

Real Estate Finance & Investment

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REA 631

Lecture(4)
Time Value of Money in
Real Estate Investment



Learning Objectives

By the end of this chapter, students will be able to:

- 1 Apply Present Value (PV) and Future Value (FV) formulas
- 2 Distinguish between simple and compound interest
- 3 Compute monthly mortgage payments using annuities
- 4 Convert between nominal, effective, and real interest rates

Why Time Value of Money Matters



EGP 1 today is worth more than EGP 1 tomorrow.



Inflation

Prices rise — money loses purchasing power.



Opportunity

Money can earn returns; idle cash cannot.



Risk

Future cash is uncertain; risk needs reward.

TVM in Real Estate Decisions

Every real estate decision is a TVM problem:



Mortgage Loans

Convert a loan today into equal monthly payments tomorrow.



Property Valuation

Discount future rents back to today's value.



Development Projects

Compare today's costs with future sale proceeds.



Down Payment Saving

Plan monthly savings to reach a future down payment.

Three Building Blocks of TVM

Every TVM calculation involves three core ingredients:

 PV

Present Value

Worth of money today

 r

Discount Rate

Required return per period

 n

Time Periods

Number of years or months

Risk-Free Rate + Risk Premium

How do we choose the discount rate r in our formulas?

$$\text{Discount Rate} = \text{Risk-Free Rate} + \text{Risk Premium}$$



Risk-Free Rate

Yield on safe government bonds (e.g., Egyptian T-bills, US Treasuries).

Egypt T-bills 2025: ~24% · US Treasuries 2025: ~4%



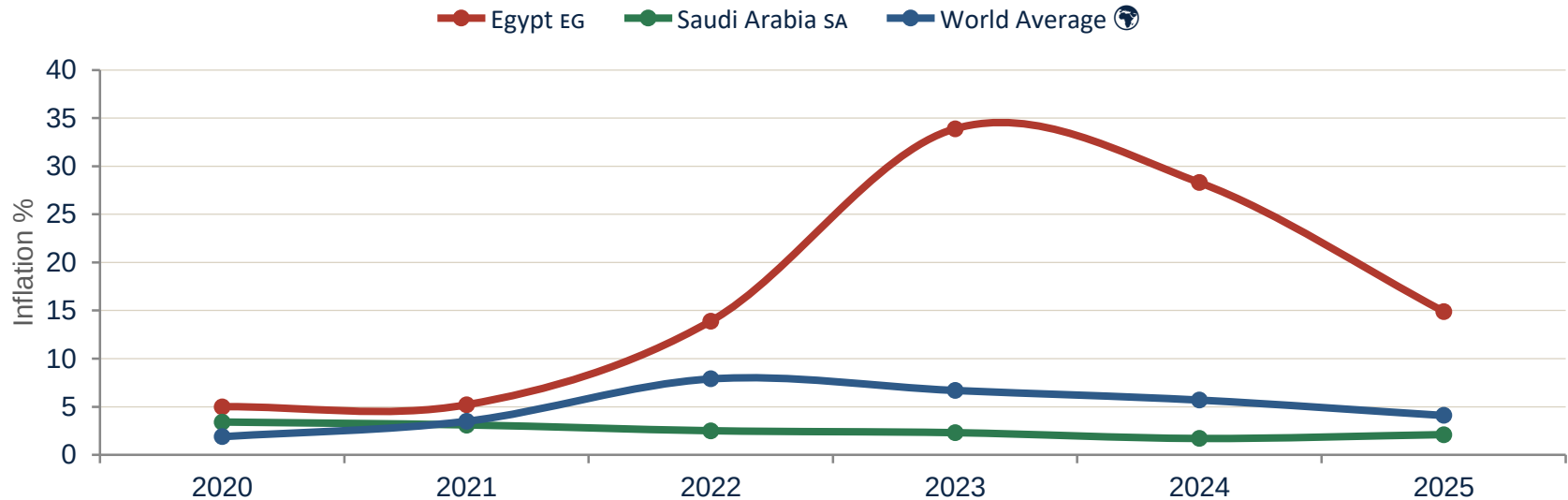
Risk Premium

Extra return demanded for project-specific risks: liquidity, location, tenant default, market.

Real estate premium typically: 3% – 8% above risk-free rate

Inflation Rates 2020–2025

Why this matters: inflation eats real returns on real estate investments.



