

Introduction to Financing

Prepared by Dr. Sara Ahmed

Lecture 1





مجموعة جامعة
الجميع الذكية
للتعليم القابضة

MIDOCEAN
UNIVERSITY
BY EVERYONE'S SMART UNIVERSITY

Time Value of Money

Lecture 1

Prepared by
Dr. Sara Ahmed

www.midocean.ae



Topic

01

OUTLINE

- 1 Future Value and Compounding
- 2 Present Value and Discounting
- 3 More on Present and Future Values

Solving for:

Implied interest rate

Number of periods

Basic Definitions

- Present Value (PV)
 - The current value of future cash flows discounted at the appropriate discount rate
 - Value at $t=0$ on a time line
- Future Value (FV)
 - The amount an investment is worth after one or more periods.
 - “Later” money on a time line

Basic Definitions

- Interest rate (r)
 - **Discount rate**
 - Cost of capital
 - Opportunity cost of capital
 - Required return
 - Terminology depends on usage

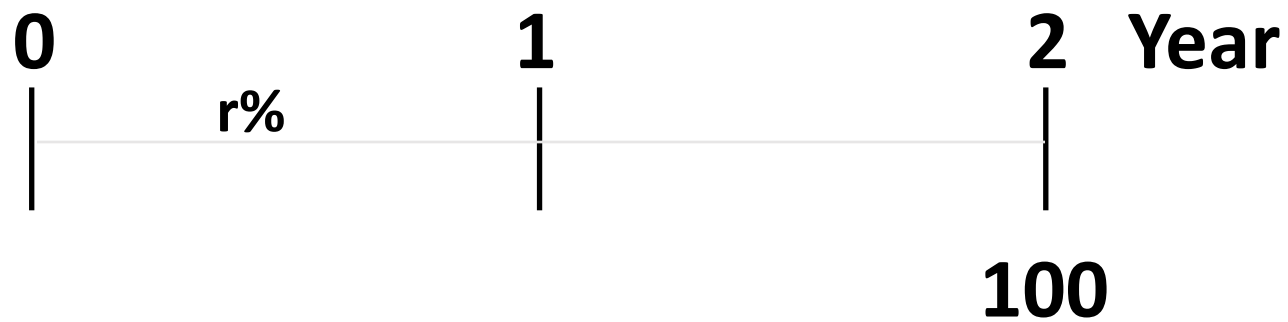
Time Line of Cash Flows

- **Tick marks** at ends of periods
 - Time 0 is today;
 - Time 1 is the end of Period 1



+CF = Cash INFLOW -CF = Cash OUTFLOW PMT = Constant CF

Time Line for a \$100 Lump Sum due at the End of Year 2.



Future Values: General Formula

$$\mathbf{FV = PV(1 + r)^t}$$

FV = future value

PV = present value

r = period interest rate, expressed
as a decimal

t = number of periods

- Future value interest factor = $(1 + r)^t$

Note: “y^x” key on your calculator

Future Values – Example 1

Suppose you invest \$100 for one year at 10% per year.

What is the future value in one year?

- Interest = $100(.10) = 10$
- Value in one year
 - = Principal + interest
 - = $100 + 10 = 110$
- Future Value (FV)
 - = $100(1 + .10) = 110$

Future Values – Example 1

Suppose you leave the money in for another year. How much will you have two years from now?

$$\begin{aligned} FV &= 100(1.10)(1.10) \\ &= 100(1.10)^2 = 121.00 \end{aligned}$$

Effects of Compounding

- Simple interest
 - Interest earned only on the original principal
- Compound interest
 - Interest earned on principal and on interest received
 - “Interest on interest” – interest earned on reinvestment of previous interest payments



Effects of Compounding

- Consider the previous example
 - FV w/simple interest
 $= 100 + 10 + 10 = 120$
 - FV w/compound interest
 $= 100(1.10)^2 = 121.00$
 - The extra 1.00 comes from the interest of $.10(10) = 1.00$ earned on the first interest payment

Texas Instruments BA-II Plus

- **0** = future value
- **.b** = present value
- **-** = period interest rate (r)
- **,** = number of periods

} One of these **MUST** be negative

, - . / 0



Texas Instruments BA-II Plus

- I/Y = period interest rate (r)
[must equal 1 for the I/Y to be the period rate ($\text{I/Y} = 1 =$ default on new BAII+)
Interest is entered as a percent, not a decimal
 - 5% interest = "5", not ".05"
- P/Y = 0 for this chapter only!
- Clear the registers before each problem
 $\text{2ND} \rightarrow \text{CLR}$
Or reenter each field

Texas Instruments BA-II Plus

- Set number of decimal places to display
 - Press **&** key,
 - Press **|** key (above “.”),
 - Enter desired decimal places (e.g., **4**).
 - Press **&U** to set the displayed choice.

