

Introduction to Finance

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MGM 237
Time Value of Money

Lecture 4



Chapter

4

Time Value of Money

Learning Outcomes

By the end of this lecture, students will be able to:

- Understand the time value of money.
- Calculate future and present values.
- Adjust rates for compounding frequency.
- Evaluate annuities and perpetuities.
- Apply TVM in financial decisions.

Why Money Has Time Value

- Money available today can be invested to earn a return.
- Waiting for money creates opportunity cost, inflation risk, and uncertainty.
- Financial decisions compare cash flows occurring at different points in time.
- Time value of money is the language of valuation: loans, bonds, stocks, projects, and retirement planning.



Teaching sentence

A cash flow is not fully comparable until we place it on the same date.

Opportunity Cost of Capital

- The opportunity cost of capital is the return investors give up when funds are committed to a project or asset.
- It is the minimum acceptable return for investments with similar risk.
- In TVM, the interest rate or discount rate represents this opportunity cost.
- A higher risk cash flow should generally be discounted at a higher rate.

Core valuation idea

Value today depends on expected cash flow, time, risk, and required return.

Decision	Question answered by cost of capital
Investment	Is the return high enough?
Financing	What is the cost of funds?
Valuation	What rate should discount cash flows?

Example

If a bank deposit earns 8%, investing in a project returning only 5% creates an opportunity loss before considering risk.

Simple Interest

Simple interest

Explanation: Interest = Principal $\times$ rate $\times$ time

Short: $I = P \times r \times t$

Future value: $FV = P + I$

- Interest is calculated only on the original amount.
- The interest base does not grow over time.
- Useful for short-term notes and introductory comparison with compound interest.

Example - simple interest

Principal = \$10,000

Rate = 8%

Time = 3 years

Interest = $10,000 \times 8\% \times 3 = \$2,400$

Future value = \$12,400

Interpretation

The investor earns the same \$800 interest every year because interest is not reinvested.

Compound Interest

Compound interest

Explanation: $FV = \text{Present amount} \times \text{compound growth factor}$

Short: $FV = PV \times (1 + i)^n$

- Compounding means interest earns interest.
- The account balance becomes the new base for future interest.
- Small rate differences become powerful over long periods.
- Factor tables summarize the compound growth factor $(1 + i)^n$.

Using the factor table

Present value = \$10,000

Periodic rate = 8%

Number of periods = 3 years

From Table 1: future value factor = 1.25971

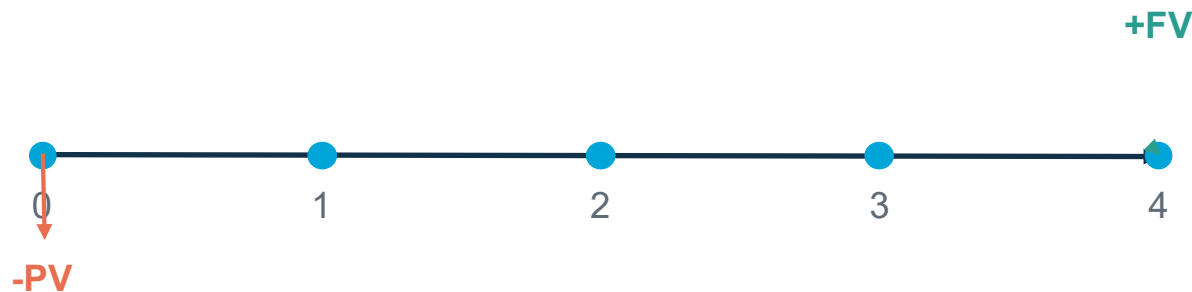
Future value = $10,000 \times 1.25971 = \$12,597.10$

Teaching point

Compound interest produces a higher future value than simple interest when interest is reinvested.

Future Value Concept

- Future value (FV) answers: What will today's amount become in the future?
- FV is used in saving plans, investment growth, sinking funds, and project terminal values.
- Moving forward in time is called compounding.
- The longer the time or the higher the rate, the higher the future value.



Core action

To find FV, multiply the present amount by a future value factor.

Future Value Formula and Table Logic

Single amount

Explanation: $FV = \text{Present amount} \times \text{table factor}$

Short: $FV = PV \times FVIF(i, n)$

$FVIF(i, n) = (1 + i)^n$

- **Step 1:** identify the interest rate (i).
- **Step 2:** identify the number of periods (n).
- **Step 3:** locate the factor in Table 1.
- **Step 4:** multiply the original amount by the factor.

TABLE 1 Future Value of \$1

$FV = \$1 (1 + i)^n$

n/i	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	20.0%
1	1.01000	1.01500	1.02000	1.02500	1.03000	1.03500	1.04000	1.04500	1.05000	1.05500	1.06000	1.07000	1.08000	1.09000	1.10000	1.11000	1.12000	1.20000
2	1.02010	1.03022	1.04040	1.05063	1.06090	1.07123	1.08160	1.09203	1.10250	1.11303	1.12360	1.14490	1.16640	1.18810	1.21000	1.23210	1.25440	1.44000
3	1.03030	1.04568	1.06121	1.07689	1.09273	1.10872	1.12486	1.14117	1.15763	1.17424	1.19102	1.22504	1.25971	1.29503	1.33100	1.36763	1.40493	1.72800
4	1.04060	1.06136	1.08243	1.10381	1.12551	1.14752	1.16986	1.19252	1.21551	1.23882	1.26248	1.31080	1.36049	1.41158	1.46410	1.51807	1.57352	2.07360
5	1.05101	1.07728	1.10408	1.13141	1.15927	1.18769	1.21665	1.24618	1.27628	1.30696	1.33823	1.40255	1.46933	1.53862	1.61051	1.68506	1.76234	2.48832
6	1.06152	1.09344	1.12616	1.15969	1.19405	1.22926	1.26532	1.30226	1.34010	1.37884	1.41852	1.50073	1.58687	1.67710	1.77156	1.87041	1.97382	2.98598
7	1.07214	1.10984	1.14869	1.18869	1.22987	1.27228	1.31593	1.36086	1.40710	1.45468	1.50363	1.60578	1.71382	1.82804	1.94872	2.07616	2.21068	3.58318
8	1.08286	1.12649	1.17166	1.21840	1.26677	1.31681	1.36857	1.42210	1.47746	1.53469	1.59385	1.71819	1.85093	1.99256	2.14359	2.30454	2.47596	4.29982
9	1.09369	1.14339	1.19509	1.24886	1.30477	1.36290	1.42331	1.48610	1.55133	1.61909	1.68948	1.83846	1.99900	2.17189	2.35795	2.55804	2.77308	5.15978
10	1.10462	1.16054	1.21899	1.28008	1.34392	1.41060	1.48024	1.55297	1.62889	1.70814	1.79085	1.96715	2.15892	2.36736	2.59374	2.83942	3.10585	6.19174
11	1.11567	1.17795	1.24337	1.31209	1.38423	1.45997	1.53945	1.62285	1.71034	1.80209	1.89830	2.10485	2.33164	2.58043	2.85312	3.15176	3.47855	7.43008
12	1.12683	1.19562	1.26824	1.34489	1.42576	1.51107	1.60103	1.69588	1.79586	1.90121	2.01220	2.25219	2.51817	2.81266	3.13843	3.49845	3.89598	8.91610
13	1.13809	1.21355	1.29361	1.37851	1.46853	1.56396	1.66507	1.77220	1.88565	2.00577	2.13293	2.40985	2.71962	3.06580	3.45227	3.88328	4.36349	10.69932
14	1.14947	1.23176	1.31948	1.41297	1.51259	1.61869	1.73168	1.85194	1.97993	2.11609	2.26090	2.57853	2.93719	3.34173	3.79750	4.31044	4.88711	12.83918
15	1.16097	1.25023	1.34587	1.44830	1.55797	1.67535	1.80094	1.93528	2.07893	2.23248	2.39656	2.75903	3.17217	3.64248	4.17725	4.78459	5.47357	15.40702
16	1.17258	1.26899	1.37279	1.48451	1.60471	1.73399	1.87298	2.02237	2.18287	2.35526	2.54035	2.95216	3.42594	3.97031	4.59497	5.31089	6.13039	18.48843

Table 1

Future Value of \$1: used when one amount is invested now and grows to a future amount.

Future Value of a Single Amount

Solved with factor table

Problem: You invest \$15,000 for 5 years at 6% annually.
What is the future value?

