

Master of Financial Technology

FNT 615
Finance Concepts





Time Value of Money

Chapter 5



Chapter 04

Intended Learning Outcomes

After reading this chapter, you should be able to:

1. Explain the mechanics of compounding and bringing the value of money back to the present.
2. Understand Annuities
3. Determine the future or present value of a sum when there are nonannual compounding periods.
4. Determine the present value of an uneven stream of payments and understand perpetuities.

Annuties

Annuities



An annuity is a **series of equal dollar payments** for a specified number of years.



When we talk about annuities, we are referring to ordinary annuities unless otherwise noted.



With an ordinary annuity the payments occur at the end of each period. Because annuities occur frequently in finance—for example, as bond interest payments—we treat them specially.



Although compounding and determining the present value of an annuity can be dealt with using the methods we have just described, these processes can be time-consuming, especially for larger annuities.



Thus, we have modified the single cash flow formulas to deal directly with annuities.

Annuities

An **annuity** is a stream of equal periodic cash flows, over a specified time period. These cash flows can be *inflows* of returns earned on investments or *outflows* of funds invested to earn future returns.

- An **ordinary (deferred) annuity** is an annuity for which the cash flow occurs at the *end* of each period
- An **annuity due** is an annuity for which the cash flow occurs at the *beginning* of each period.
- NB An annuity due will always be greater than an otherwise equivalent ordinary annuity because interest will compound for an additional period.

Compound Annuities

A compound annuity involves depositing or investing an equal sum of money at the end of each year for a certain number of years and allowing it to grow.

Perhaps we are saving money for education, a new car, or a vacation home.

In any case, we want to know how much our savings will have grown at some point in the future.

Compound Annuities

- Actually, we can find the answer by using equation (5-1), our compounding equation, and compounding each of the individual deposits to its future value.
- For example, if to provide for a college education we are going to deposit \$500 at the end of each year for the next 5 years in a bank where it will earn 6 percent interest, how much will we have at the end of 5 years? Compounding each of these values using equation (5-1), we find that we will have \$2,818.50 at the end of 5 years.

$$\begin{aligned} \text{FV}_5 &= \$500(1 + 0.06)^4 + \$500(1 + 0.06)^3 + \$500(1 + 0.06)^2 + \$500(1 + 0.06) + \\ &\quad \$500 \\ &= \$500(1.262) + \$500(1.191) + \$500(1.124) + \$500(1.060) + \$500 \\ &= \$631.00 + \$595.50 + \$562.00 + \$530.00 + \$500.00 \\ &= \$2,818.50 \end{aligned}$$

Compound Annuities

- By examining the mathematics involved and the graph of the movement of money through time in Table 5-1, we can see that all we are really doing is adding up the future values of different cash flows that initially occurred in different time periods.
- Fortunately, there is also an equation that helps us calculate the future value of an annuity:

$$\begin{aligned}\text{Future value of an annuity} &= PMT \left[\frac{\text{future value factor} - 1}{r} \right] \\ &= PMT \left[\frac{(1 + r)^n - 1}{r} \right]\end{aligned}$$

Compound Annuities

TABLE 5-1 Growth of a 5-Year, \$500 Annuity Compounded at 6 Percent

YEAR	0	1	2	3	4	5
	$r = 6\%$					
Dollar deposits at end of year		500	500	500	500	500
						\$ 500.00
						530.00
						562.00
						595.50
						631.00
Future value of the annuity						<u>\$2,818.50</u>

Compound Annuities

SOLVING FOR THE PRESENT VALUE WITH TWO FLOWS IN DIFFERENT YEARS

There are several different ways you can solve this problem—using the mathematical formulas, a financial calculator, or a spreadsheet—each one giving you the same answer.

1. Using the Mathematical Formula. Substituting the values of $n = 5$, $r = 4$ percent, and $FV_5 = \$500$; and $n = 10$, $r = 4$ percent, and $FV_{10} = \$1,000$ into equation (5-2) and adding these values together, we find

$$\begin{aligned}\text{Present value} &= \$500 \left[\frac{1}{(1 + 0.04)^5} \right] + \$1,000 \left[\frac{1}{(1 + 0.04)^{10}} \right] \\ &= \$500(0.822) + \$1,000(0.676) \\ &= \$411 + \$676 = \$1,087\end{aligned}$$

Compound Annuities

To simplify our discussion, we will refer to the value in brackets in equation (5-3) as the **annuity future value factor**. It is defined as $\left[\frac{(1 + r)^n - 1}{r} \right]$.