Texas Instruments BA-II Plus

- Be sure “payment per period” is set to “1”
 - Press **&** key,
 - Press **[** (above -),
 - Enter “1”,
 - Press **!**
 - Press **&U**

TI BAII+: Set Time Value Parameters

- Be sure calculator is set for cash flows at the END of each period
- To set **END** (for cash flows occurring at the end of the period),
 - Press **&** key,
 - Press **]** (above **/**).
 - This is a toggle switch. The default is END.
 - To change to BEGIN, hit **&V** (above **!**) to go back and forth.
 - Press **&U** to set the displayed choice.

Future Values – Example 2

- Suppose you invest the \$100 from the previous example for 5 years. How much would you have?

Formula Solution:

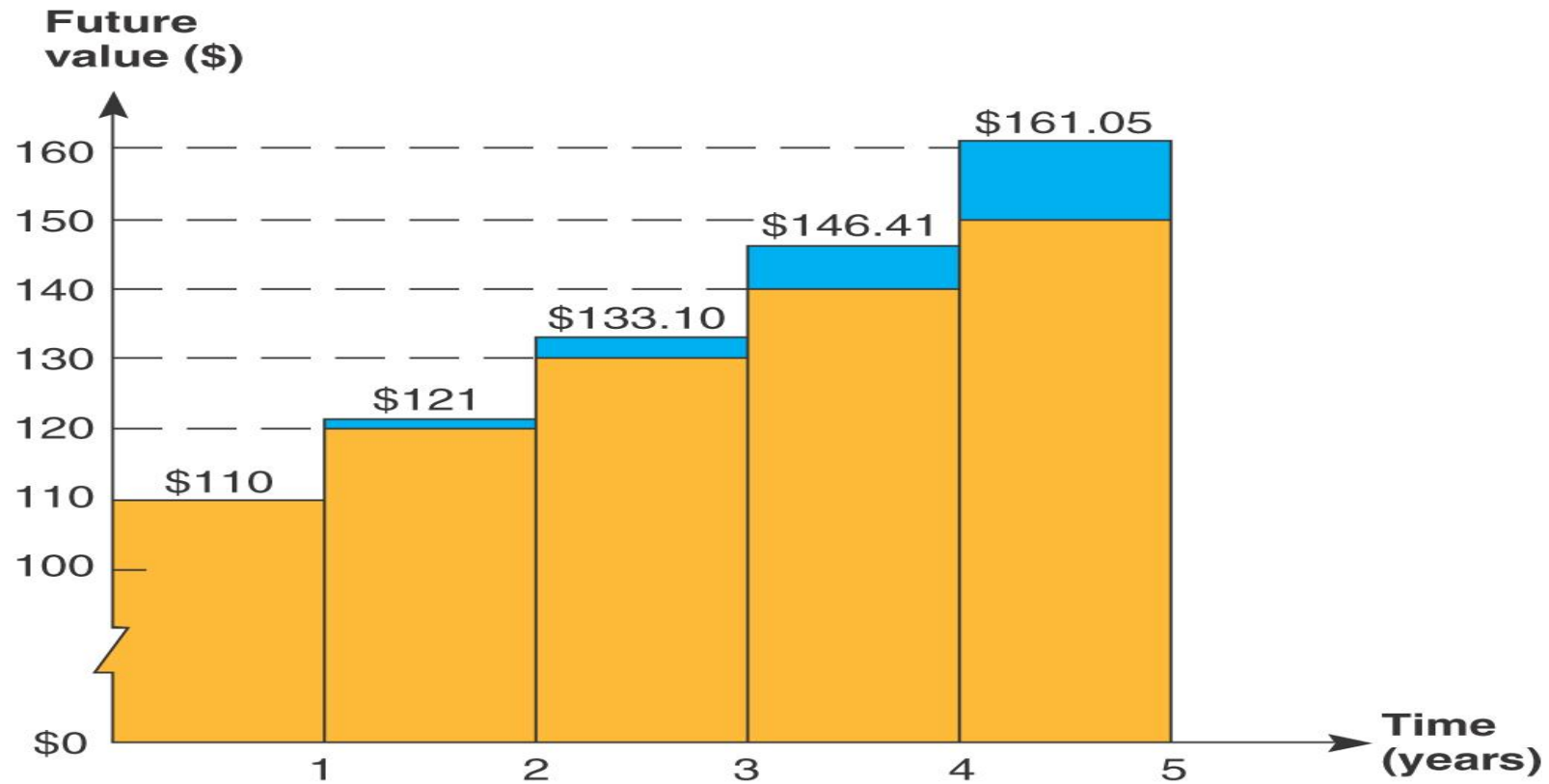
$$\begin{aligned}\text{FV} &= \text{PV}(1+r)^t \\ &= 100(1.10)^5 \\ &= 100(1.6105) \\ &= 161.05\end{aligned}$$

