

Business Mathematics

Prepared by Dr. Alfaisal Hasan

MTH 112
Business Mathematics

Lecture NO 3



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Chapter

Topics

- Chapter 1: Mathematical preliminaries
- Chapter 2: The straight line and applications
- **Chapter 3: Functions and Graphs**
- Chapter 4: Limits and Continuity
- Chapter 5: Financial Mathematics
- Chapter 6: Differentiation
- Chapter 7: Methods of Differentiation
- Chapter 8: Function of several variables
- Chapter 9: Integration and applications
- Chapter 10: Linear algebra and applications

Exponential Functions

Objectives

- To use the properties of exponents to:
 - Simplify exponential expressions.
 - Solve exponential equations.
- To sketch graphs of exponential functions.

Exponential Functions

- A polynomial function has the basic form: $f(x) = x^3$
- An exponential function has the basic form: $f(x) = 3^x$
- An exponential function has the variable in the exponent, not in the base.
- General Form of an Exponential Function:
$$f(x) = N^x, N > 0$$

Exponential Functions

Properties of Exponents

$$A^X \cdot A^Y = A^{X+Y}$$

$$\frac{A^X}{A^Y} = A^{X-Y}$$

$$(A^X)^Y = A^{XY}$$

$$A^{-X} = \frac{1}{A^X}$$

$$(AB)^X = A^X B^X$$

$$\frac{1}{A^{-X}} = A^X$$

$$\left(\frac{A}{B}\right)^X = \frac{A^X}{B^X}$$

$$A^{x/y} = \sqrt[y]{A^X} = \left(\sqrt[y]{A}\right)^X$$

Exponential Functions

■ Simplify: $2^2 \cdot 2^3 = 2^5 = 32$

$$2^2 \cdot 2^{-6} = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

$$(2^3)^2 = 2^6 = 64$$

Exponential Functions

■ Simplify: $\left(\frac{2}{3}\right)^{-3} = \frac{2^{-3}}{3^{-3}} = \frac{3^3}{2^3} = \frac{27}{8}$

$$\frac{3^7}{3^9} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$(2^{1/2})(8^{1/2}) = (2 \cdot 8)^{1/2} = 16^{1/2} = \sqrt{16} = 4$$

Exponential Functions

■ Solve: $5^x = 125$

$$5^x = 5^3$$

$$x = 3$$

Exponential Functions

■ Solve: $7^{(x-1)} = 7^{-1/2}$

$$x - 1 = -\frac{1}{2}$$
$$x = \frac{1}{2}$$

Exponential Functions

■ Solve: $8^x = 2$

$$(2^3)^x = 2^1$$

$$2^{3x} = 2^1$$

$$3x = 1$$

$$x = \frac{1}{3}$$

Exponential Functions

■ Solve: $8^x = 4$

$$(2^3)^x = 2^2$$

$$2^{3x} = 2^2$$

$$3x = 2$$

$$x = \frac{2}{3}$$

Exponential Functions

■ Solve: $\left(\frac{1}{3}\right)^x = 27$

$$\left(3^{-1}\right)^x = 27$$

$$3^{-x} = 3^3$$

$$-x = 3$$

$$x = -3$$

Exponential Functions

■ Solve: $x^{1/3} = 27$

$$\left(x^{1/3}\right)^3 = 27^3$$

$$x = 19,683$$

Not considered an exponential equation, because the variable is now in the base.

Exponential Functions

■ Solve:

Not considered an exponential equation, because the variable is in the base.