Using this new notation, we can rewrite equation (5-3) as follows:

$$FV_n = PMT \left[\frac{(1 + r)^n - 1}{r} \right] = PMT(\text{annuity future value factor})$$

Rather than asking how much we will accumulate if we deposit an equal sum in a savings account each year, a more common question to ask is how much we must deposit each year to accumulate a certain amount of savings. This problem frequently occurs with respect to saving for large expenditures.

For example, if we know that we need \$10,000 for college in 8 years, how much must we deposit in the bank at the end of each year at 6 percent interest to have the college money ready? In this case we know the values of n , r , and FV_n in equation (5-3); what we do not know is the value of PMT . Substituting these example values in equation (5-3), we find

$$\begin{aligned} \$10,000 &= PMT \left[\frac{(1 + 0.06)^8 - 1}{0.06} \right] \\ \$10,000 &= PMT(9.8975) \\ \frac{\$10,000}{9.8975} &= PMT \end{aligned}$$

Thus, we must deposit \$1,010.36 in the bank at the end of each year for 8 years at 6 percent interest to accumulate \$10,000 at the end of 8 years.

EXAMPLE 5.7**Calculating deposit amount to accumulate \$5,000**

How much must we deposit in an 8 percent savings account at the end of each year to accumulate \$5,000 at the end of 10 years?

STEP 1: FORMULATE A SOLUTION STRATEGY

In order to determine the amount we must deposit, we have to use equation (5-3) as follows:

$$FV = PMT \left[\frac{(1 + r)^n - 1}{r} \right] \quad (5-3)$$

STEP 2: CRUNCH THE NUMBERS

Substituting into equation (5-3), we compute the future value as follows:

$$\$5,000 = PMT \left[\frac{(1 + 0.06)^8 - 1}{0.06} \right]$$

$$\$5,000 = PMT(14.4866)$$

$$\frac{5,000}{14.4866} = PMT = \$345.15$$

STEP 3: ANALYZE YOUR RESULTS

Thus, we must deposit \$345.15 per year for 10 years at 8 percent to accumulate \$5,000.

The Present Value of an Annuity

- Pension payments, insurance obligations, and the interest owed on bonds all involve annuities. To compare these three types of investments we need to know the present value of each.
- For example, if we wish to know what \$500 received at the end of each of the next 5 years is worth today given a discount rate of 6 percent, we can simply substitute the appropriate values into equation (5-2), such that

$$\begin{aligned} PV &= \$500 \left[\frac{1}{(1 + 0.06)^1} \right] + \$500 \left[\frac{1}{(1 + 0.06)^2} \right] + \$500 \left[\frac{1}{(1 + 0.06)^3} \right] \\ &\quad + \$500 \left[\frac{1}{(1 + 0.06)^4} \right] + \$500 \left[\frac{1}{(1 + 0.06)^5} \right] \\ &= \$500(0.943) + \$500(0.890) + \$500(0.840) + \$500(0.792) + \$500(0.747) \\ &= \$2,106 \end{aligned}$$

The Present Value of an Annuity

- Thus, the present value of this annuity is \$2,106.00.
- By examining the mathematics involved and the graph of the movement of money through time in Table 5-2, we can see that all we are really doing is adding up the present values of different cash flows that initially occurred in different time periods.
- Fortunately, there is also an equation that helps us calculate the present value of an annuity:

$$\begin{aligned}\text{Present value of an annuity} &= PMT \left[\frac{1 - \text{present value factor}}{r} \right] \\ &= PMT \left[\frac{1 - (1 + r)^{-n}}{r} \right]\end{aligned}$$

TABLE 5-2 Illustration of a 5-Year, \$500 Annuity Discounted to the Present at 6 Percent

YEAR	<i>r</i> = 6%					
	0	1	2	3	4	5
Dollars received at end of year		500	500	500	500	500
	\$ 471.50	←				
	445.00	←				
	420.00	←				
	396.00	←				
	373.50	←				
Present value of the annuity	<u>\$2,106.00</u>					

To simplify our discussion, we will refer to the value in the brackets in equation (5-4) as the **annuity present value factor**. It is defined as $\left[\frac{1 - (1 + r)^{-n}}{r} \right]$.

EXAMPLE 5.8**Calculating the present value of an annuity**

What is the present value of a 10-year \$1,000 annuity discounted back to the present at 5 percent?

STEP 1: FORMULATE A SOLUTION STRATEGY

The present value of the annuity can be computed using equation (5-4) as follows:

$$PV = PMT \left[\frac{1 - (1 + r)^{-n}}{r} \right] \quad (5-4)$$

STEP 2: CRUNCH THE NUMBERS

Substituting into equation (5-4), we compute the present value as follows:

$$\begin{aligned} PV &= \$1,000 \left[\frac{1 - (1 + 0.05)^{-10}}{0.05} \right] \\ &= \$1,000(7.722) \\ &= \$7,722 \end{aligned}$$

STEP 3: ANALYZE YOUR RESULTS

Thus, the present value of this annuity is \$7,722.