Keystrokes: 1.1 ; 5 N <100 N 161.0510

Table 4.1

TABLE 4.1 Future value of \$100 at 10 percent	Year	Beginning Amount	Interest Earned	Ending Amount
	1	\$100.00	\$10.00	\$110.00
	2	110.00	11.00	121.00
	3	121.00	12.10	133.10
	4	133.10	13.31	146.41
	5	146.41	14.64	161.05
			Total interest	\$61.05

Figure 4.1



Growth of \$100 original amount at 10% per year. Blue shaded area represents the portion of the total that results from compounding of interest.

Texas Instruments BA-II Plus

- To calculate FV: 10% 5 years PV=\$100

<u>Key Entry</u>	<u>Display</u>
• &^	0.00
5 ,	5.00
10 -	10.00
100 S.	-100.00
0 /	0.00
% 0	161.05



Excel Spreadsheet Functions

- Excel TVM functions:
 - =FV(rate,nper,pmt,pv)
 - =PV(rate,nper,pmt,fv)
 - =RATE(nper,pmt,pv,fv)
 - =NPER(rate,pmt,pv,fv)
- Use the formula icon (f_x) when you can't remember the exact formula
- Click on the Excel icon to open a spreadsheet containing four different examples.

Future Values – Example 3

Suppose you had a relative deposit \$10 at 5.5% interest 200 years ago. How much would the investment be worth today?

Formula Solution:

$$\begin{aligned} \text{FV} &= \text{PV}(1+r)^t \\ &= 10(1.055)^{200} \\ &= 10(44718.984) \\ &= 447,189.84 \end{aligned}$$

Calculator Solution

200	,
5.5	-
10	S.
0	/
%0	= 447,189.84

Excel Solution: =FV(Rate,Nper,Pmt,PV)

$$=FV(0.055,200,0,-10) = 447189.84$$

NOTE: Rate = decimal

Future Value: General Growth Formula

Suppose your company expects to increase unit sales of widgets by 15% per year for the next 5 years. If you currently sell 3 million widgets in one year, how many widgets do you expect to sell in 5 years?

Formula Solution:

$$\begin{aligned}\text{FV} &= \text{PV}(1+r)^t \\ &= 3(1.15)^5 \\ &= 3(2.0114) \\ &= 6.0341 \text{ million}\end{aligned}$$

Calculator Solution

$$\begin{aligned}&5 \quad , \\ &15 \quad - \\ &3 \quad . \\ &0 \quad / \\ &\%0 = -6.0341\end{aligned}$$

Excel Solution: `=FV(Rate,Nper,Pmt,PV)`

$$= \text{FV}(0.15, 5, 0, 3) = -6.0341$$

Future Value: Important Relationship I

For a given interest rate:

- The longer the time period,
- The higher the future value

$$FV = PV(1 + r)^t$$

For a given r , as t increases, FV increases

Future Value

Important Relationship II

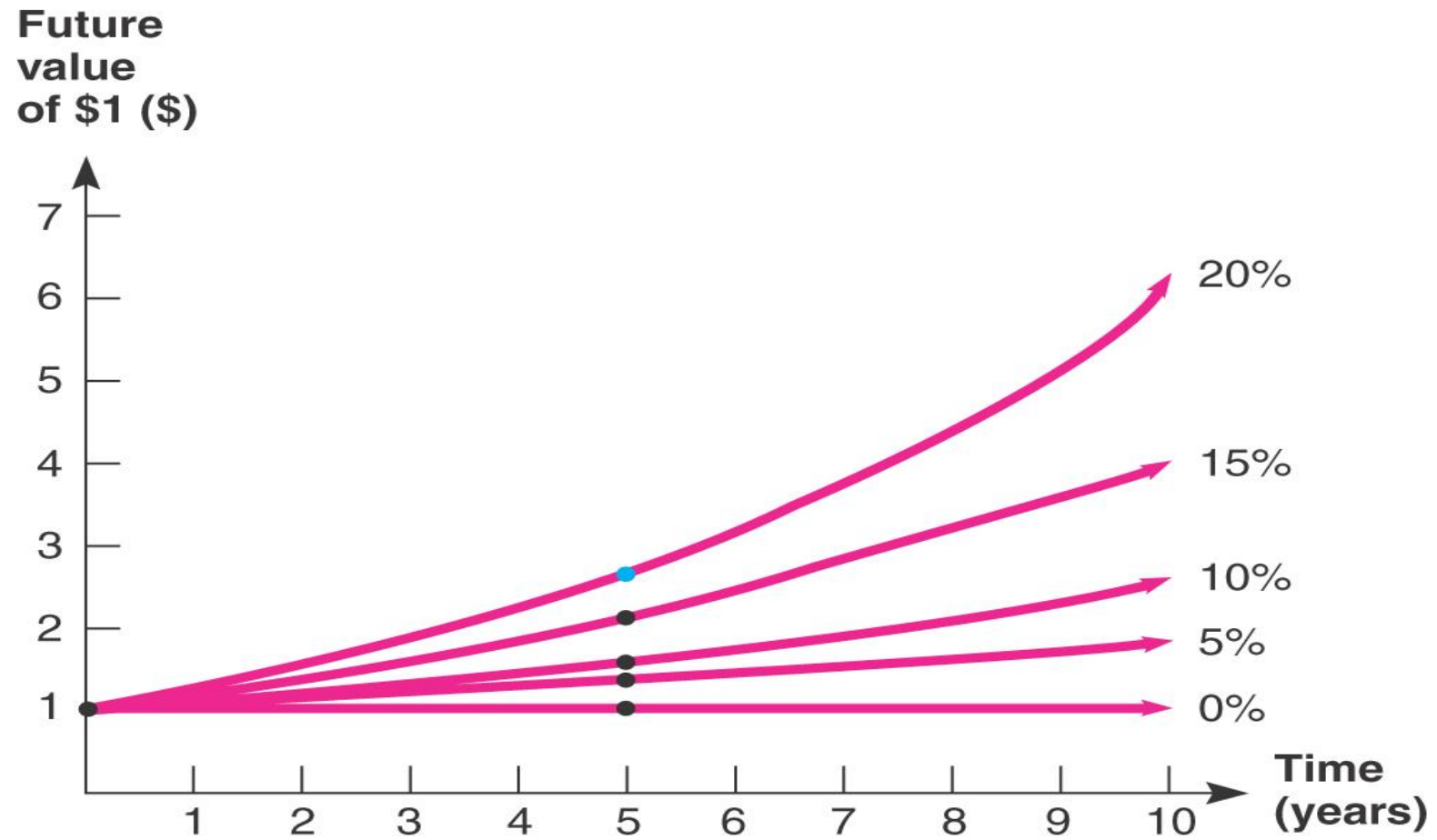
For a given time period:

- The higher the interest rate,
- The larger the future value

$$FV = PV(1 + r)^t$$

For a given t , as r increases, FV increases

Figure 4.2



Quick Quiz: Part 1

- What is the difference between simple interest and compound interest?
- Suppose you have \$500 to invest and you believe that you can earn 8% per year over the next 15 years
- How much would you have at the end of 15 years using compound interest?
- How much would you have using simple interest?

Present Values

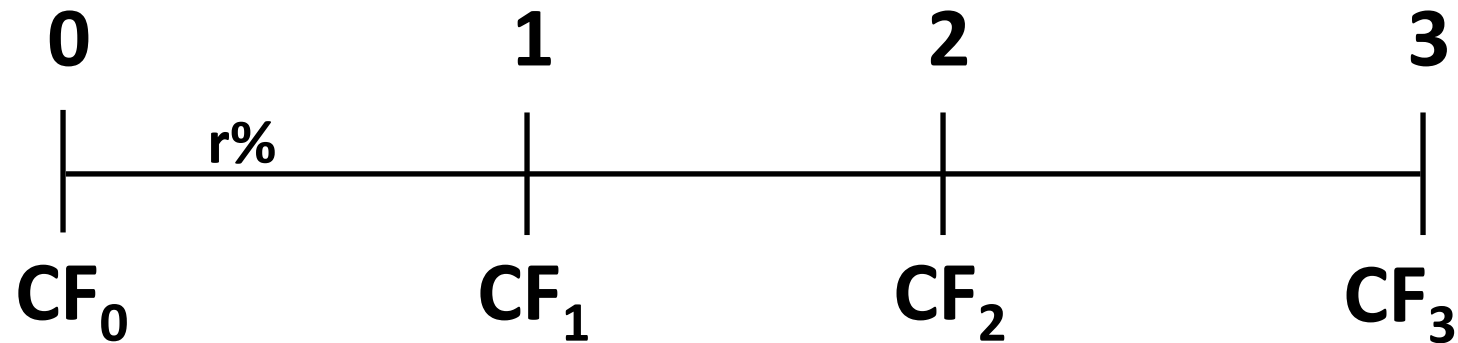
- The current value of future cash flows discounted at the appropriate discount rate
- Value at $t=0$ on a time line
- Answers the questions:
 - How much do I have to invest today to have some amount in the future?
 - What is the current value of an amount to be received in the future?