Egypt 2023: 33.9% | Saudi 2025: 2.1% | Global avg 2025: 4.1% — Source: IMF, World Bank, CBE, GASTAT

Future Value (FV)

What is Future Value?

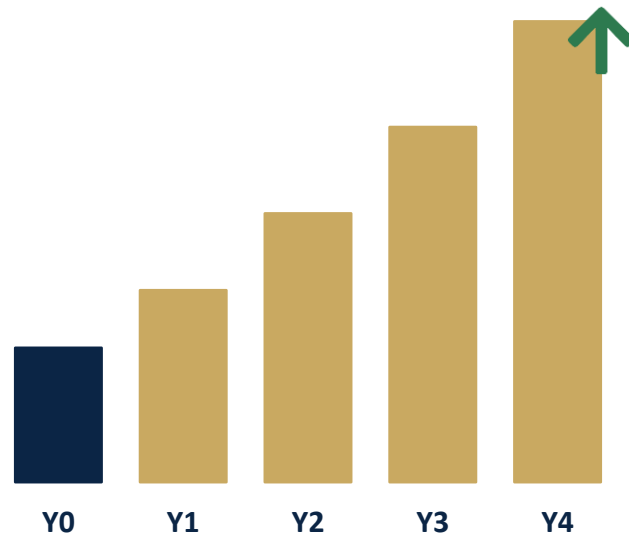
FV is the value of money TODAY compounded forward to a FUTURE date — adding earned interest year by year.

Formula

$$FV = PV \times (1 + r)^n$$

PV = today's amount · r = rate · n = years

Visualization — money grows over time



Future Value of EGP 500,000



Problem: You invest EGP 500,000 today at 12% per year. What will it be worth in 10 years?

Step-by-step solution:

1 Identify $PV = 500,000$ $r = 12\% = 0.12$ $n = 10$

2 Apply formula $FV = PV \times (1 + r)^n$

3 Substitute $FV = 500,000 \times (1.12)^{10} = 500,000 \times 3.1058$

4 Result $FV \approx \text{EGP } 1,552,924$

Your money triples in 10 years thanks to compounding!

Present Value (PV)

What is Present Value?

PV is the value TODAY of money you will receive in the FUTURE — "discounted" backwards by removing interest.

Formula

$$PV = FV \div (1 + r)^n$$

FV = future amount · r = rate · n = years

Discounting back to today

PV
(today)

FV
(future)

$\div (1+r)^n$

Money in the future is worth less today.



Problem: The future value of an investment equal EGP 1,552,924 after 10 years at 12% per year. What will it be worth today?

Step-by-step solution:

1 Identify $FV = 1,552,924 \quad \cdot \quad r = 12\% = 0.12 \quad \cdot \quad n = 10$

2 Apply formula $PV = FV / (1 + r)^n$

3 Substitute $PV = 1,552,924 / (1.12)^{10} = 1,552,924 / 3.1058$

4 Result $PV \approx \text{EGP } 500,000$

Compounding vs. Discounting

↑ **COMPOUNDING** → Forward in time

PV → FV

Use FV when:

- Saving for a future down payment
- Forecasting property appreciation
- Projecting reserve fund balances

↓ **DISCOUNTING** ← Back to today

PV ← FV

Use PV when:

- Valuing future rental income today
- Pricing a property's expected sale
- Comparing investment alternatives