$$x^{3/4} = 8$$

$$\left(x^{3/4}\right)^{4/3} = 8^{4/3}$$

$$x = \left(\sqrt[3]{8}\right)^4$$

$$x = (2)^4$$

$$x = 16$$

Given that $f(x) = 4^x$

Find the value of $f(-2)$

$$f(-2) = 4^{-2}$$

$$f(-2) = \frac{1}{4^2} = \frac{1}{16}$$

Given that $f(x) = 4^x$

Find the value of $f(-1)$

$$f(-1) = 4^{-1}$$

$$f(-1) = \frac{1}{4^1} = \frac{1}{4}$$

Given that $f(x) = 4^x$

Find the value of $f(3)$

$$f(3) = 4^3$$

$$f(3) = 64$$

Given that $f(x) = 4^x$

Find the value of $f(-3)$

$$f(-3) = 4^{-3}$$

$$f(-3) = \frac{1}{4^3} = \frac{1}{64}$$

Given that $f(x) = 4^x$

Find the value of $f(0)$

$$f(0) = 4^0$$

$$f(0) = 1$$

Given that $f(x) = \left(\frac{1}{3}\right)^x$

Find the value of $f(0)$

$$f(0) = \left(\frac{1}{3}\right)^0$$

$$f(0) = 1$$

Given that $f(x) = \left(\frac{1}{3}\right)^x$
Find the value of $f(1)$

$$f(1) = \left(\frac{1}{3}\right)^1$$

$$f(1) = \frac{1}{3}$$

Given that $f(x) = \left(\frac{1}{3}\right)^x$
Find the value of $f(2)$

$$f(2) = \left(\frac{1}{3}\right)^2$$

$$f(2) = \frac{1}{9}$$

Given that $f(x) = \left(\frac{1}{3}\right)^x$
Find the value of $f(-1)$

$$f(-1) = \left(\frac{1}{3}\right)^{-1}$$

$$f(-1) = \frac{1^{-1}}{3^{-1}} = \frac{3^1}{1^1} = 3$$

Given that $f(x) = \left(\frac{1}{3}\right)^x$
Find the value of $f(-2)$

$$f(-2) = \left(\frac{1}{3}\right)^{-2}$$

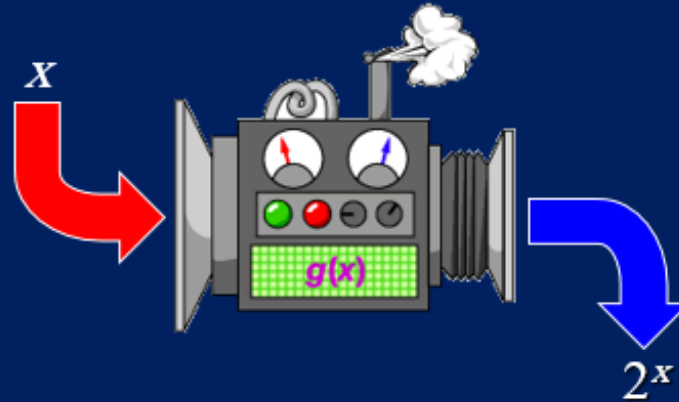
$$f(-2) = \frac{1^{-2}}{3^{-2}} = \frac{3^2}{1^2} = 9$$

Exponential Functions

- General Form of an Exponential Function:

$$f(x) = N^x, N > 0$$

$$g(x) = 2^x$$



$$g(3) = 8$$

$$g(2) = 4$$

$$g(1) = 2$$

$$g(0) = 1$$

$$g(-1) = 2^{-1} = \frac{1}{2}$$

$$g(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

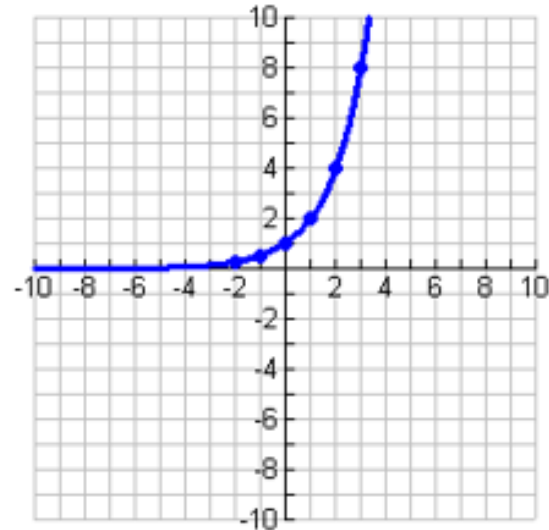
Exponential Functions

$$g(x) = 2^x$$

x	$g(x)$
2	4
1	2
0	1
-1	$\frac{1}{2}$
-2	$\frac{1}{4}$

Exponential Functions

$$g(x) = 2^x$$



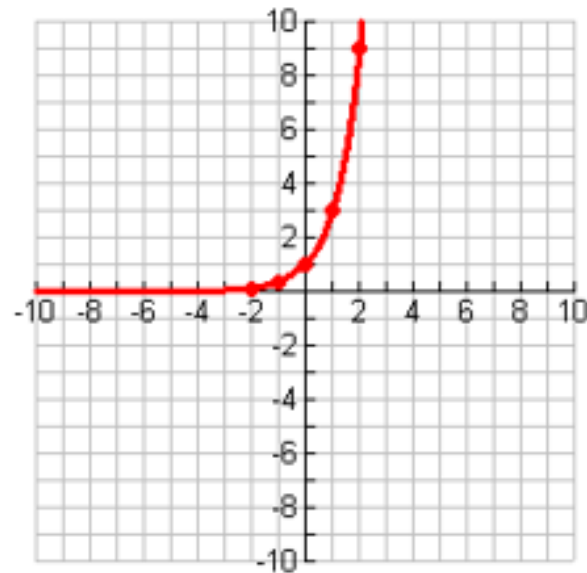
Exponential Functions

$$h(x) = 3^x$$

x	$h(x)$
2	9
1	3
0	1
-1	$\frac{1}{3}$
-2	$\frac{1}{9}$

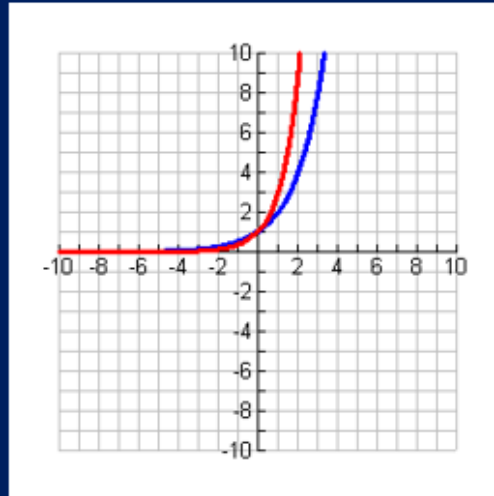
Exponential Functions

$$h(x) = 3^x$$



Exponential Functions

$$g(x) = 2^x \text{ (blue)}$$

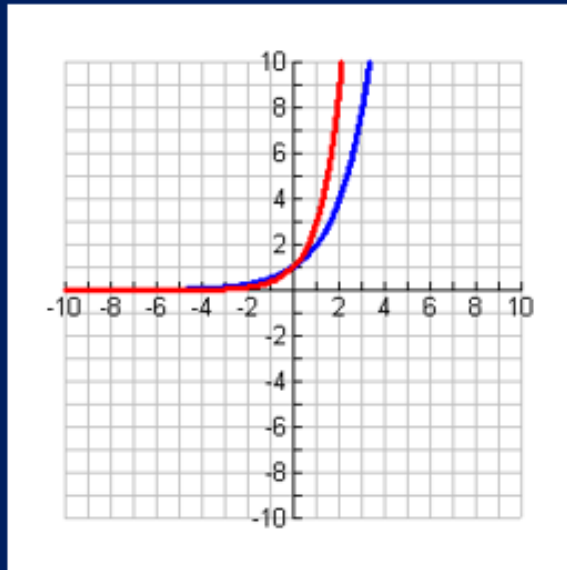


$$h(x) = 3^x \text{ (red)}$$

- Exponential functions with positive bases greater than 1 have graphs that are increasing.
- The function never crosses the x -axis because there is nothing we can plug in for x that will yield a zero answer.
- The x -axis is a left horizontal asymptote.

Exponential Functions

$$g(x) = 2^x \text{ (blue)}$$



$$h(x) = 3^x \text{ (red)}$$

- A smaller base means the graph rises more gradually.
- A larger base means the graph rises more quickly.
- Exponential functions will not have negative bases.