The Present Value of an Annuity

- When we solve for PMT, the financial interpretation of this action would be: How much can be withdrawn, perhaps as a pension or to make loan payments, from an account that earns r percent compounded annually for each of the next n years if we wish to have nothing left at the end of n years?
- For example, if we have \$5,000 in an account earning 8 percent interest, how large of an annuity can we draw out each year if we want nothing left at the end of 5 years? In this case the present value, PV, of the annuity is \$5,000, $n = 5$ years, $r = 8$ percent, and PMT is unknown. Substituting this into equation (5-4), we find

$$\$5,000 = PMT \left[\frac{1 - (1 + 0.08)^{-5}}{0.08} \right] = PMT(3.993)$$

$$\$1,252 = PMT$$

Thus, this account will fall to zero at the end of 5 years if we withdraw \$1,252 at the end of each year.

Annuities Due

- Annuities due are really just ordinary annuities in which all the annuity payments have been shifted forward by 1 year.
- Compounding them and determining their future and present value is actually quite simple.
- With an annuity due, each annuity payment occurs at the beginning of each period rather than at the end of the period.
- Let's first look at how this affects our compounding calculations.
- Because an annuity due merely shifts the payments from the end of the year to the beginning of the year, we now compound the cash flows for one additional year.

Future value of an annuity due = future value of an annuity $\times (1 + r)$

$$FV_n(\text{annuity due}) = PMT \left[\frac{(1 + r)^n - 1}{r} \right] (1 + r) \quad (5-5)$$

In an earlier example on saving for college, we calculated the value of a 5-year ordinary annuity of \$500 invested in the bank at 6 percent to be \$2,818.50. If we now assume this to be a 5-year annuity due, its future value increases from \$2,818.50 to \$2,987.66.

$$\begin{aligned} FV_5 &= \$500 \left[\frac{(1 + 0.06)^5 - 1}{0.06} \right] (1 + 0.06) \\ &= \$500(5.637093)(1.06) \\ &= \$2,987.66 \end{aligned}$$

Annuities Due

- Likewise, with the present value of an annuity due, we simply receive each cash flow 1 year earlier—that is, we receive it at the beginning of each year rather than at the end of each year.
- Thus, because each cash flow is received 1 year earlier, it is discounted back for one less period. To determine the present value of an annuity due, we merely need to find the present value of an ordinary annuity and multiply that by $(1 + r)$, which in effect cancels out 1 year's discounting.

Present value of an annuity due = present value of an annuity $\times (1 + r)$

$$PV(\text{annuity due}) = PMT \left[\frac{1 - (1 + r)^{-n}}{r} \right] (1 + r) \quad (5-6)$$

Reexamining the earlier college saving example in which we calculated the present value of a 5-year ordinary annuity of \$500 given a discount rate of 6 percent, we now find that if it is an annuity due rather than an ordinary annuity, the present value increases from \$2,106 to \$2,232.55.

$$\begin{aligned} PV &= \$500 \left[\frac{1 - (1 + 0.06)^{-5}}{0.06} \right] (1 + 0.06) \\ &= \$500(4.21236)(1.06) \\ &= \$2,232.55 \end{aligned}$$

EXAMPLE 5.9**Calculating the Lotto payments on \$2 million**

Forty-three states run lotteries and most of those lotteries are similar: You must select 6 out of 44 numbers correctly in order to win the jackpot. If you come close, there are some significantly lesser prizes, which we ignore for now. For each million dollars in the lottery jackpot, you receive \$50,000 per year for 20 years, and your chance of winning is 1 in 7.1 million. A recent advertisement for a particular state lottery went as follows: “Okay, you got two kinds of people. You’ve got the kind who play Lotto all the time, and the kind who play Lotto some of the time. You know, like only on a Saturday when they stop in at the store on the corner for some peanut butter cups and diet soda and the jackpot happens to be really big. I mean, my friend Ned? He’s like ‘Hey, it’s only \$2 million this week.’ Well, hellloooo, anybody home? I mean, I don’t know about you, but I wouldn’t mind having a measly 2 mill coming *my* way. . . .”

What is the present value of these payments?