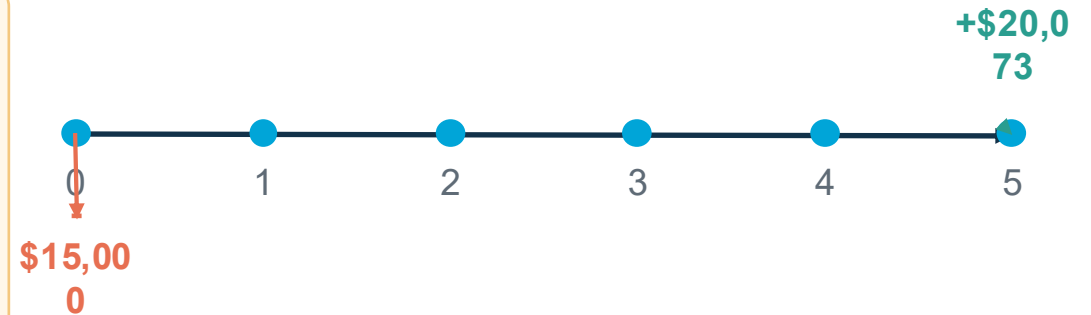
Table used: Table 1 - Future Value of \$1

Locate: row number of periods = 5, column periodic rate = 6%

Factor = 1.33823

Future value = $\$15,000 \times 1.33823 = \$20,073.45$

Short formula used: $FV = PV \times FVIF(i, n)$



Interpretation

After five years, the investment grows from \$15,000 to about \$20,073 because both principal and interest are compounded annually.

Compounding Frequency

Compounding may occur annually, semiannually, quarterly, monthly, daily, or continuously.

When compounding is more frequent, interest is credited more often.

For table-based problems, first convert the quoted annual rate into a periodic rate, then convert years into total periods.
 Periodic rate = quoted annual rate $\div$ compounding periods per year.

Number of periods = years $\times$ compounding periods per year.

Frequency	Periodic rate	Periods over 5 years
Annual	r	5
Semiannual	$r \div 2$	10
Quarterly	$r \div 4$	20
Monthly	$r \div 12$	60

Exam rule

Always match the rate with the period. If the table column is 4%, the period must be one 4% compounding period, not necessarily one year.

Annual Compounding

Annual compounding

Number of periods = number of years
Periodic rate = annual interest rate

- Interest is added once at the end of each year.
- The annual interest rate is used directly in the factor table.
- The number of periods equals the number of years.
- Most introductory TVM examples begin with annual compounding.

Annual example

Present value = \$10,000

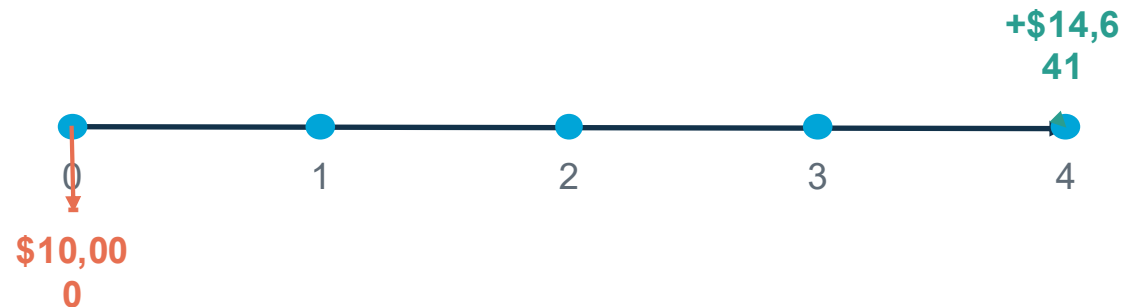
Annual rate = 10%

Time = 4 years

Table 1 factor at Number of periods = 4 and Periodic rate = 10% = 1.46410

Future value = $10,000 \times 1.46410 = \$14,641$

Short formula used: $FV = PV \times FVIF(i, n)$



Semiannual Compounding

Semiannual conversion

$$\text{Periodic rate} = \text{annual rate} \div 2$$
$$\text{Number of periods} = \text{years} \times 2$$

If the nominal annual rate is 8%, the semiannual rate is 4%.

If the investment lasts 3 years, the number of periods is 6.

Then use Table 1 with Periodic rate = 4% and Number of periods = 6.

Solved with table

Present value = \$10,000

Nominal rate = 8%, semiannual

Time = 3 years

Periodic rate = $8\% \div 2 = 4\%$

Number of periods = $3 \times 2 = 6$

Table 1 factor = 1.26532

Future value = \$12,653.20

Short formula used: $FV = PV \times FVIF(i, n)$

Interpretation

The problem has six compounding periods, not three, because interest is credited twice each year.

Quarterly and Monthly Compounding

Compounding	m	Periodic rate	Number of periods
Quarterly	4	$\text{APR} \div 4$	$\text{Years} \times 4$
Monthly	12	$\text{APR} \div 12$	$\text{Years} \times 12$
General rule	m	$\text{APR} \div m$	$\text{Years} \times m$

Using quarterly periods

Short formula used: $\text{FV} = \text{PV} \times \text{FVIF}(i, n)$

Quarterly example:

Present value = \$10,000, quoted annual rate = 8%,
time = 5 years

Periodic rate = $8\% \div 4 = 2\%$ per quarter

Number of periods = $5 \times 4 = 20$ quarters

Table 1 factor at Number of periods = 20, Periodic
rate = 2% = 1.48595

Future value = \$14,859.50

Important

If the exact periodic rate or period is not available in the factor tables, a formula, Excel, or calculator is needed. For this course, examples will be designed to match the provided tables.

Present Value Concept

- Present value (PV) answers: What is a future cash flow worth today?
- Moving backward in time is called discounting.
- PV is essential for valuing bonds, stocks, loans, leases, and investment projects.
- A dollar received later is worth less today when the discount rate is positive.

+PV

+\$FV



Core action

To find PV, multiply the future amount by a present value factor.

Present Value Formula and Table Logic

Single amount

Explanation: $PV = \text{Future amount} \times \text{table factor}$

Short: $PV = FV \times PVIF(i, n)$

$PVIF(i, n) = 1 \div (1 + i)^n$

- Use Table 2 for one future lump sum.
- Find the row for n and the column for i .
- The factor is always less than 1 when the rate is positive.
- Multiply the future value by the factor.