Exponential Functions

The Number e

A base often associated with exponential functions is:

$$e \approx 2.71828169$$

Exponential Functions

The Number e

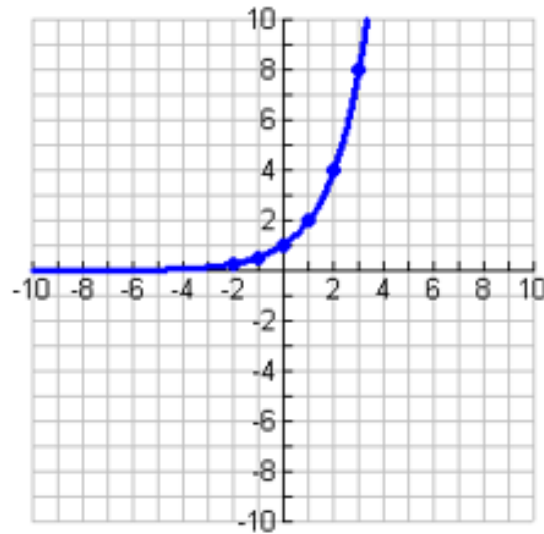
- Euler's number
- Leonhard Euler
(pronounced "oiler")
- Swiss mathematician
and physicist



Exponential Functions

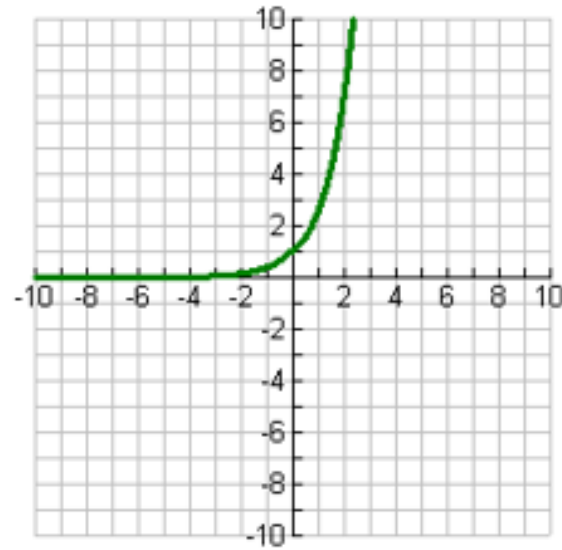
Remember

$$g(x) = 2^x$$



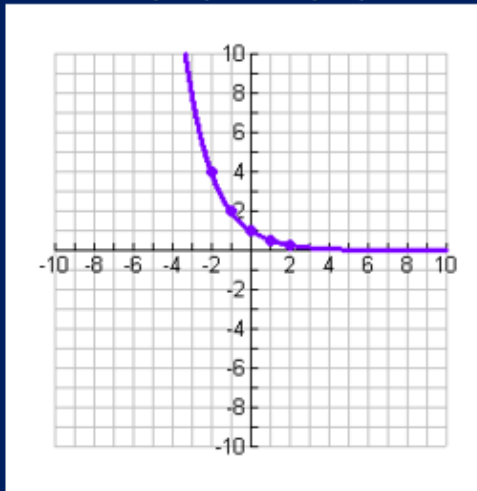
Exponential Functions

$$f(x) = e^x$$



Exponential Functions

$$j(x) = \left(\frac{1}{2}\right)^x$$



- Exponential functions with positive bases less than 1 have graphs that are decreasing.

Exponential Functions

Why study exponential functions?

- Exponential functions are used in our real world to measure growth, interest, and decay.
- Growth obeys exponential functions.
- Ex: rumors, human population, bacteria, computer technology, nuclear chain reactions, compound interest
- Decay obeys exponential functions.
- Ex: Carbon-14 dating, half-life, Newton's Law of Cooling

Exponential Functions

You can use exponential functions to model the way certain populations increase or decrease over time.

Increasing populations can be modeled by exponential growth function.

Exponential Functions

Can you tell by looking at an equation whether the function models exponential growth or decay?

$$y = ab^x$$

When $b > 1$, the function models exponential growth,

and b is the **growth factor**.

Exponential Functions

When $0 < b < 1$,
the function models exponential **decay**,
and b is the **decay factor**.
It also called negative growth.