STEP 1: FORMULATE A SOLUTION STRATEGY

The answer to this question depends on what assumption you make about the time value of money. In this case, let’s assume that your required rate of return on an investment with this level of risk is 10 percent. Keeping in mind that the Lotto is an annuity due—that is, on a \$2 million lottery you would get \$100,000 immediately and \$100,000 at the end of each of the next 19 years. Thus, the present value of this 20-year annuity due discounted back to present at 10 percent becomes:

$$PV(\text{annuity due}) = PMT \left[\frac{1 - (1 + r)^{-n}}{r} \right]$$

STEP 2: CRUNCH THE NUMBERS

Solving this is a two-step process. In step 1, you treat the jackpot as if it were an ordinary annuity and find the present value. Then multiply the present value times $(1 + r)$, in this case times 1.10.

Substituting into equation (5-6), we compute the present value as follows:

$$\begin{aligned} PV &= \$100,000 \left[\frac{1 - (1 + 0.10)^{-20}}{0.10} \right] (1 + 0.10) \\ &= \$100,000(8.51356)(1.10) \\ &= \$936,492 \end{aligned}$$

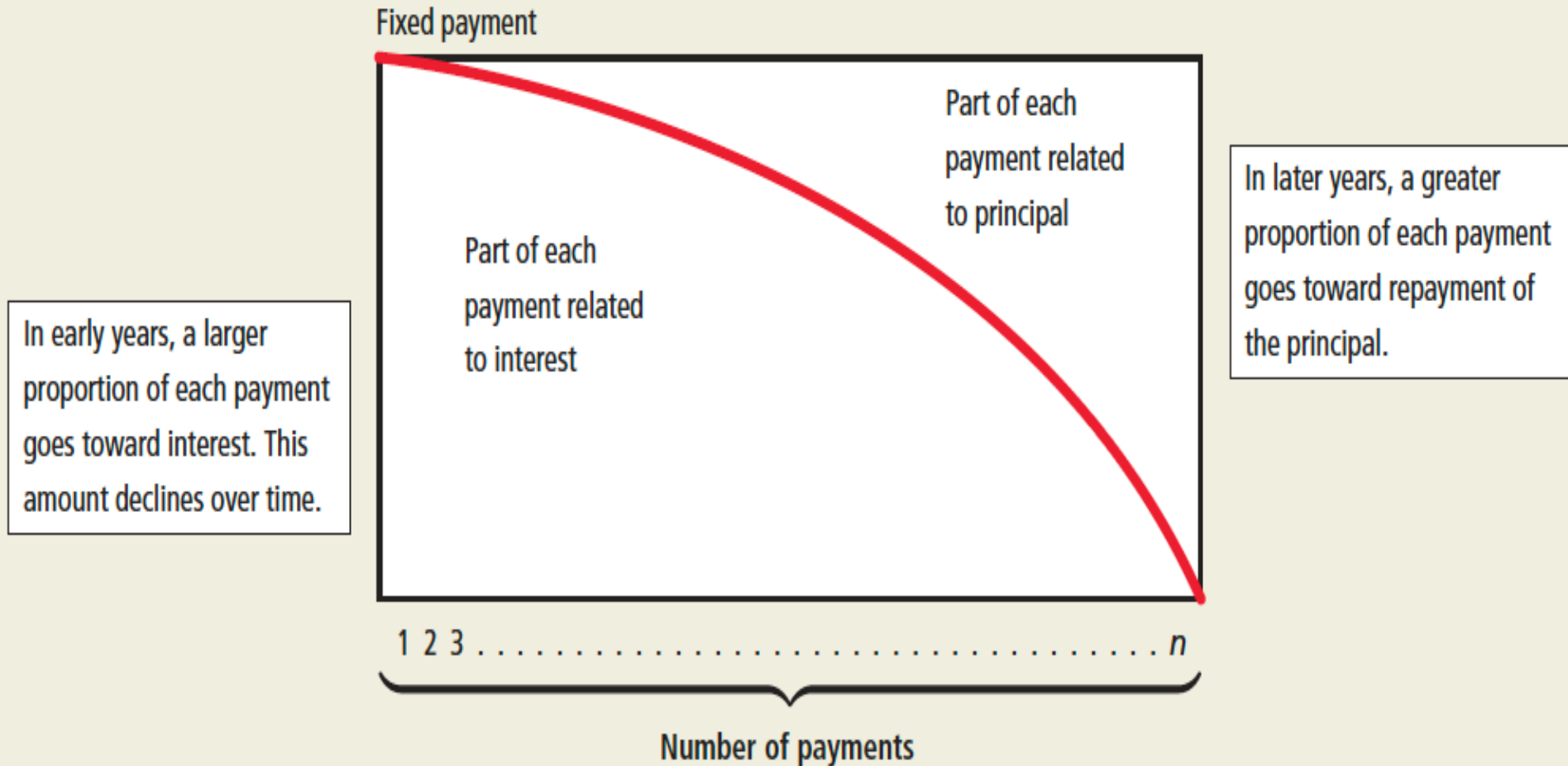
STEP 3: ANALYZE YOUR RESULTS

Thus, the present value of the \$2 million Lotto jackpot is less than \$1 million if 10 percent is the discount rate. Moreover, because the chance of winning is only 1 in 7.1 million, the expected value of each dollar “invested” in the lottery is only $(1/7.1 \text{ million}) \times (\$936,492) = 13.19\text{¢}$. That is, for every dollar you spend on the lottery you should expect to get, on average, about 13 cents back—not a particularly good deal. Although this ignores the minor payments for coming close, it also ignores taxes. In this case, it looks like “my friend Ned” is doing the right thing by staying clear of the lottery. Obviously, the main value of the lottery is entertainment. Unfortunately, without an understanding of the time value of money, it can sound like a good investment.

Amortized Loans

- The procedure of solving for PMT, the annuity payment value when r , n , and PV are known, is also used to determine what payments are associated with paying off a loan in equal installments over time.
- Loans that are paid off this way, in equal periodic payments, are called amortized loans.
- Actually, the word amortization comes from the Latin word meaning “about to die.” When you pay off a loan using regular, fixed payments, that loan is amortized.
- Although the payments are fixed, different amounts of each payment are applied toward the principal and the interest.
- With each payment, you owe a bit less toward the principal.
- As a result, the amount that has to go toward the interest payment declines with each payment, whereas the portion of each payment that goes toward the principal increases. Figure 5-4 illustrates the process of amortization.

FIGURE 5-4 The Amortization Process



Example

- For example, suppose a firm wants to purchase a piece of machinery.