Simple vs. Compound Interest

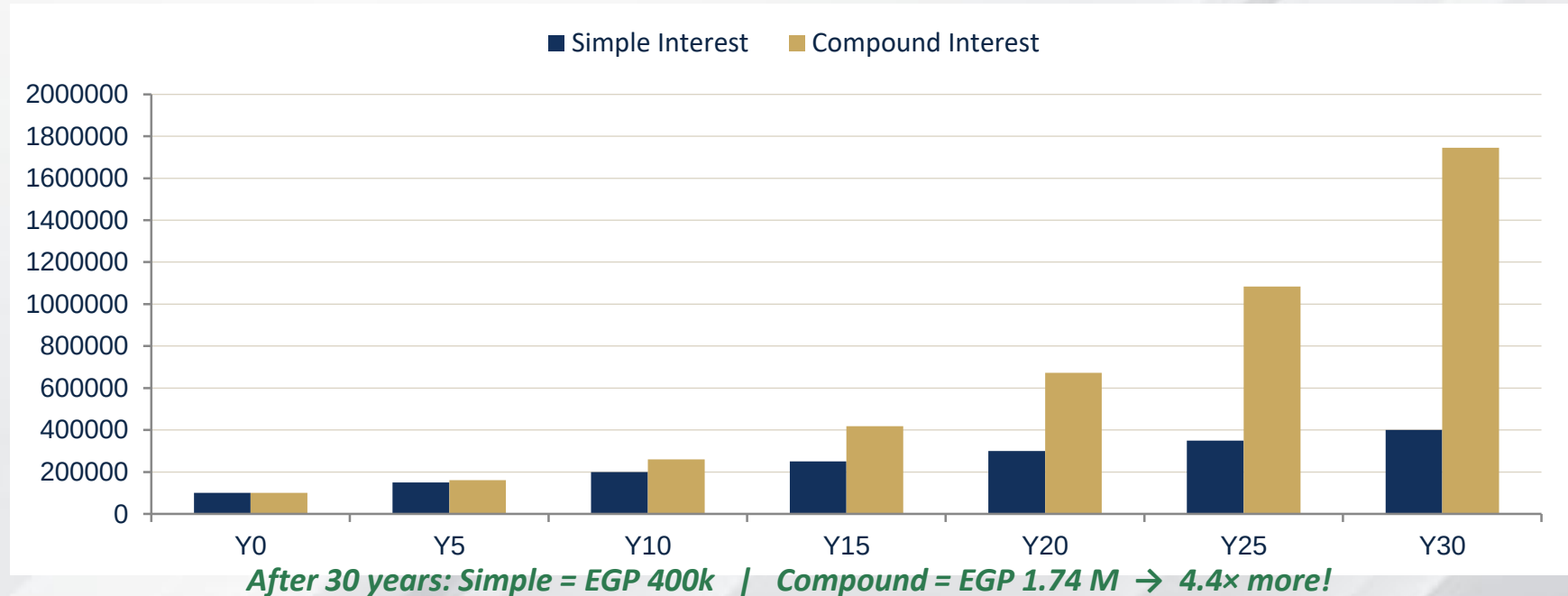
FEATURE	SIMPLE INTEREST	COMPOUND INTEREST
Interest is earned on	Original principal only	Principal + accumulated interest
Formula	$FV = PV \times (1 + r \cdot n)$	$FV = PV \times (1 + r)^n$
EGP 100,000 at 10% — 5 yrs	EGP 150,000	EGP 161,051
Used in	Short-term loans, T-bills	Mortgages, investments, savings



KEY INSIGHT: Compound interest = "interest on interest" — it's why time + reinvestment build wealth.

The Power of Compounding

EGP 100,000 invested at 10% for 30 years — see the difference:



More Periods = More Growth

Same 12% annual rate. Same EGP 100,000. The frequency changes the result:

FREQUENCY	PERIODS / YEAR (m)	FORMULA	FV after 1 year
Annual	1	$(1 + 0.12)^1$	112,000
Semi-annual	2	$(1 + 0.06)^2$	112,360
Quarterly	4	$(1 + 0.03)^4$	112,551
Monthly	12	$(1 + 0.01)^{12}$	112,683
Daily	365	$(1 + 0.12/365)^{365}$	112,747

Formula: $FV = PV \times (1 + r/m)^{(m \cdot n)}$

Nominal (APR) vs. Effective (EAR)

NOMINAL • APR

The stated, advertised yearly rate.

It does NOT account for compounding within the year.

Example: 18% APR on a mortgage

EFFECTIVE • EAR

The TRUE annual rate after compounding.

It captures "interest on interest" within the year.

Always $EAR \geq APR$ (equal only if $m = 1$)

CONVERSION FORMULA

$$EAR = (1 + APR / m)^m - 1$$

Example: APR 18%, monthly compounding $\rightarrow$ EAR $\approx$ 19.56%

What is an Annuity?



An annuity is a series of equal payments made at regular intervals.

Ordinary Annuity

Payments at the END of each period.

- Most mortgages
- Most loan repayments
- Bond coupon payments



Annuity Due

Payments at the BEGINNING of each period.