TABLE 2 Present Value of \$1

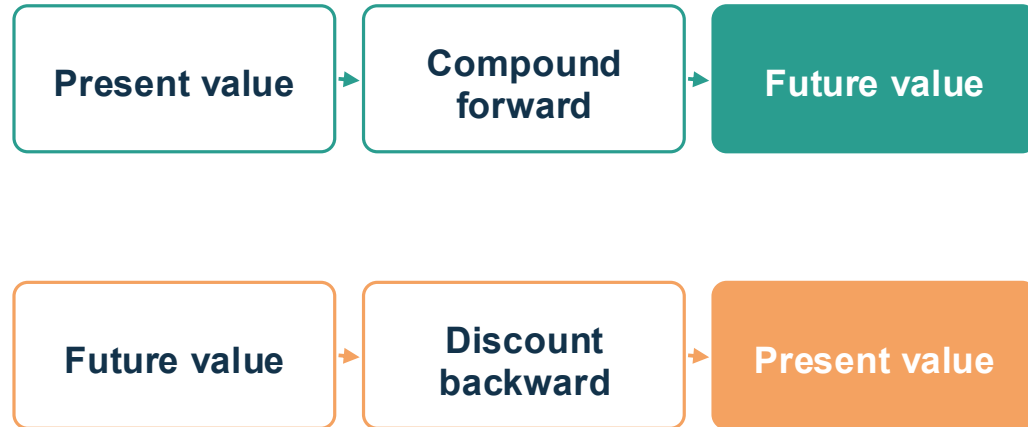
$$PV = \frac{\$1}{(1 + i)^n}$$

n/i	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	20.0%
1	0.99010	0.98522	0.98039	0.97561	0.97087	0.96618	0.96154	0.95694	0.95238	0.94787	0.94340	0.93458	0.92593	0.91743	0.90909	0.90090	0.89286	0.83333
2	0.98030	0.97066	0.96117	0.95181	0.94260	0.93351	0.92456	0.91573	0.90703	0.89845	0.89000	0.87344	0.85734	0.84168	0.82645	0.81162	0.79719	0.69444
3	0.97059	0.95632	0.94232	0.92860	0.91514	0.90194	0.88900	0.87630	0.86384	0.85161	0.83962	0.81630	0.79383	0.77218	0.75131	0.73119	0.71178	0.57870
4	0.96098	0.94218	0.92385	0.90595	0.88849	0.87144	0.85480	0.83856	0.82270	0.80722	0.79209	0.76290	0.73503	0.70843	0.68301	0.65873	0.63552	0.48225
5	0.95147	0.92826	0.90573	0.88385	0.86261	0.84197	0.82193	0.80245	0.78353	0.76513	0.74726	0.71299	0.68058	0.64993	0.62092	0.59345	0.56743	0.40188
6	0.94205	0.91454	0.88797	0.86230	0.83748	0.81350	0.79031	0.76790	0.74622	0.72525	0.70496	0.66634	0.63017	0.59627	0.56447	0.53464	0.50663	0.33490
7	0.93272	0.90103	0.87056	0.84127	0.81309	0.78599	0.75992	0.73483	0.71068	0.68744	0.66506	0.62275	0.58349	0.54703	0.51316	0.48166	0.45235	0.27908
8	0.92348	0.88771	0.85349	0.82075	0.78941	0.75941	0.73069	0.70319	0.67684	0.65160	0.62741	0.58201	0.54027	0.50187	0.46651	0.43393	0.40388	0.23257
9	0.91434	0.87459	0.83676	0.80073	0.76642	0.73373	0.70259	0.67290	0.64461	0.61763	0.59190	0.54393	0.50025	0.46043	0.42410	0.39092	0.36061	0.19381
10	0.90529	0.86167	0.82035	0.78120	0.74409	0.70892	0.67556	0.64393	0.61391	0.58543	0.55839	0.50835	0.46319	0.42241	0.38554	0.35218	0.32197	0.16151
11	0.89632	0.84893	0.80426	0.76214	0.72242	0.68495	0.64958	0.61620	0.58468	0.55491	0.52679	0.47509	0.42888	0.38753	0.35049	0.31728	0.28748	0.13459
12	0.88745	0.83639	0.78849	0.74356	0.70138	0.66178	0.62460	0.58966	0.55684	0.52598	0.49697	0.44401	0.39711	0.35553	0.31863	0.28584	0.25668	0.11216
13	0.87866	0.82403	0.77303	0.72542	0.68095	0.63940	0.60057	0.56427	0.53032	0.49856	0.46884	0.41496	0.36770	0.32618	0.28966	0.25751	0.22917	0.09346
14	0.86996	0.81185	0.75788	0.70773	0.66112	0.61778	0.57748	0.53997	0.50507	0.47257	0.44230	0.38782	0.34046	0.29925	0.26333	0.23199	0.20462	0.07789
15	0.86135	0.79985	0.74301	0.69047	0.64186	0.59689	0.55526	0.51672	0.48102	0.44793	0.41727	0.36245	0.31524	0.27454	0.23939	0.20900	0.18270	0.06491
16	0.85282	0.78803	0.72845	0.67362	0.62317	0.57671	0.53391	0.49447	0.45811	0.42458	0.39365	0.33873	0.29189	0.25187	0.21763	0.18829	0.16312	0.05409

Table 2

Present Value of \$1: used to discount one future cash flow back to today.

Discounting versus Compounding



- Compounding moves money forward in time.
- Discounting moves money backward in time.
- The same interest rate links PV and FV.
- Correct timing is the first step in solving any TVM problem.

Reciprocal idea

If \$1 today becomes \$1.33823 in five years at 6%, then \$1.33823 received in five years is worth \$1 today at 6%.

The same rate works in opposite directions.

Table connection

Table 1 grows \$1 forward. Table 2 discounts \$1 backward.

Discount Rate Interpretation

- The discount rate is the return required to accept a future cash flow today.
- It reflects time, risk, inflation expectations, and alternative investment opportunities.
- Higher discount rates reduce present value.
- Riskier cash flows are usually discounted at higher rates.

Discount rate	Effect on PV	Reason
Low rate	Higher PV	Cash flow is safer or opportunity cost is low
High rate	Lower PV	Cash flow is riskier or opportunity cost is high

Managerial meaning

The discount rate is not just a mathematical input. It is a financial judgment about risk and required return.

Relationship between PV, FV, Rate, and Time

Variable	Meaning	Effect if it increases
PV	Amount today	FV increases
FV	Amount in the future	PV required increases
i	Interest or discount rate	FV increases; PV decreases
n	Number of periods	FV increases; PV decreases

Two sides of the same relationship

Explanation: FV grows money forward; PV discounts money backward

Short: $FV = PV \times (1+i)^n$; $PV = FV \div (1+i)^n$

Interpretation

PV, FV, i, and n are linked. If you know three variables, you can solve for the fourth.

Table approach

Tables are fastest when i and n are given and the factor is directly available.

Solving for Future Value

When PV, i, and n are known

Explanation: $FV = \text{Present amount} \times \text{table factor}$

Short: $FV = PV \times FVIF(i, n)$

Factor table solution

Problem: \$25,000 is invested for 10 years at 7%.

Table used: Table 1

Row: Number of periods = 10

Column: Periodic rate = 7%

Factor = 1.96715

Future value = $25,000 \times 1.96715 = \$49,178.75$

Short formula used: $FV = PV \times FVIF(i, n)$

TABLE 1 Future Value of \$1

$$FV = \$1(1 + i)^n$$

n/i	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	20.0%
1	1.01000	1.01500	1.02000	1.02500	1.03000	1.03500	1.04000	1.04500	1.05000	1.05500	1.06000	1.07000	1.08000	1.09000	1.10000	1.11000	1.12000	1.20000
2	1.02010	1.03022	1.04040	1.05063	1.06090	1.07123	1.08160	1.09203	1.10250	1.11303	1.12360	1.14490	1.16640	1.18810	1.21000	1.23210	1.25440	1.44000
3	1.03030	1.04568	1.06121	1.07689	1.09273	1.10872	1.12486	1.14117	1.15763	1.17424	1.19102	1.22504	1.25971	1.29503	1.33100	1.36763	1.40493	1.72800
4	1.04060	1.06136	1.08243	1.10381	1.12551	1.14752	1.16986	1.19252	1.21551	1.23882	1.26248	1.31080	1.36049	1.41158	1.46410	1.51807	1.57352	2.07360
5	1.05101	1.07728	1.10408	1.13141	1.15927	1.18769	1.21665	1.24618	1.27628	1.30696	1.33823	1.40255	1.46933	1.53862	1.61051	1.68506	1.76234	2.48832
6	1.06152	1.09344	1.12616	1.15969	1.19405	1.22926	1.26532	1.30226	1.34010	1.37884	1.41852	1.50073	1.58687	1.67710	1.77156	1.87041	1.97382	2.98598
7	1.07214	1.10984	1.14869	1.18869	1.22987	1.27228	1.31593	1.36086	1.40710	1.45468	1.50363	1.60578	1.71382	1.82804	1.94872	2.07616	2.21068	3.58318
8	1.08286	1.12649	1.17166	1.21840	1.26677	1.31681	1.36857	1.42210	1.47746	1.53469	1.59385	1.71819	1.85093	1.99256	2.14359	2.30454	2.47596	4.29982
9	1.09369	1.14339	1.19509	1.24886	1.30477	1.36290	1.42331	1.48610	1.55133	1.61909	1.68948	1.83846	1.99900	2.17189	2.35795	2.55804	2.77308	5.15978
10	1.10462	1.16054	1.21899	1.28008	1.34392	1.41060	1.48024	1.55297	1.62889	1.70814	1.79085	1.96715	2.15892	2.36736	2.59374	2.83942	3.10585	6.19174
11	1.11567	1.17795	1.24337	1.31209	1.38423	1.45997	1.53945	1.62285	1.71034	1.80209	1.89830	2.10485	2.33164	2.58043	2.85312	3.15176	3.47855	7.43008
12	1.12683	1.19562	1.26824	1.34489	1.42576	1.51107	1.60103	1.69588	1.79586	1.90121	2.01220	2.25219	2.51817	2.81266	3.13843	3.49845	3.89598	8.91610
13	1.13809	1.21355	1.29361	1.37851	1.46853	1.56396	1.66507	1.77220	1.88565	2.00577	2.13293	2.40985	2.71962	3.06580	3.45227	3.88328	4.36349	10.69932
14	1.14947	1.23176	1.31948	1.41297	1.51259	1.61869	1.73168	1.85194	1.97993	2.11609	2.26090	2.57853	2.93719	3.34173	3.79750	4.31044	4.88711	12.83918
15	1.16097	1.25023	1.34587	1.44830	1.55797	1.67535	1.80094	1.93528	2.07893	2.23248	2.39656	2.75903	3.17217	3.64248	4.17725	4.78459	5.47357	15.40702
16	1.17258	1.26899	1.37279	1.48451	1.60471	1.73399	1.87298	2.02237	2.18287	2.35526	2.54035	2.95216	3.42594	3.97031	4.59497	5.31089	6.13039	18.48843
17	1.18432	1.28680	1.39681	1.51451	1.64100	1.77699	1.92300	2.08000	2.24800	2.42800	2.62000	3.05000	3.55000	4.12000	4.77000	5.50000	6.32000	20.10000

Conclusion

The investment almost doubles because the factor is close to 2.0 after 10 years at 7%.

Solving for Present Value

When FV, i, and n are known

Explanation: $PV = \text{Future amount} \times \text{table factor}$

Short: $PV = FV \times PVIF(i, n)$

Factor table solution

Problem: You expect to receive \$50,000 in 5 years.
Required return is 8%. What is the value today?