Exponential Functions

Now tell me just by looking at the equation if it represent exponential growth or decay.

$$y = 15(1.45)^x \quad \text{Exponential Growth}$$

$$y = 7.3(0.8)^x \quad \text{Exponential Decay}$$

Given the function $y = 3(2.25)^x$
Is exponential decay or growth

$2.25 > 1.0$ **Exponential growth**

Given the function $y = 7(5.16)^x$
Is exponential decay or growth

$5.16 > 1.0$ **Exponential growth**

Given the function $y = 8(0.9)^x$
Is exponential decay or growth

$0.9 < 1.0$ **Exponential decay**

Given the function $y = 2(0.14)^x$
Is exponential decay or growth

$0.14 < 1.0$ **Exponential decay**

Logarithmic Functions

For $x > 0$, $b > 0$ and b not equal to 1 the logarithm of x with base b is defined by the following:

$$\log_b x = y \Leftrightarrow x = b^y$$

Given that exponential form $(5)^3 = 125$

Find the logarithmic form

$$\log_5 125 = 3$$

Given that exponential form $(2)^5 = 32$

Find the logarithmic form

$$\log_2 32 = 5$$

Given that logarithmic form $\log_4 64 = 3$

Find the exponential form

$$(4)^3 = 64$$

Logarithmic Functions

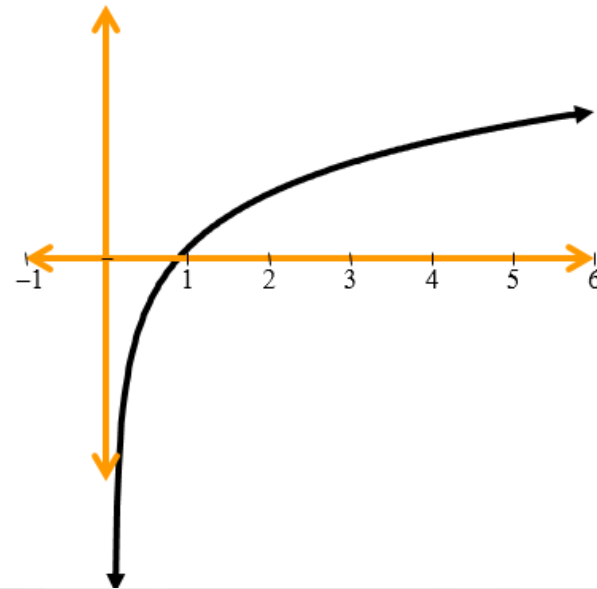
Properties of Logarithmic Function

- Domain: $\{x > 0\}$
- Range: all real numbers
- x intercept: $(1, 0)$
- No y intercept
- Approaches y axis as vertical asymptote
- Base determines shape.

Logarithmic Functions

Shape of logarithmic graphs

- For $b > 1$, the graph rises from left to right.



Logarithmic Functions

Common Logarithmic Function
The logarithmic function with
base 10

$$\log_{10} x = y \Leftrightarrow \log x = y$$

Logarithmic Functions

Natural logarithmic function
The logarithmic function with a
base of e

$$\log_e x = y \Leftrightarrow \ln x = y$$

Logarithmic Functions

Calculator Keys

- [LOG]
- [LN]

Logarithmic Functions

$$\log_b 1 = 0$$

$$\log_b b = 1$$

$$\log_b x = \log_b y \Leftrightarrow x = y$$

Logarithmic Functions

For $x > 0$, $y > 0$, $b > 0$ and $b \neq 1$
Product rule of Logarithms

$$\log_b (xy) = \log_b x + \log_b y$$

Logarithmic Functions

For $x > 0$, $y > 0$, $b > 0$ and $b \neq 1$
Product rule of Logarithms

$$\log_b \left(\frac{x}{y} \right) = \log_b x - \log_b y$$

Logarithmic Functions

For $x > 0$, $y > 0$, $b > 0$ and $b \neq 1$
Power rule for Logarithms

$$\log_b x^r = r (\log_b x)$$

Logarithmic Functions

- Apply the product, quotient, and power properties of logarithms.
- Combine and Expand logarithmic expressions

Logarithmic Functions

$$\log_b (xy) = \log_b x + \log_b y$$

$$\log_b \left(\frac{x}{y} \right) = \log_b x - \log_b y$$

$$\log_b x^r = r (\log_b x)$$

Logarithmic Functions

$$\ln(xy) = \ln x + \ln y$$

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

$$\ln x^r = r(\ln x)$$

Exponential and Logarithmic Functions

- A **logarithm**, or **log**, is defined in terms of an exponential.