Present Values

- *Present Value = the current value of an amount to be received in the future*
 - Why is it worth less than face value?
 - Opportunity cost
 - Risk & Uncertainty
- Discount Rate = f (time, risk)

Time Line of Cash Flows

- **Tick marks** at ends of periods
 - Time 0 is today;
 - Time 1 is the end of Period 1



+CF = Cash INFLOW -CF = Cash OUTFLOW PMT = Constant CF

Present Values

$$\mathbf{FV = PV(1 + r)^t}$$

- Rearrange to solve for PV

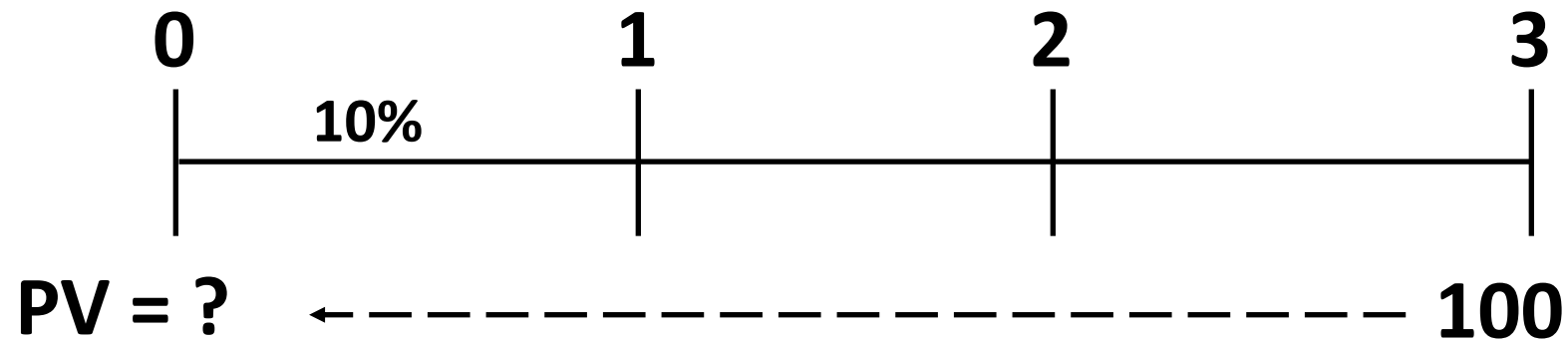
$$\mathbf{PV = FV / (1+r)^t}$$

$$\mathbf{PV = FV(1+r)^{-t}}$$

- “Discounting” = finding the present value of one or more future amounts.

What's the PV of \$100 due in 3 Years if $r = 10\%$?

Finding PVs is discounting, and it's the reverse of compounding.



Formula: $PV = FV(1+r)^{-t} = 100(1.10)^{-3} = \75.13

Calculator: 3, 10- 0/ 1000 %. = -75.13

Excel: =PV(.10,3,0,100) = -75.13

Present Value: Example 1

Single Period

Suppose you need \$10,000 in one year for the down payment on a new car. If you can earn 7% annually, how much do you need to invest today?

Formula Solution:

$$\begin{aligned} PV &= FV(1+r)^{-t} \\ &= 10,000(1.07)^{-1} \\ &= 10,000/1.07 \\ &= 9,345.79 \end{aligned}$$

Calculator Solution

1 ,
7 -
0 /
10000 0
%. = -9345.79

Excel Solution: =PV(Rate,Nper,Pmt,FV)

$$=PV(0.07,1,0,10000) = -9345.79$$

Present Values: Example 2

Multi-Periods

You want to begin saving for your daughter's college education and you estimate that she will need \$150,000 in 17 years. If you feel confident that you can earn 8% per year, how much do you need to invest today?

Formula Solution:

$$\begin{aligned}\text{PV} &= \text{FV}(1+r)^{-t} \\ &= 150,000(1.08)^{-17} \\ &= 150,000 / (1.08)^{17} \\ &= -40,540.34\end{aligned}$$

Calculator Solution:

17 ,
8 -
0 /
150000 0
%. = -40540.34

Excel Solution: =PV(Rate,Nper,Pmt,FV)

$$=PV(0.08,17,0,150000) = -40540.34$$

Present Values: Example 3

Multi-Periods

Your parents set up a trust fund for you 10 years ago that is now worth \$19,671.51. If the fund earned 7% per year, how much did your parents invest?