Table used: Table 2

Row: Number of periods = 5

Column: Periodic rate = 8%

Factor = 0.68058

Present value = $50,000 \times 0.68058 = \$34,029$

Short formula used: $PV = FV \times PVIF(i, n)$

TABLE 2 Present Value of \$1

$$PV = \frac{\$1}{(1 + i)^n}$$

n/i	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	20.0%
1	0.99010	0.98522	0.98039	0.97561	0.97087	0.96618	0.96154	0.95694	0.95238	0.94787	0.94340	0.93458	0.92593	0.91743	0.90909	0.90090	0.89286	0.83333
2	0.98030	0.97066	0.96117	0.95181	0.94260	0.93351	0.92456	0.91573	0.90703	0.89845	0.89000	0.87344	0.85734	0.84168	0.82645	0.81162	0.79719	0.69444
3	0.97059	0.95632	0.94232	0.92860	0.91514	0.90194	0.88900	0.87630	0.86384	0.85161	0.83962	0.81630	0.79383	0.77218	0.75131	0.73119	0.71178	0.57870
4	0.96098	0.94218	0.92385	0.90595	0.88849	0.87144	0.85480	0.83856	0.82270	0.80722	0.79209	0.76290	0.73503	0.70843	0.68301	0.65873	0.63552	0.48225
5	0.95147	0.92826	0.90573	0.88385	0.86261	0.84197	0.82193	0.80245	0.78353	0.76513	0.74726	0.71299	0.68058	0.64993	0.62092	0.59345	0.56743	0.40188
6	0.94205	0.91454	0.88797	0.86230	0.83748	0.81350	0.79031	0.76790	0.74622	0.72525	0.70496	0.66634	0.63017	0.59627	0.56447	0.53464	0.50663	0.33490
7	0.93272	0.90103	0.87056	0.84127	0.81309	0.78599	0.75992	0.73483	0.71068	0.68744	0.66506	0.62275	0.58349	0.54703	0.51316	0.48166	0.45235	0.27908
8	0.92348	0.88771	0.85349	0.82075	0.78941	0.75941	0.73069	0.70319	0.67684	0.65160	0.62741	0.58201	0.54027	0.50187	0.46651	0.43393	0.40388	0.23257
9	0.91434	0.87459	0.83676	0.80073	0.76642	0.73373	0.70259	0.67290	0.64461	0.61763	0.59190	0.54393	0.50025	0.46043	0.42410	0.39092	0.36061	0.19381
10	0.90529	0.86167	0.82035	0.78120	0.74409	0.70892	0.67556	0.64393	0.61391	0.58543	0.55839	0.50835	0.46319	0.42241	0.38554	0.35218	0.32197	0.16151
11	0.89632	0.84893	0.80426	0.76214	0.72242	0.68495	0.64958	0.61620	0.58468	0.55491	0.52679	0.47509	0.42888	0.38753	0.35049	0.31728	0.28748	0.13459
12	0.88745	0.83639	0.78849	0.74356	0.70138	0.66178	0.62460	0.58966	0.55684	0.52598	0.49697	0.44401	0.39711	0.35553	0.31863	0.28584	0.25668	0.11216
13	0.87866	0.82403	0.77303	0.72542	0.68095	0.63940	0.60057	0.56427	0.53032	0.49856	0.46884	0.41496	0.36770	0.32618	0.28966	0.25751	0.22917	0.09346
14	0.86996	0.81185	0.75788	0.70773	0.66112	0.61778	0.57748	0.53997	0.50507	0.47257	0.44230	0.38782	0.34046	0.29925	0.26333	0.23199	0.20462	0.07789
15	0.86135	0.79985	0.74301	0.69047	0.64186	0.59689	0.55526	0.51672	0.48102	0.44793	0.41727	0.36245	0.31524	0.27454	0.23939	0.20900	0.18270	0.06491
16	0.85282	0.78803	0.72845	0.67362	0.62317	0.57671	0.53391	0.49447	0.45811	0.42458	0.39365	0.33873	0.29189	0.25187	0.21763	0.18829	0.16312	0.05409

Conclusion

A future amount is worth less today because investors require an 8% return for waiting.

Solving for Interest Rate

Table search method

Explanation: Factor = Future amount ÷ Present amount

Short: $FVIF = FV \div PV$

Solved by locating the factor

Problem: \$10,000 grows to \$13,382.30 in 5 years.

What rate was earned?

Step 1: Factor = $13,382.30 \div 10,000 = 1.33823$

Step 2: Go to Table 1, row number of periods = 5

Step 3: Find the factor closest to 1.33823

Answer: 6%

Why Table 1?

The problem moves from present amount to future amount, so the factor must be a future value factor.

n = 5	5%	6%	7%
FV factor	1.27628	1.33823	1.40255

Exam tip

If the exact factor is not available, choose the closest factor or use interpolation only if required by the instructor.

Solving for Number of Periods

Table search method

Explanation: Factor = Future amount ÷ Present amount

Short: FVIF = FV ÷ PV

Solved by locating the row

Problem: How long will it take \$20,000 to grow to \$31,877 at 6%?

Step 1: Factor = $31,877 \div 20,000 = 1.59385$

Step 2: Go to Table 1, column periodic rate = 6%

Step 3: Find factor 1.59385

Answer: 8 years

Column 6%	n = 7	n = 8	n = 9
FV factor	1.50363	1.59385	1.68948

Interpretation

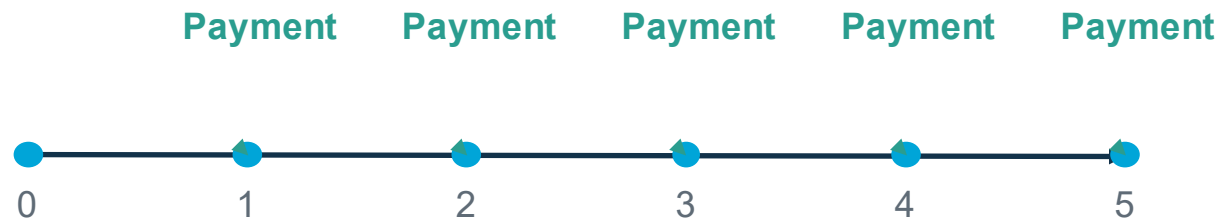
At 6%, the balance reaches the required future value after approximately eight annual compounding periods.

Common mistake

Do not confuse years with periods when compounding is not annual.

Annuity Definition

- An annuity is a stream of equal cash flows paid at regular intervals.
- Examples: rent payments, loan installments, pension receipts, annual deposits, and bond coupons.
- Annuities require special tables because there is more than one cash flow.
- The timing of payments determines whether it is an ordinary annuity or annuity due.



Ordinary Annuity

Timing rule

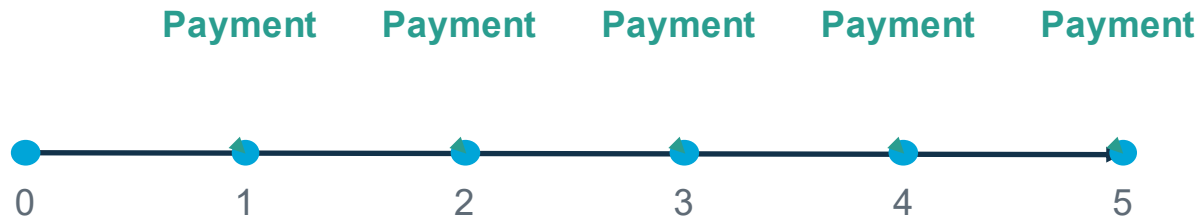
Payments occur at the END of each period

Examples

End-of-month rent paid after using the apartment;
annual savings deposit made at year-end.

Table selection

Use ordinary annuity tables when the phrase says
“end of each year/period” or is silent about timing.



- Most financial tables assume ordinary annuity unless stated otherwise.
- Loan payments are commonly treated as ordinary annuities.
- Bond coupons are usually valued as ordinary annuities.
- Use Table 3 for future value and Table 4 for present value.

Annuity Due

Timing rule

Payments occur at the BEGINNING of each period

Payment Payment Payment Payment Payment



- Rent payments are often paid at the beginning of the month.
- Insurance premiums may be paid at the start of coverage.
- Annuity due values are higher than ordinary annuity values when all else is equal.
- Use Table 5 for future value and Table 6 for present value.

Key comparison

Ordinary annuity: first payment at period 1.
Annuity due: first payment at period 0.

Financial effect

Because each payment occurs earlier, annuity due has one extra compounding or one less discounting period.

Future Value of an Ordinary Annuity

Ordinary annuity

Explanation: Annuity FV = Equal payment × annuity FV factor

Short: $FVA = PMT \times FVIFA(i, n)$

Solved with table

Problem: Deposit \$2,000 at the end of each year for 5 years at 6%.