- Apartment rent payments
- Lease contracts
- Insurance premiums



Present Value of an Annuity (PVA)

How much should I pay TODAY for a stream of EQUAL future cash flows?

$$PVA = PMT \times [1 - (1 + r)^{-n}] \div r$$

PMT = each payment · r = rate per period · n = number of payments

Real estate uses:



Mortgage Pricing

Loan amount = PV of all monthly payments



Rental Valuation

Property value = PV of expected rents



Bond Pricing

Bond price = PV of coupon stream

Valuing 15 Years of Rental Income



Problem: A property promises EGP 100,000 per year for 15 years. At a 10% discount rate, what is its value today?

1 Identify

$$PMT = 100,000 \quad \cdot \quad r = 10\% = 0.10 \quad \cdot \quad n = 15$$

2 Apply formula

$$PVA = PMT \times [1 - (1 + r)^{-n}] \div r$$

3 Substitute

$$PVA = 100,000 \times [1 - (1.10)^{-15}] \div 0.10$$

4 Compute

$$PVA = 100,000 \times 7.6061$$

5 Result

$$PVA \approx \text{EGP } 760,608$$

Maximum price to pay today for this property: EGP 760,608

Future Value of an Annuity (FVA)

If I save the SAME amount every period, how much will I have in the FUTURE?

$$\text{FVA} = \text{PMT} \times [(1 + r)^n - 1] \div r$$

PMT = each deposit · r = rate per period · n = number of deposits



Quick Example — Saving for a Down Payment

Deposit EGP 50,000 at the end of each year for 8 years at 10% annual interest.

$$\text{FVA} = 50,000 \times [(1.10)^8 - 1] \div 0.10 = 50,000 \times 11.4359 \approx \text{EGP } 571,795$$

→ You contributed 400,000; interest earned you ~ 171,795

Calculating a Mortgage Payment



Problem: An EGP 2,000,000 mortgage at 18% annual interest, repaid monthly over 20 years. What is the monthly payment?

1 Convert to monthly $r = 18\% / 12 = 1.5\% = 0.015$ $n = 20 \times 12 = 240$

2 Annuity formula $PV = PMT \times [1 - (1+r)^{-n}] \div r$

3 Solve for PMT $PMT = PV \div \{ [1 - (1+r)^{-n}] \div r \}$

4 Substitute $PMT = 2,000,000 \div 64.7956$

5 Result Monthly payment $\approx$ EGP 30,866

Total paid over 20 years $\approx$ EGP 7.41 M $\rightarrow$ EGP 5.41 M is interest!

Cash Flows That Last Forever

 *A perpetuity is a stream of EQUAL payments that continues FOREVER.*

Perpetuity Formula

$$PV = PMT / r$$

PMT = constant cash flow · r = rate

In Real Estate

$$\text{Property Value} = \text{NOI} / \text{Cap Rate}$$

This is the perpetuity formula in disguise!



Example: Property generates EGP 200,000 / year forever. Cap rate = 10%.

$$\text{Property Value} = 200,000 \div 0.10 = \text{EGP } 2,000,000$$

Real vs. Nominal Returns

Fisher Equation

$$(1 + \text{nominal}) = (1 + \text{real}) \times (1 + \text{inflation})$$

Approximation: real $\approx$ nominal – inflation (works when both are small)

Worked Example 4 – Egyptian property at 14% nominal, 10% inflation:

$$\text{Real return} = (1 + 0.14) / (1 + 0.10) - 1$$

$$\text{Real return} = 1.14 / 1.10 - 1$$

$$\text{Real return} = 1.0364 - 1$$

$$\text{Real return} \approx 3.64\%$$

Lesson: a "high" 14% nominal return is actually only ~3.6% in real, purchasing-power terms.

Egyptian Inflation vs. Property Returns

Assumed nominal property return: 18% per year. See how inflation changes the picture.

YEAR	EGYPT INFLATION	NOMINAL RETURN	REAL RETURN	PURCHASING POWER
2020	5.0%	18%	12.4%	Strong gain ↑
2021	5.2%	18%	12.2%	Strong gain ↑
2022	13.9%	18%	3.6%	Modest gain →
2023	33.9%	18%	-11.9%	Real LOSS ↓
2024	28.3%	18%	-8.0%	Real LOSS ↓
2025	14.9%	18%	2.7%	Modest gain →

⚠ In high-inflation years (2023 – 2024), Egyptian investors actually LOST purchasing power on "good" 18% returns.