Formula Solution:

$$\begin{aligned}\text{PV} &= \text{FV}(1+r)^{-t} \\ &= 19,671.51(1.07)^{-10} \\ &= 19,671.51/(1.07)^{10} \\ &= -10,000\end{aligned}$$

Calculator Solution:

10 ,
7 -
0 /
19671.51 0
%. = -10000

Excel Solution: =PV(Rate,Nper,Pmt,FV)

$$=PV(0.07,10,0,19671.51) = -10000$$

Present Value: Important Relationship I

For a given interest rate:

- The longer the time period,
- The lower the present value

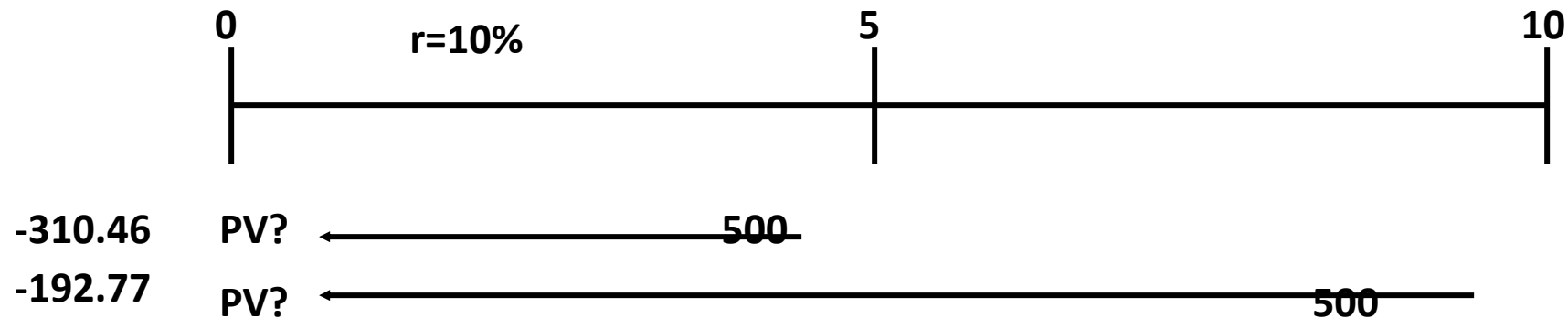
$$PV = \frac{FV}{(1+r)^t}$$

For a given r , as t increases, PV decreases

Present Value:

Important Relationship I

What is the present value of \$500 to be received in 5 years? 10 years? The discount rate is 10%



5 yrs: $PV = 500 / (1.10)^5 = -310.46$ $(1.10)^5 = 1.6105$

10 yrs: $PV = 500 / (1.10)^{10} = -192.77$ $(1.10)^{10} = 2.5937$

Present Value

Important Relationship II

For a given time period:

- The higher the interest rate,
- The smaller the present value

$$PV = \frac{FV}{(1+r)^t}$$

For a given t, as r increases, PV decreases

Present Value: Important Relationship II

What is the present value of \$500 received in 5 years if the interest rate is 10%? 15%?

Rate = 10%

Calculator Solution:

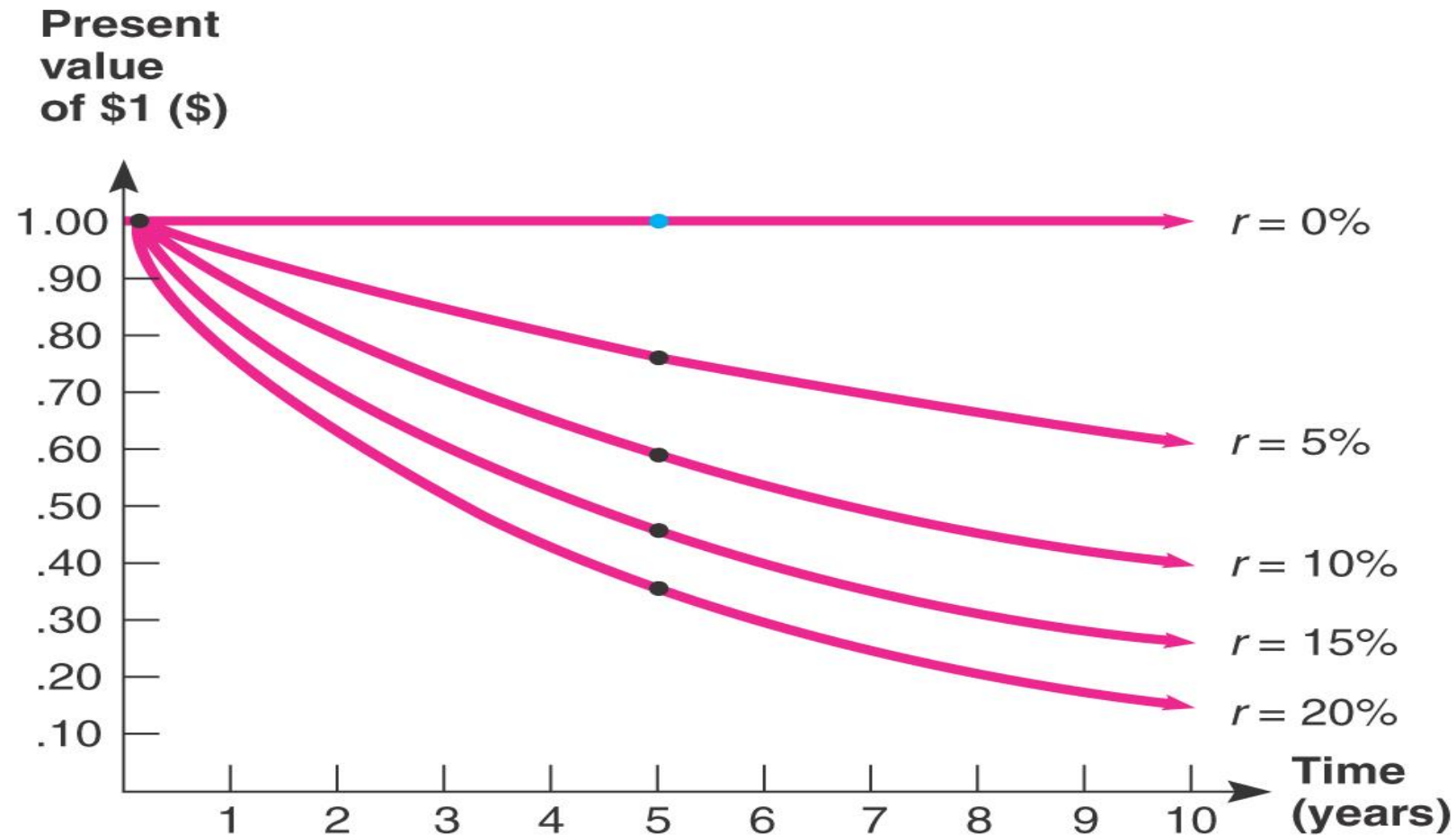
5 ,
10 -
0 /
500 0
%. = -310.46

Rate = 15%

Calculator Solution:

5 ,
15 -
0 /
500 0
%. = -248.59

Figure 4.3



Quick Quiz: Part 2

- What is the relationship between present value and future value?
- Suppose you need \$15,000 in 3 years. If you can earn 6% annually, how much do you need to invest today?
- If you could invest the money at 8%, would you have to invest more or less than at 6%? How much?

The Basic PV Equation - Refresher

$$PV = FV / (1 + r)^t$$

There are four parts to this equation

- PV, FV, r and t
- Know any three, solve for the fourth
- Be sure and remember the sign convention

+CF = Cash INFLOW -CF = Cash OUTFLOW

Discount Rate

- To find the implied interest rate, rearrange the basic PV equation and solve for r:

$$FV = PV(1 + r)^t$$

$$r = (FV / PV)^{1/t} - 1$$

- If using formulas with a calculator, make use of both the y^x and the $1/x$ keys

Discount Rate – Example 1

You are looking at an investment that will pay \$1200 in 5 years if you invest \$1000 today. What is the implied rate of interest?

- Formula:

$$r = (1200 / 1000)^{1/5} - 1 = .03714 = 3.714\%$$

- Calculator – the sign convention matters!!!

5 ,

1000 **S.** (you pay \$1,000 today)

0 /

1200 **0** (you receive \$1,200 in 5 years)

% - = 3.714%

- Excel: =RATE(5,0,-1000,1200) = 0.03714

Discount Rate – Example 2

Suppose you are offered an investment that will allow you to double your money in 6 years. You have \$10,000 to invest. What is the implied rate of interest?

$$\frac{20000}{10000} = 2$$
$$2 = (1 + r)^6$$
$$2^{1/6} = 1 + r$$
$$r = 2^{1/6} - 1 = 0.1225$$

%- = 12.25%

Excel: =RATE(6,0,-10000,20000) = 0.1225

Discount Rate – Example 3

Suppose you have a 1-year old son and you want to provide \$75,000 in 17 years towards his college education. You currently have \$5,000 to invest. What interest rate must you earn to have the \$75,000 when you need it?

