0000
```

Extra Practice – Example 5 Solution

```
>> A = [1 2 3; 4 5 6; 7 8 0];  
>> inv(A)  
ans =  
    -1.7778    0.8889   -0.1111  
     1.5556   -0.7778    0.2222  
    -0.1111    0.2222   -0.1111
```

which is similar to:

$$A^{-1} = \frac{1}{9} \begin{bmatrix} -16 & 8 & -1 \\ 14 & -7 & 2 \\ -1 & 2 & -1 \end{bmatrix}$$

Vectors and Matrices

Vectors and Matrices

- Vectors (arrays) are defined as
- `>> v = [1, 2, 4, 5]`
- `>> w = [1; 2; 4; 5]`
- Matrices (2D arrays) defined similarly
- `>> A = [1,2,3;4,-5,6;5,-6,7]`

$$v = [1 \ 2 \ 4 \ 5]$$

$$w = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -5 & 6 \\ 5 & -6 & 7 \end{bmatrix}$$

Vectors and Matrices

❑ $t = 1:10$

$t =$

1 2 3 4 5 6 7 8 9 10

❑ $k = 2:-0.5:-1$

$k =$

2 1.5 1 0.5 0 -0.5 -1

❑ $B = [1:4; 5:8]$

$x =$

1 2 3 4
5 6 7 8

Extra Practice – Example 6

Given A and B:

>> A = [1 2 3;4 5 6;7 8 9]									>> B = [3 5 2; 5 2 8; 3 6 9]								
A =									B =								
1	2	3							3	5	2						
4	5	6							5	2	8						
7	8	9							3	6	9						

Extra Practice – Example 6 solution

Addition

```
>> X = A + B
```

X =

4	7	5
9	7	14
10	14	18

Subtraction

```
>> Y = A - B
```

Y =

-2	-3	1
-1	3	-2
4	2	0

Product

```
>> Z = A * B
```

Z =

22	27	45
55	66	102
88	105	159

Transpose

```
>> T = A'
```

T =

1	4	7
2	5	8
3	6	9

Extra Practice – Example 7

$$-x_1 + x_2 + 2x_3 = 2$$

$$3x_1 - x_2 + x_3 = 6$$

$$-x_1 + 3x_2 + 4x_3 = 4$$

Extra Practice – Example 7 solution

```
» A = [-1 1 2; 3 -1 1;-1 3 4];
```

```
» b = [2;6;4];
```

```
» x = inv(A)*b
```

```
x =
```

```
1.0000
```

```
-1.0000
```

```
2.0000
```

```
» x = A\b
```

```
x =
```

```
1.0000
```

```
-1.0000
```

```
2.0000
```

$$-x_1 + x_2 + 2x_3 = 2$$

$$3x_1 - x_2 + x_3 = 6$$

$$-x_1 + 3x_2 + 4x_3 = 4$$

Lecture Summary

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