Table used: Table 3

Row: Number of periods = 5

Column: Periodic rate = 6%

Factor = 5.6371

Future value = $2,000 \times 5.6371 = \$11,274.20$

Short formula used: $FVA = PMT \times FVIFA(i, n)$

TABLE 3 Future Value of an Ordinary Annuity of \$1

$$FVA = \frac{(1+i)^n - 1}{i}$$

n/i	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	20.0%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0150	2.0200	2.0250	2.0300	2.0350	2.0400	2.0450	2.0500	2.0550	2.0600	2.0700	2.0800	2.0900	2.1000	2.1100	2.1200	2.2000
3	3.0301	3.0452	3.0604	3.0756	3.0909	3.1062	3.1216	3.1370	3.1525	3.1680	3.1836	3.2149	3.2464	3.2781	3.3100	3.3421	3.3744	3.6400
4	4.0604	4.0909	4.1216	4.1525	4.1836	4.2149	4.2465	4.2782	4.3101	4.3423	4.3746	4.4399	4.5061	4.5731	4.6410	4.7097	4.7793	5.3680
5	5.1010	5.1523	5.2040	5.2563	5.3091	5.3625	5.4163	5.4707	5.5256	5.5811	5.6371	5.7507	5.8666	5.9847	6.1051	6.2278	6.3528	7.4416
6	6.1520	6.2296	6.3081	6.3877	6.4684	6.5502	6.6330	6.7169	6.8019	6.8881	6.9753	7.1533	7.3359	7.5233	7.7156	7.9129	8.1152	9.9299
7	7.2135	7.3230	7.4343	7.5474	7.6625	7.7794	7.8983	8.0192	8.1420	8.2669	8.3938	8.6540	8.9228	9.2004	9.4872	9.7833	10.0890	12.9159
8	8.2857	8.4328	8.5830	8.7361	8.8923	9.0517	9.2142	9.3800	9.5491	9.7216	9.8975	10.2598	10.6366	11.0285	11.4359	11.8594	12.2997	16.4991
9	9.3685	9.5593	9.7546	9.9545	10.1591	10.3685	10.5828	10.8021	11.0266	11.2563	11.4913	11.9780	12.4876	13.0210	13.5795	14.1640	14.7757	20.7989
10	10.4622	10.7027	10.9497	11.2034	11.4639	11.7314	12.0061	12.2882	12.5779	12.8754	13.1808	13.8164	14.4866	15.1929	15.9374	16.7220	17.5487	25.9587
11	11.5668	11.8633	12.1687	12.4835	12.8078	13.1420	13.4864	13.8412	14.2068	14.5835	14.9716	15.7836	16.6455	17.5603	18.5312	19.5614	20.6546	32.1504
12	12.6825	13.0412	13.4121	13.7956	14.1920	14.6020	15.0258	15.4640	15.9171	16.3856	16.8699	17.8885	18.9771	20.1407	21.3843	22.7132	24.1331	39.5805
13	13.8093	14.2368	14.6803	15.1404	15.6178	16.1130	16.6268	17.1599	17.7130	18.2868	18.8821	20.1406	21.4953	22.9534	24.5227	26.2116	28.0291	48.4966
14	14.9474	15.4504	15.9739	16.5190	17.0863	17.6770	18.2919	18.9321	19.5986	20.2926	21.0151	22.5505	24.2149	26.0192	27.9750	30.0949	32.3926	59.1959
15	16.0969	16.6821	17.2934	17.9319	18.5989	19.2957	20.0236	20.7841	21.5786	22.4087	23.2760	25.1290	27.1521	29.3609	31.7725	34.4054	37.2797	72.0351
16	17.2579	17.9324	18.6393	19.3802	20.1569	20.9710	21.8245	22.7193	23.6575	24.6411	25.6725	27.8881	30.3243	33.0034	35.9497	39.1899	42.7533	87.4421

Table 3

Future Value of an Ordinary Annuity of \$1: end-of-period equal payments.

Present Value of an Ordinary Annuity

Ordinary annuity

Explanation: Annuity PV = Equal payment
× annuity PV factor

Short: $PVA = PMT \times PVIFA(i, n)$

Solved with table

Problem: Receive \$3,000 at the end of each year for 5 years. Discount rate is 8%.

Table used: Table 4

Row: Number of periods = 5

Column: Periodic rate = 8%

Factor = 3.99271

Present value = $3,000 \times 3.99271 = \$11,978.13$

Short formula used: $PVA = PMT \times PVIFA(i, n)$

TABLE 4 Present Value of an Ordinary Annuity of \$1

$$PVA = \frac{1 - \frac{1}{(1+i)^n}}{i}$$

n/i	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	20.0%
1	0.99010	0.98522	0.98039	0.97561	0.97087	0.96618	0.96154	0.95694	0.95238	0.94787	0.94340	0.93458	0.92593	0.91743	0.90909	0.90090	0.89286	0.83333
2	1.97040	1.95588	1.94156	1.92742	1.91347	1.89969	1.88609	1.87267	1.85941	1.84632	1.83339	1.80802	1.78326	1.75911	1.73554	1.71252	1.69005	1.52778
3	2.94099	2.91220	2.88388	2.85602	2.82861	2.80164	2.77509	2.74896	2.72325	2.69793	2.67301	2.62432	2.57710	2.53129	2.48685	2.44371	2.40183	2.10648
4	3.90197	3.85438	3.80773	3.76197	3.71710	3.67308	3.62990	3.58753	3.54595	3.50515	3.46511	3.38721	3.31213	3.23972	3.16987	3.10245	3.03735	2.58873
5	4.85343	4.78264	4.71346	4.64583	4.57971	4.51505	4.45182	4.38998	4.32948	4.27028	4.21236	4.10020	3.99271	3.88965	3.79079	3.69590	3.60478	2.99061
6	5.79548	5.69719	5.60143	5.50813	5.41719	5.32855	5.24214	5.15787	5.07569	4.99553	4.91732	4.76654	4.62288	4.48592	4.35526	4.23054	4.11141	3.32551
7	6.72819	6.59821	6.47199	6.34939	6.23028	6.11454	6.00205	5.89270	5.78637	5.68297	5.58238	5.38929	5.20637	5.03295	4.86842	4.71220	4.56376	3.60459
8	7.65168	7.48593	7.32548	7.17014	7.01969	6.87396	6.73274	6.59589	6.46321	6.33457	6.20979	5.97130	5.74664	5.53482	5.33493	5.14612	4.96764	3.83716
9	8.56602	8.36052	8.16224	7.97087	7.78611	7.60769	7.43533	7.26879	7.10782	6.95220	6.80169	6.51523	6.24689	5.99525	5.75902	5.53705	5.32825	4.03097
10	9.47130	9.22218	8.98259	8.75206	8.53020	8.31661	8.11090	7.91272	7.72173	7.53763	7.36009	7.02358	6.71008	6.41766	6.14457	5.88923	5.65022	4.19247
11	10.36763	10.07112	9.78685	9.51421	9.25262	9.00155	8.76048	8.52892	8.30641	8.09254	7.88687	7.49867	7.13896	6.80519	6.49506	6.20652	5.93770	4.32706
12	11.25508	10.90751	10.57534	10.25776	9.95400	9.66333	9.38507	9.11858	8.86325	8.61852	8.38384	7.94269	7.53608	7.16073	6.81369	6.49236	6.19437	4.43922
13	12.13374	11.73153	11.34837	10.98319	10.63496	10.30274	9.98565	9.68285	9.39357	9.11708	8.85268	8.35765	7.90378	7.48690	7.10336	6.74987	6.42355	4.53268
14	13.00370	12.54338	12.10625	11.69091	11.29607	10.92052	10.56312	10.22283	9.89864	9.58965	9.29498	8.74547	8.24424	7.78615	7.36669	6.98187	6.62817	4.61057
15	13.86505	13.34323	12.84926	12.38138	11.93794	11.51741	11.11839	10.73955	10.37966	10.03758	9.71225	9.10791	8.55948	8.06069	7.60608	7.19087	6.81086	4.67547

Table 4

Present Value of an Ordinary Annuity of \$1: equal end-of-period payments.

Future Value of Annuity Due

Annuity due

Explanation: FV of annuity due =
Payment × annuity-due FV factor

Short: $FVAD = PMT \times FVAD \text{ factor}$

Solved with table

Problem: Deposit \$2,000 at the beginning of each year for 5 years at 6%.