Calculator:

17 ,;5000 S.;0 /; 75000 0 %- = 17.27%

Excel: =RATE(17,0,-5000,75000) = 0.1727

Quick Quiz: Part 3

- What are some situations in which you might want to compute the implied interest rate?
- Suppose you are offered the following investment choices:
 - You can invest \$500 today and receive \$600 in 5 years. The investment is considered low risk.
 - You can invest the \$500 in a bank account paying 4% annually.
 - What is the implied interest rate for the first choice and which investment should you choose?

Finding the Number of Periods

- Start with basic equation and solve for t:

$$\mathbf{FV = PV(1 + r)^t}$$

$$\mathbf{t = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+r)}}$$

Calculator: %,

Excel: =NPER(Rate, Pmt, PV, FV)

Number of Periods – Example

You want to purchase a new car and you are willing to pay \$20,000. If you can invest at 10% per year and you currently have \$15,000, how long will it be before you have enough money to pay cash for the car?

Calculator Solution:

10 -; 15000 **S** .;20000 **0**;

%, = 3.02 years

Excel: =NPER(0.10,0,-15000,20000)= 3.02

Number of Periods – Example

$$t = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+r)}$$

- Formula Solution:
 - $FV/PV = 20,000/15,000 = 1.333$
 - $\ln(1.333) = 0.2877$
 - $\ln(1.10) = 0.0953$
 - $t = 0.2877/0.0953 = 3.0189$

Quick Quiz: Part 4

- When might you want to compute the number of periods?
- Suppose you want to buy some new furniture for your family room. You currently have \$500 and the furniture you want costs \$600. If you can earn 6%, how long will you have to wait if you don't add any additional money?

Example: Work the Web

- Many financial calculators are available online
- Click on the web surfer to go to the present value portion of the Moneychimp web site and work the following example:
 - You need \$40,000 in 15 years. If you can earn 9.8% interest, how much do you need to invest today?
 - You should get \$9,841

Table 4.4

I. Symbols

PV = Present value, what future cash flows are worth today

FV_t = Future value, what cash flows are worth in the future

r = Interest rate, rate of return, or discount rate per period typically , but not always, one year

t = Number of periods typically , but not always, the number of years

C = Cash amount

II. Future value of C invested at r percent per period for t periods

$$FV_t = C \times (1 + r)^t$$

The term $(1 + r)^t$ is called the *future value factor*.

III. Present value of C to be received in t periods at r percent per period

$$PV = C/(1 + r)^t$$

The term $1/(1 + r)^t$ is called the *present value factor*.

IV. The basic present value equation giving the relationship between present and future value is

$$PV = FV_t/(1 + r)^t$$

Quick Quiz 1 Solution

- Invest \$500 at 8% per year over 15 years.
 - How much would you have at the end of 15 years using compound interest?
 - 15, 8 - 500 S. 0 / %0 1586.08
 - $500(1.08)^{15} = 1586.08$
 - $=FV(.08, 15, 0, -500)$
 - How much would you have using simple interest?
 - $500 + 15(500)(.08) = 1,100$

Quick Quiz 2 Solution

- You need \$15,000 in 3 years. You can earn 6% annually, how much do you need to invest today?

3 , 6 - 15000 0 0 / %. -12594.29

$$PV = 15000 / (1.06)^3 = 15000 / (1.191016) =$$

$$= 15000 \times 0.83962 = 12594.29$$

$$= PV(.06, 3, 0, 15000)$$

Quick Quiz 2 Solution

- You need \$15,000 in 3 years. If you could invest the money at 8%, would you have to invest more or less than at 6%? How much?

3 , 8 - 15000 0 0 / %. -11907.48

$$PV = 15000 / (1.08)^3 = 15000 / (1.125971)$$

$$= 15000 \times (0.79383) = 11907.48$$

$$= PV(.08, 3, 0, 15000)$$

$$\text{Difference} = \$686.81$$

Quick Quiz 3 Solution

Investment choices:

- Invest \$500 today and receive \$600 in 5 years. The investment is considered low risk.
- Invest the \$500 in a bank account paying 4% annually.
- What is the implied interest rate for the first choice and which investment should you choose?

5, 500 \$, 0 / 600 0 %- 3.714%

$$r = (600/500)^{1/5} - 1 = 3.714\%$$

$$= \text{RATE}(5, 0, -500, 600)$$

The bank account pays a higher rate.

Quick Quiz 4 Solution

Suppose you want to buy some new furniture for Your family room. You currently have \$500 and the furniture you want costs \$600. If you can earn 6%, how long will you have to wait if you don't add any additional money?

6 - 500 \$, 0 / 600 0 %, 3.13 years

$t = \ln(600/500) / \ln(1.06) = 3.13 \text{ years}$

$=\text{NPER}(.06, 0, -500, 600)$



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Thank You