If $b^x = a$, then $\log_b a = x$

If $5^2 = 25$, then $\log_5 25 = 2$

Exponential and Logarithmic Functions

Solve each logarithmic equation.

$$\ln 3x = 2$$

Exponential and Logarithmic Functions

$$3x = e^2$$

Inverse Property

$$x = \frac{1}{3}e^2$$

Multiply each side by $\frac{1}{3}$.

$$x \approx 2.46$$

Use a calculator.

The solution is $x = \frac{1}{3}e^2$
 ≈ 2.46

Check this in the original equation.

Exponential and Logarithmic Functions

Solve the equation

$$\log 2 + \log x = \log 24$$

Solve the equation

$$\log 2 + \log x = \log 24$$

$$\log 2 + \log x = \log 24$$

$$\log(2x) = \log 24$$

$$2x = 24$$

$$\frac{2x}{2} = \frac{24}{2}$$

$$x = 12$$

Exponential and Logarithmic Functions

Solve the equation

$$\log_5 x - 2\log_5 7 = \log_5 125$$

Exponential and Logarithmic Functions

$$\log_5 x - 2\log_5 7 = \log_5 125$$

$$\log_5 x - \log_5 7^2 = \log_5 125$$

$$\log_5 x - \log_5 49 = \log_5 125$$

Exponential and Logarithmic Functions

$$\log_5 \frac{x}{49} = \log_5 125$$

$$\frac{x}{49} = 125$$

$$x = 125 \times 49 = 6125$$

Exponential and Logarithmic Functions

Solve the equation

$$\log_2 x + 2\log_2 5 = 2\log_2(x - 6)$$

Exponential and Logarithmic Functions

$$\log_2 x + 2\log_2 5 = 2\log_2(x - 6)$$

$$\log_2 x + \log_2 5^2 = \log_2(x - 6)^2$$

$$\log_2 x + \log_2 25 = \log_2(x - 6)^2$$

Exponential and Logarithmic Functions

$$\log_2(25x) = \log_2(x - 6)^2$$

$$25x = (x - 6)^2$$

$$25x = x^2 - 12x + 36$$

$$0 = x^2 - 37x + 36$$

Exponential and Logarithmic Functions

$$(x - 36)(x - 1) = 0$$

$$x - 36 = 0 \quad x - 1 = 0$$

$$x = 36$$

$$\textcircled{x = 1}$$

Reject solutions
that would mean
taking the log of
a negative number

Given the expression $\ln\left(\frac{y^2}{x}\right)$
Simplify it

$$\begin{aligned}\ln\left(\frac{y^2}{x}\right) &= \ln y^2 - \ln x \\ &= 2\ln y - \ln x\end{aligned}$$

Un Solved Examples

Given that $f(x) = 5^x$

Find the value of $f(-2)$

Un Solved Examples

Given that $f(x) = 5^x$

Find the value of $f(0)$

Un Solved Examples

Given that $f(x) = 5^x$

Find the value of $f(1)$

Un Solved Examples

Given that $f(x) = 5^x$

Find the value of $f(-1)$

Un Solved Examples

Given that $f(x) = \left(\frac{1}{4}\right)^x$

Find the value of $f(-1)$

Un Solved Examples

Given that $f(x) = \left(\frac{1}{4}\right)^x$

Find the value of $f(0)$

Un Solved Examples

Given that exponential form $(16)^{\frac{1}{2}} = 4$

Find the logarithmic form

Un Solved Examples

Given that logarithmic form $\log_8 16 = \frac{3}{4}$

Find the exponential form

Un Solved Examples

Given that logarithmic form $\log 100 = 2$

Find the exponential form

Un Solved Examples

Given the function $y = 28(0.3)^x$

Is exponential decay or growth

Un Solved Examples

Given the function $y = 2(4.3)^x$

Is exponential decay or growth

Un Solved Examples

Given the expression $\ln \frac{y}{x^3}$
Simplify it

Un Solved Examples

Solve the equation $\ln\left(\frac{x}{10}\right) = \ln(15)$



Thank You