Sources: World Bank, IMF, CBE, FocusEconomics

Three Markets, Three Realities

How a real estate investor's environment differs across regions:

EGYPT EG



Mortgage rate: ~18 – 28%



Avg inflation 2020-25: ~16.9%



High inflation, EGP devaluation

SAUDI ARABIA SA



Mortgage rate: ~4.1 – 5.0%



Avg inflation 2020-25: ~2.5%



Currency pegged to USD (stable)

WORLD AVERAGE



Advanced economies: 3 – 7%



Global avg 2020-25: ~4.9%



Fed cycle drives global rates

Sources: SAMA, IMF World Economic Outlook, CFR Inflation Tracker

INVESTMENT DECISIONS Net Present Value

Two Cases of Cash Flows

Unequal Cash Flows

Each year's cash flow is different — discount every year on its own.

$$NPV = -C_0 + CF_1/(1+r)^1 + CF_2/(1+r)^2 + \dots + CF_n/(1+r)^n$$

Use when: rents, resale, or income change from year to year (the realistic case in real estate).

Step: discount each year separately, add the PVs, then subtract the cost C_0 .

Equal Cash Flows (Annuity)

The same cash flow repeats every year — use the annuity shortcut.

$$NPV = -C_0 + CF \times [1 - (1+r)^{-n}] \div r$$

Use when: a fixed rent / equal instalment is received for n equal periods.

The bracket $[1 - (1+r)^{-n}] \div r$ is the present-value annuity factor.

Worked Exercise

The Problem

You buy a small rental property for **\$200,000**.

Expected net cash flows:

Year 1: \$50,000

Year 2: \$70,000

Year 3: \$90,000

Year 4: \$100,000 (incl. resale)

Required return $r = 10\%$

Cash flows differ each year → use the unequal-cash-flow formula.

The Solution

Year	Cash flow	PV factor @10%	Present value
1	\$50,000	0.9091	\$45,455
2	\$70,000	0.8264	\$57,851
3	\$90,000	0.7513	\$67,618
4	\$100,000	0.6830	\$68,301
$\Sigma PV =$			\$239,225

$$NPV = \$239,225 - \$200,000 = +\$39,225$$



NPV > 0 → Accept. The property adds \$39,225 of value over the 10% required return.

Worked Exercise · Annuity

The Problem

You buy a rental apartment for **\$100,000**.

It earns the **same net rent every year**:

\$20,000 / year
for **n = 10 years**

Required return **r = 8%**

Cash flows are equal → use the annuity formula (no year-by-year table needed).

The Solution

$$NPV = -C_0 + CF \times [1 - (1+r)^{-n}] \div r$$

- 1 Annuity factor = $[1 - (1.08)^{-10}] \div 0.08 = 6.7101$
- 2 PV of rents = $\$20,000 \times 6.7101 = \$134,202$

$$NPV = \$134,202 - \$100,000 = +\$34,202$$

- ✓ **NPV > 0 → Accept.** The apartment adds \$34,202 of value above the 8% required return.

Common TVM Mistakes to Avoid



Mixing periods

Using annual rate with monthly payments — always match r and n to the same period.



Ignoring inflation

Comparing nominal returns across countries without adjusting for inflation.



Wrong sign in Excel

Forgetting that loan amount is positive but PMT is negative (or vice versa) breaks PMT/PV.



APR vs. EAR mix-up

Quoting APR (18%) but actually paying EAR (~19.6%) when compounding monthly.



18-Questions

Solved Question Bank

Foundations of TVM

Q1

MCQ

The core idea behind "time value of money" is that:

- A) Money loses value because banks charge fees.
- B) **EGP 1 today can earn interest, so it is worth more than EGP 1 in the future.**
- C) Inflation is always higher than interest rates.
- D) Real estate prices never go down.

Answer: B. TVM is built on opportunity cost — money today can be invested to earn a return.

Q2

T / F

Compound interest pays interest on both the original principal AND any previously earned interest.

Answer: TRUE. Compound = "interest on interest". Simple interest applies only to the original principal.

Future & Present Value

Q3

MCQ

EGP 500,000 invested today at 12% per year becomes approximately how much after 10 years?

- A) EGP 1,100,000
- B) EGP 1,552,924
- C) EGP 1,800,000
- D) EGP 2,500,000

Answer: B. $FV = 500,000 \times (1.12)^{10} = 500,000 \times 3.1058 \approx \text{EGP } 1,552,924$ (Worked Example 1).