Table used: Table 5

Row: Number of periods = 5

Column: Periodic rate = 6%

Factor = 5.9753

Future value = $2,000 \times 5.9753 = \$11,950.60$

Short formula used: $FVAD = PMT \times FVAD \text{ factor}$

TABLE 5 Future Value of an Annuity Due of \$1

$$FVAD = \left[\frac{(1+i)^n - 1}{i} \right] \times (1+i)$$

n/i	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	20.0%
1	1.0100	1.0150	1.0200	1.0250	1.0300	1.0350	1.0400	1.0450	1.0500	1.0550	1.0600	1.0700	1.0800	1.0900	1.1000	1.1100	1.1200	1.2000
2	2.0301	2.0452	2.0604	2.0756	2.0909	2.1062	2.1216	2.1370	2.1525	2.1680	2.1836	2.2149	2.2464	2.2781	2.3100	2.3421	2.3744	2.6400
3	3.0604	3.0909	3.1216	3.1525	3.1836	3.2149	3.2465	3.2782	3.3101	3.3423	3.3746	3.4399	3.5061	3.5731	3.6410	3.7097	3.7793	4.3680
4	4.1010	4.1523	4.2040	4.2563	4.3091	4.3625	4.4163	4.4707	4.5256	4.5811	4.6371	4.7507	4.8666	4.9847	5.1051	5.2278	5.3528	6.4416
5	5.1520	5.2296	5.3081	5.3877	5.4684	5.5502	5.6330	5.7169	5.8019	5.8881	5.9753	6.1533	6.3359	6.5233	6.7156	6.9129	7.1152	8.9299
6	6.2135	6.3230	6.4343	6.5474	6.6625	6.7794	6.8983	7.0192	7.1420	7.2669	7.3938	7.6540	7.9228	8.2004	8.4872	8.7833	9.0890	11.9159
7	7.2857	7.4328	7.5830	7.7361	7.8923	8.0517	8.2142	8.3800	8.5491	8.7216	8.8975	9.2598	9.6366	10.0285	10.4359	10.8594	11.2997	15.4991
8	8.3685	8.5593	8.7546	8.9545	9.1591	9.3685	9.5828	9.8021	10.0266	10.2563	10.4913	10.9780	11.4876	12.0210	12.5795	13.1640	13.7757	19.7989
9	9.4622	9.7027	9.9497	10.2034	10.4639	10.7314	11.0061	11.2882	11.5779	11.8754	12.1808	12.8164	13.4866	14.1929	14.9374	15.7220	16.5487	24.9587
10	10.5668	10.8633	11.1687	11.4835	11.8078	12.1420	12.4864	12.8412	13.2068	13.5835	13.9716	14.7836	15.6455	16.5603	17.5312	18.5614	19.6546	31.1504
11	11.6825	12.0412	12.4121	12.7956	13.1920	13.6020	14.0258	14.4640	14.9171	15.3856	15.8699	16.8885	17.9771	19.1407	20.3843	21.7132	23.1331	38.5805
12	12.8093	13.2368	13.6803	14.1404	14.6178	15.1130	15.6268	16.1599	16.7130	17.2868	17.8821	19.1406	20.4953	21.9534	23.5227	25.2116	27.0291	47.4966
13	13.9474	14.4504	14.9739	15.5190	16.0863	16.6770	17.2919	17.9321	18.5986	19.2926	20.0151	21.5505	23.2149	25.0192	26.9750	29.0949	31.3926	58.1959
14	15.0969	15.6821	16.2934	16.9319	17.5989	18.2957	19.0236	19.7841	20.5786	21.4087	22.2760	24.1290	26.1521	28.3609	30.7725	33.4054	36.2797	71.0351
15	16.2579	16.9324	17.6393	18.3802	19.1569	19.9710	20.8245	21.7193	22.6575	23.6411	24.6725	26.8881	29.3243	32.0034	34.9497	38.1899	41.7533	86.4421
16	17.4304	18.2014	19.0121	19.8647	20.7616	21.7050	22.6975	23.7417	24.8404	25.9964	27.2129	29.8402	32.7502	35.9737	39.5447	43.5008	47.8837	104.9306

Why higher?

Each payment is invested one period earlier than in an ordinary annuity.

Present Value of Annuity Due

Annuity due

Explanation: PV of annuity due = Payment × annuity-due PV factor

Short: PVAD = PMT × PVAD factor

Solved with table

Problem: Receive \$3,000 at the beginning of each year for 5 years at 8%.

Table used: Table 6

Row: Number of periods = 5

Column: Periodic rate = 8%

Factor = 4.31213

Present value = 3,000 × 4.31213 =
\$12,936.39

Short formula used: PVAD = PMT × PVAD factor

TABLE 6 Present Value of an Annuity Due of \$1

$$PVAD = \left[\frac{1 - \frac{1}{(1+i)^n}}{i} \right] \times (1+i)$$

n/i	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	20.0%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.000
2	1.99010	1.98522	1.98039	1.97561	1.97087	1.96618	1.96154	1.95694	1.95238	1.94787	1.94340	1.93458	1.92593	1.91743	1.90909	1.90090	1.89286	1.833
3	2.97040	2.95588	2.94156	2.92742	2.91347	2.89969	2.88609	2.87267	2.85941	2.84632	2.83339	2.80802	2.78326	2.75911	2.73554	2.71252	2.69005	2.527
4	3.94099	3.91220	3.88388	3.85602	3.82861	3.80164	3.77509	3.74896	3.72325	3.69793	3.67301	3.62432	3.57710	3.53129	3.48685	3.44371	3.40183	3.106
5	4.90197	4.85438	4.80773	4.76197	4.71710	4.67308	4.62990	4.58753	4.54595	4.50515	4.46511	4.38721	4.31213	4.23972	4.16987	4.10245	4.03735	3.588
6	5.85343	5.78264	5.71346	5.64583	5.57971	5.51505	5.45182	5.38998	5.32948	5.27028	5.21236	5.10020	4.99271	4.88965	4.79079	4.69590	4.60478	3.990
7	6.79548	6.69719	6.60143	6.50813	6.41719	6.32855	6.24214	6.15787	6.07569	5.99553	5.91732	5.76654	5.62288	5.48592	5.35526	5.23054	5.11141	4.325
8	7.72819	7.59821	7.47199	7.34939	7.23028	7.11454	7.00205	6.89270	6.78637	6.68297	6.58238	6.38929	6.20637	6.03295	5.86842	5.71220	5.56376	4.604
9	8.65168	8.48593	8.32548	8.17014	8.01969	7.87396	7.73274	7.59589	7.46321	7.33457	7.20979	6.97130	6.74664	6.53482	6.33493	6.14612	5.96764	4.837
10	9.56602	9.36052	9.16224	8.97087	8.78611	8.60769	8.43533	8.26879	8.10782	7.95220	7.80169	7.51523	7.24689	6.99525	6.75902	6.53705	6.32825	5.030
11	10.47130	10.22218	9.98259	9.75206	9.53020	9.31661	9.11090	8.91272	8.72173	8.53763	8.36009	8.02358	7.71008	7.41766	7.14457	6.88923	6.65022	5.192
12	11.36763	11.07112	10.78685	10.51421	10.25262	10.00155	9.76048	9.52892	9.30641	9.09254	8.88687	8.49867	8.13896	7.80519	7.49506	7.20652	6.93770	5.327
13	12.25508	11.90751	11.57534	11.25776	10.95400	10.66333	10.38507	10.11858	9.86325	9.61852	9.38384	8.94269	8.53608	8.16073	7.81369	7.49236	7.19437	5.439
14	13.13374	12.73153	12.34837	11.98318	11.63496	11.30274	10.98565	10.68285	10.39357	10.11708	9.85268	9.35765	8.90378	8.48690	8.10336	7.74987	7.42355	5.532

Why higher?

Receiving money at the beginning of each period makes the stream more valuable today.

Perpetuity Concept

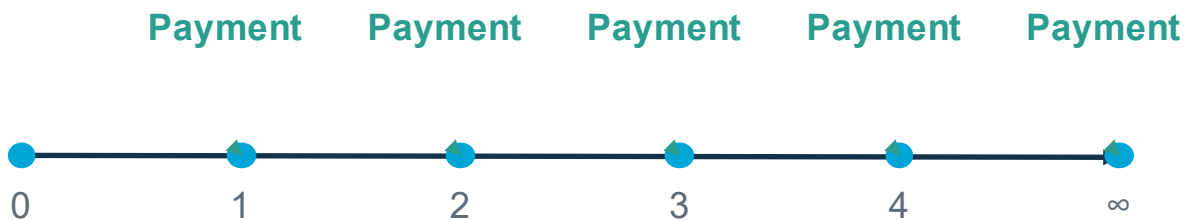
- A perpetuity is an equal payment expected to continue indefinitely.
- It is a simplified valuation model for preferred stock and some long-lived cash flows.
- Because payments never stop, there is no finite n value.
- Perpetuity problems use a formula rather than a finite factor table.

Examples

Preferred dividends, perpetual scholarships, endowment funds, and constant annual maintenance savings.

Key assumption

The payment is constant and the required return is positive.



Present Value of Perpetuity

Perpetuity

Explanation: $PV = \text{Constant payment} \div \text{required return}$

Short: $PV = PMT \div i$

Payment is the equal cash flow received each period forever.

i is the required return or discount rate per period.

The higher the required return, the lower the present value.

Perpetuity is a useful simplification, but real cash flows may grow or stop.

Perpetuity example

Problem: Preferred stock pays \$5 every year forever. Required return is 10%.

Present value = $5 \div 0.10 = \$50$

Interpretation: An investor requiring 10% would pay about \$50 for a perpetual \$5 annual cash flow.

Short formula used: $PV = PMT \div i$

Exam note

Do not use annuity tables for perpetuity because there is no fixed number of periods.

Loan Amortization Concept

- A loan is amortized when it is repaid through equal installments over time.
- Each payment includes interest on the outstanding balance plus principal repayment.
- Interest is larger in early payments because the beginning balance is high.
- Principal repayment increases over time as the balance declines.