Q4

MCQ

Which of the following increases the FUTURE value of a single deposit?

- A) A higher interest rate
- B) A longer time horizon
- C) More frequent compounding
- D) All of the above

Answer: D. All three boost FV. They all enlarge the $(1 + r/m)^{(m \cdot n)}$ factor.

Discounting & Annuities

Q5

MCQ

A property promises EGP 100,000 per year for 15 years at a 10% discount rate. Its present value is closest to:

- A) EGP 500,000
- B) EGP 760,608**
- C) EGP 1,000,000
- D) EGP 1,500,000

Answer: B. $PVA = 100,000 \times [1 - (1.10)^{-15}] / 0.10 \approx \text{EGP } 760,608$ (Worked Example 2).

Q6

T / F

An annuity due has its payments occurring at the BEGINNING of each period, while an ordinary annuity has them at the END.

Answer: TRUE. Apartment rents = annuity due (paid at start). Most mortgages = ordinary annuity (paid at end).

Mortgage Math

Q7

MCQ

An EGP 2,000,000 mortgage at 18% annual interest, repaid monthly over 20 years, requires a monthly payment of approximately:

- A) EGP 17,500
- B) EGP 24,000
- C) EGP 30,866
- D) EGP 42,000

Answer: C. $r = 1.5\%/month$, $n = 240$. $PMT = 2,000,000 / 64.7956 \approx \text{EGP } 30,866$ (Worked Example 3).

Q8

T / F

If you double the term of a fixed-rate mortgage from 20 to 40 years, the total interest paid will less than double, and may stay roughly the same.

Answer: FALSE. Doubling the term INCREASES total interest dramatically (often more than 2×) because the balance is paid down more slowly.

APR, EAR & Inflation

Q10

MCQ

If your nominal return is 14% and inflation is 10%, your REAL return is approximately:

- A) 24%
- B) 4%
- C) 3.64%
- D) -4%

Answer: C. Fisher: $\text{real} = (1.14 / 1.10) - 1 \approx 3.64\%$ (Worked Example 4).

Perpetuities & Cap Rates

Q11

MCQ

A property generates EGP 200,000 per year forever, with a 10% cap rate. Its value is:

- A) EGP 200,000
- B) EGP 1,000,000
- C) EGP 2,000,000
- D) EGP 20,000,000

Answer: C. $PV \text{ of perpetuity} = PMT / r = 200,000 / 0.10 = \text{EGP } 2,000,000.$

Q12

T / F

The formula "Property Value = NOI / Cap Rate" is mathematically a perpetuity formula.

Answer: TRUE. It assumes constant NOI forever, exactly the perpetuity assumption: $PV = PMT / r.$

Mixed Review

Q13 · TF

Egypt's high inflation in 2023 ($\approx 33.9\%$) means an 18% nominal return produced a NEGATIVE real return.

✓ TRUE. Real = $(1.18/1.339) - 1 \approx -11.9\%$.

Q14 · TF

Saudi Arabia's mortgage rates are typically lower than Egypt's because Saudi inflation and policy rates are lower.

✓ TRUE. SAMA repo rate $\approx 4.25\%$ (2025); Saudi mortgage rates $\approx 4-5\%$.

Q15 · MCQ

The discount rate used in real estate valuation typically equals:

A) Risk-free rate only B) Inflation rate only C) Risk-free rate + risk premium D) Mortgage rate

✓ C. $r = \text{risk-free} + \text{premium for project-specific risk}$.

Q16 · MCQ

If you save EGP 50,000 / year for 8 years at 10%, your accumulated balance is closest to:

A) EGP 400,000 B) EGP 571,795 C) EGP 700,000 D) EGP 800,000

✓ B. $FVA = 50,000 \times [(1.10)^8 - 1] / 0.10 \approx 571,795$.

Q17 · TF

If the discount rate increases, the present value of future cash flows decreases.

✓ TRUE. Higher $r \rightarrow$ bigger denominator $(1+r)^n \rightarrow$ smaller PV.

Q18 · MCQ

Global inflation peaked in 2022 at about ____ before declining.

A) 3% B) 5% C) 7.9% D) 15%

✓ C. Per IMF/World Bank: world CPI averaged 7.93% in 2022.

Thank You