Payment logic

Explanation: $\text{Loan} = \text{Equal installment} \times \text{annuity PV factor}$

Short: $\text{Loan} = \text{PMT} \times \text{PVIFA}(i,n)$
 $\text{PMT} = \text{Loan} \div \text{PVIFA}(i,n)$

Using the annuity present value table

Loan = \$20,000

Rate = 8%

Term = 5 years

Use Table 4: present value annuity factor = 3.99271

Payment = $20,000 \div 3.99271 = \$5,009.13$

Installment Payments

Amortization formulas

Explanation:

$\text{Interest} = \text{Beginning balance} \times \text{rate}$

$\text{Principal} = \text{Payment} - \text{Interest}$

Short: $\text{INT} = \text{Beg. Bal.} \times i$; $\text{Principal} = \text{PMT} - \text{INT}$

Interpretation

The borrower pays the same installment each year, but the composition changes: interest decreases and principal increases.

Table link

The installment amount comes from the present value of an ordinary annuity table.

First payment

From the previous loan example:

Beginning balance = \$20,000

Payment = \$5,009.13

Rate = 8%

Interest = $20,000 \times 8\% = \$1,600$

Principal = $5,009.13 - 1,600 = \$3,409.13$

Ending balance = \$16,590.87

Loan Repayment Schedule

Year	Beg. balance	Payment	Interest	Principal	End balance
1	\$20,000.00	\$5,009.13	\$1,600.00	\$3,409.13	\$16,590.87
2	\$16,590.87	\$5,009.13	\$1,327.27	\$3,681.86	\$12,909.01
3	\$12,909.01	\$5,009.13	\$1,032.72	\$3,976.41	\$8,932.60
4	\$8,932.60	\$5,009.13	\$714.61	\$4,294.52	\$4,638.08
5	\$4,638.08	\$5,009.13	\$371.05	\$4,638.08	\$0.00

Pattern

Interest decreases every year because it is calculated on a smaller remaining balance.

Decision use

Managers use amortization schedules to plan cash payments and assess debt capacity.

Effective Annual Rate

Effective annual rate

Explanation: EAR is the true annual rate after compounding

Short: $EAR = (1 + APR/m)^m - 1$

The effective annual rate measures the actual annual rate earned or paid after compounding.

It is higher than quoted annual rate when compounding occurs more than once per year.

effective annual rate allows comparison between loans or investments with different compounding frequencies.

Compounding periods per year = number of times interest is credited in one year.

Monthly compounding

quoted annual rate = 12%, compounded monthly

Compounding periods per year = 12

Effective annual rate = $(1 + 1\%)^{12} - 1$

effective annual rate = 12.68%

The true annual cost is 12.68%, not exactly 12%.

Short formula used: $EAR = (1 + APR/m)^m - 1$

Interpretation

More frequent compounding increases the effective annual rate.

Annual Percentage Rate

Term	Meaning	Use
APR	Quoted annual rate before adjusting for compounding	Common in loans and credit products
Periodic rate	APR divided by compounding periods per year	Used with tables/calculators
EAR	Actual annual rate after compounding	Best for comparison

The quoted annual rate is not always the true cost

quoted annual rate tells you the quoted rate.
effective annual rate tells you the effective annual result after compounding.

Comparison example

Short formula used: $EAR = (1 + APR/m)^m - 1$

quoted annual rate = 10%, semiannual compounding

Periodic rate = $10\% \div 2 = 5\%$

Periods per year = 2

Effective annual rate = $(1.05)^2 - 1 = 10.25\%$

Time Value of Money in Bond Valuation

Bond valuation

$$\text{Bond price} = \text{PV of coupons} + \text{PV of face value}$$

- Coupons form an ordinary annuity.
- Face value is a single future amount.
- Use Table 4 for coupon annuity PV.
- Use Table 2 for principal PV.

Interpretation

If the required return is higher than the coupon rate, the bond sells below face value.

Table-based bond price

Face value = \$1,000

Coupon = \$80 per year

Maturity = 5 years

Required return = 10%

Present value of coupons = $80 \times 3.79079 = \$303.26$

Present value of face value = $1,000 \times 0.62092 = \$620.92$

Bond price = \$924.18

Short formula used: Bond Price = PV coupons + PV face value

Time Value of Money in Stock Valuation

- A stock price reflects expected future cash flows to shareholders.
- Dividends may be valued as growing or non-growing cash flows.
- Unlike bonds, stock cash flows are usually less certain.
- Higher expected growth increases value; higher required return decreases value.

Teaching point

Stock valuation is TVM applied to uncertain long-term cash flows.

Stock valuation

Constant-growth model:

Explanation: $\text{Stock price} = \text{Next dividend} \div (\text{required return} - \text{growth})$

Short: $P_0 = D_1 \div (r - g)$

Example

Next expected dividend = \$3.00

Required return = 11%

Growth rate = 5%

Stock price today = $3 \div (0.11 - 0.05) = \$50$

This is a growing perpetuity, not a finite annuity.

Short formula used: $P_0 = D_1 \div (r - g)$

Time Value of Money in Capital Budgeting

Investment decision

Explanation: $NPV = PV \text{ of future inflows} - \text{initial investment}$

Short: $NPV = PV \text{ inflows} - \text{initial outlay}$

Accept projects with positive Net present value if other strategic conditions are satisfied.

Cash flows must be discounted because they occur in the future.

A project may look profitable in total cash but fail after discounting.

The discount rate should reflect project risk.

Using the annuity present value table

Initial cost = \$45,000

Annual cash inflow = \$12,000 for 5 years

Required return = 10%

Use Table 4: present value annuity factor = 3.79079

Present value of inflows = $12,000 \times 3.79079 =$

\$45,489.48

Net present value = \$489.48

Short formula used: $NPV = PV \text{ inflows} - \text{Initial outlay}$

Decision

The project is marginally acceptable because Net present value is positive.

TVM Using Excel

Task	Excel function	Table equivalent
Future value single amount	=FV(rate,nper,0,-pv)	Table 1
Present value single amount	=PV(rate,nper,0,-fv)	Table 2
Future value annuity	=FV(rate,nper,-pmt,0)	Table 3 or 5
Present value annuity	=PV(rate,nper,-pmt,0)	Table 4 or 6
Loan payment	=PMT(rate,nper,-pv)	Table 4 logic

Course approach

Numerical questions will mainly use the present and future value factor tables.

Excel as a check

Excel is useful for verification, larger period counts, unusual interest rates, and complex cash flows.

Warning

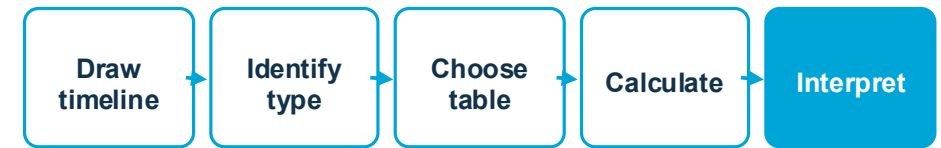
Excel signs matter: cash inflows and outflows should have opposite signs.

TVM Using Financial Calculator Logic

Key	Meaning	Typical input
N	Number of periods	Years × compounding periods
I/Y	Interest rate per period	APR ÷ m
PV	Present value	Current amount or loan principal
PMT	Equal payment	Annuity or loan payment
FV	Future value	Target value or maturity value

Before calculating

Draw a timeline and identify whether each cash flow is a single amount, ordinary annuity, annuity due, or perpetuity.



Sign convention

Money paid out and money received should have opposite signs in calculators and spreadsheets.

Worked Example 1: Future Value

Solution

Question: An investor deposits \$25,000 today in an account earning 7% annually for 10 years. Find the future value.

Step 1: This is a single amount growing forward.

Step 2: Use Table 1 - Future Value of \$1.

Step 3: Locate row number of periods = 10 and column periodic rate = 7%.

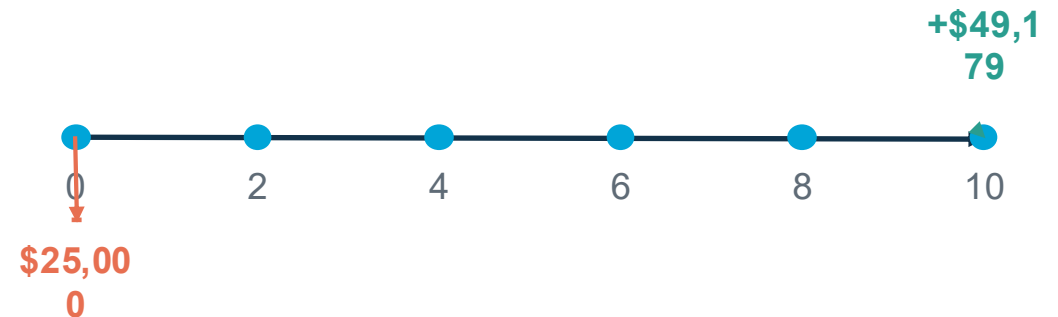
Step 4: Factor = 1.96715.

Step 5: Future value = $25,000 \times 1.96715 = \$49,178.75$.

Answer

\$49,178.75

Short formula used: $FV = PV \times FVIF(i, n)$



Interpretation

The factor is the value of \$1 after 10 years at 7%. Multiplying converts it to the value of \$25,000.

Worked Example 2: Present Value

Solution

Question: A company expects to receive \$80,000 in 10 years. The required return is 9%. Find the present value.

Step 1: This is one future amount discounted back to today.

Step 2: Use Table 2 - Present Value of \$1.

Step 3: Locate row number of periods = 10 and column periodic rate = 9%.

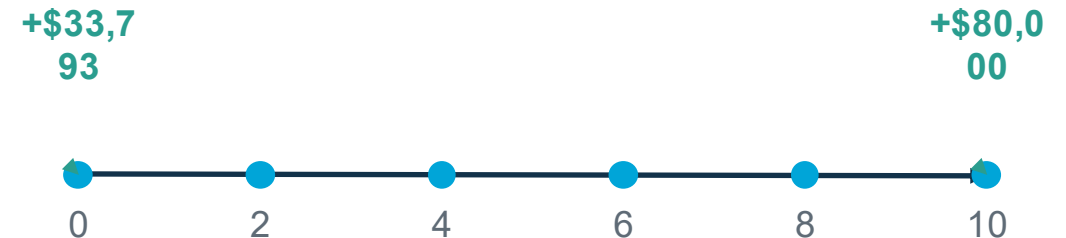
Step 4: Factor = 0.42241.

Step 5: Present value = $80,000 \times 0.42241 = \$33,792.80$.

Answer

\$33,792.80

Short formula used: $PV = FV \times PVIF(i, n)$



Interpretation

The future cash flow is worth less today because the investor requires a 9% return.

Worked Example 3: Annuity

Ordinary annuity

Question: You deposit \$6,000 every year for 10 years at 8%. Find future value if deposits are made at the end of each year.

Ordinary annuity: Use Table 3

Factor at Number of periods = 10, Periodic rate = 8% = 14.4866

Future value = $6,000 \times 14.4866 = \$86,919.60$

Annuity due

If deposits are made at the beginning of each year:

Annuity due: Use Table 5

Factor at Number of periods = 10, Periodic rate = 8% = 15.6455

Future value = $6,000 \times 15.6455 = \$93,873.00$

Short formula used: $FVA = PMT \times FVIFA(i, n)$

Interpretation

The annuity due is higher because each payment earns one additional year of interest compared with the ordinary annuity.



Common Mistakes and Exam Tips

- **Choose the correct table:**

- Table 1 for FV of one amount; Table 2 for PV of one amount.
- Table 3 for FV ordinary annuity; Table 4 for PV ordinary annuity.
- Table 5 for FV annuity due; Table 6 for PV annuity due.
- Match rate and periods:
- If compounding is semiannual, divide the rate by 2 and multiply years by 2.
- Interpret timing:
- End-of-period is ordinary annuity; beginning-of-period is annuity due.

Exam workflow

1. Draw the timeline.
2. Identify the cash flow type.
3. Select the table.
4. Locate i and n .
5. Multiply and interpret.

Do not do this

Do not use annual rate with monthly periods or use an annuity table for a single future amount.

Rounding

Keep factor values to at least four decimals during calculation; round final answer only.

Lecture Summary and Numerical Homework

- Money has time value because it can earn returns and because future cash flows involve risk and waiting.
- Compounding moves money forward; discounting moves money backward.
- Single amounts use Table 1 or Table 2.
- Ordinary annuities use Table 3 or Table 4; annuity due uses Table 5 or Table 6.
- TVM supports valuation, borrowing, investment, retirement planning, bonds, stocks, and capital budgeting.

Final message

A good financial analyst does not only calculate the number; he explains what the number means for a real decision.

Homework	Required table
1. FV of \$12,000 for 8 years at 5%	Table 1
2. PV of \$40,000 due in 12 years at 7%	Table 2
3. FV of \$2,500 end-year deposits for 10 years at 6%	Table 3
4. PV of \$4,000 end-year receipts for 8 years at 9%	Table 4
5. Loan payment on \$30,000 over 10 years at 8%	Table 4

Source of factors

Use the present and future value tables provided for this lecture.

MCQ Practice 1: Time Value Concept

Question 1

Why is money received today more valuable than the same amount received in the future?

- A. Future money is always risk-free
- B. Today's money can be invested and earn a return
- C. Inflation increases the value of future money
- D. Financial decisions ignore timing

Question focus

Theoretical focus

Concept: opportunity cost and earning capacity.

Model answer

Correct answer: B

Explanation

Money today has earning capacity. Waiting creates opportunity cost, inflation exposure, and uncertainty.

MCQ Practice 2: Choosing the Correct Table

Question 2

A student wants to find what \$10,000 invested today will become after 5 years at 6%. Which table should be used?

- A. Present Value of \$1
- B. Future Value of \$1
- C. Present Value of an Ordinary Annuity
- D. Future Value of an Ordinary Annuity

Question focus

Table selection

One amount grows forward from today to the future.

Model answer

Correct answer: B

Explanation

A single current amount moving forward uses Table 1: Future Value of \$1.

MCQ Practice 3: Compounding Frequency

Question 3

An investment earns a quoted annual rate of 12% compounded quarterly for 3 years. What periodic rate and number of periods should be used?

- A. 12% and 3 periods
- B. 6% and 6 periods
- C. 3% and 12 periods
- D. 1% and 36 periods

Question focus

Formula logic

Periodic rate = quoted annual rate $\div$ periods per year.

Number of periods = years $\times$ periods per year.

Model answer

Correct answer: C

Explanation

Quarterly means 4 times per year: $12\% \div 4 = 3\%$, and $3 \times 4 = 12$ periods.

MCQ Practice 4: Ordinary Annuity

Question 4

Which statement correctly describes an ordinary annuity?

- A. Equal payments at the beginning of each period
- B. Unequal payments at irregular dates
- C. Equal payments at the end of each period
- D. One single future cash flow

Question focus

Theoretical focus

Timing rule distinguishes ordinary annuity from annuity due.

Model answer

Correct answer: C

Explanation

Ordinary annuity payments occur at the end of each period.

MCQ Practice 5: Present Value and Rate

Question 5

If the discount rate increases while the future amount and time remain constant, what happens to present value?

- A. Present value increases
- B. Present value decreases
- C. Present value remains unchanged
- D. Present value becomes equal to future value

Question focus

Theoretical focus

Higher required return means stronger discounting.

Model answer

Correct answer: B

Explanation

A higher discount rate reduces the value today of a future cash flow.

MCQ Practice 6: Simple Interest

Question 6

A principal of \$10,000 is invested for 3 years at 8% simple interest. What is the future value?

- A. \$10,800
- B. \$12,400
- C. \$12,597.10
- D. \$14,000

Question focus

Simple calculation

Interest = Principal $\times$ Rate $\times$ Time

Future value = Principal + Interest

Model answer

Correct answer: B

Explanation

Interest = $10,000 \times 8\% \times 3 = \$2,400$. Future value
= $10,000 + 2,400 = \$12,400$.

MCQ Practice 7: Future Value of One Amount

Question 7

What is the future value of \$5,000 after 6 periods at 8% compound interest?

Given factor: Future Value of \$1 at 6 periods, 8% = 1.58687

- A. \$3,151.00
- B. \$5,800.00
- C. \$7,934.35
- D. \$8,000.00

Question focus

Table method

Future value = Present amount × Future value factor

Model answer

Correct answer: C

Explanation

$\$5,000 \times 1.58687 = \$7,934.35.$

MCQ Practice 8: Present Value of One Amount

Question 8

What is the present value of \$8,000 to be received after 8 periods at 12%?

Given factor: Present Value of \$1 at 8 periods, 12% = 0.40388

- A. \$3,231.04
- B. \$4,038.80
- C. \$6,217.20
- D. \$8,000.00

Question focus

Table method

Present value = Future amount × Present value factor

Model answer

Correct answer: A

Explanation

$\$8,000 \times 0.40388 = \$3,231.04.$

MCQ Practice 9: Future Value of Ordinary Annuity

Question 9

An investor deposits \$2,000 at the end of each period for 6 periods at 8%. What is the future value?

Given factor: FV ordinary annuity factor = 7.3359

- A. \$12,000.00
- B. \$14,671.80
- C. \$15,645.50
- D. \$16,000.00

Question focus

Table method

Future value of ordinary annuity = Equal payment $\times$ annuity factor

Model answer

Correct answer: B

Explanation

$\$2,000 \times 7.3359 = \$14,671.80.$

MCQ Practice 10: Loan Installment

Question 10

A loan of \$20,000 is repaid in 5 equal year-end installments at 8%. Present value annuity factor = 3.99271. What is the annual installment?

- A. \$3,409.13
- B. \$4,000.00
- C. \$5,009.13
- D. \$6,386.00

Question focus

Loan formula

Equal installment = Loan amount ÷ present value annuity factor

Model answer

Correct answer: C

Explanation

$\$20,000 \div 3.99271 = \$5,009.13.$



Time Value of Money

Thank You