- To do this, it borrows \$6,000 to be repaid in four equal payments at the end of each of the next 4 years, and the interest rate that is paid to the lender is 15 percent on the outstanding portion of the loan.
- To determine what the annual payments associated with the repayment of this debt will be, we simply use equation (5-4) and solve for the value of PMT, the annual annuity.
- In this case, we know PV, r, and n. PV, the present value of the annuity, is \$6,000; r, the annual interest rate, is 15 percent; and n, the number of years for which the annuity will last, is 4 years.
- PMT, the annuity payment received (by the lender and paid by the firm) at the end of each year, is unknown.
- Substituting these values into equation (5-4), we find

$$\$6,000 = PMT \left[\frac{1 - (1 + 0.15)^{-4}}{0.15} \right]$$

$$\$6,000 = PMT(2.85498)$$

$$\$2,101.59 = PMT$$

Example

- To repay the principal and interest on the outstanding loan in 4 years, the annual payments would be \$2,101.59.
- The breakdown of interest and principal payments is given in the loan amortization schedule in Table 5-3, with very minor rounding error.
- As you can see, the interest portion of the payment declines each year as the loan outstanding balance declines.

TABLE 5-3 Loan Amortization Schedule Involving a \$6,000 Loan at 15 Percent to Be Repaid in 4 Years

Year	Annuity	Interest Portion of the Annuity ^a	Repayment of the principal Portion of the Annuity ^b	Outstanding Loan Balance After the Annuity Payment
0	0	0	0	\$6,000.00
1	\$2,101.59	\$900.00	\$1,201.59	4,798.41
2	2,101.59	719.76	1,381.83	3,416.58
3	2,101.59	512.49	1,589.10	1,827.48
4	2,101.59	274.12	1,827.48	

Finding Present and Future Values with Nonannual Periods

- The same logic that applies to calculating the EAR also applies to calculating present and future values when the periods are semiannual, quarterly, or any other nonannual period.
- Previously when we moved money through time we assumed that the cash flows occurred annually and the compounding or discounting period was always annual.
- However, it need not be, as evidenced by the fact that bonds generally pay interest semiannually and most mortgages require monthly payments.



Finding Present and Future Values with Nonannual Periods

- For example, if we invest our money for 5 years at 8 percent interest compounded semiannually, we are really investing our money for ten 6-month periods during which we receive 4 percent interest each period. If it is 8 percent compounded quarterly for 5 years, we receive 2 percent interest per period for twenty 3-month periods.
- This process can easily be generalized, giving us the following formula for finding the future value of an investment for which interest is compounded in nonannual periods:

$$FV_n = PV \left[1 + \frac{r}{m} \right]^{m \cdot n}$$

where FV_n = the future value of the investment at the end of n years

n = the number of years during which the compounding occurs

r = annual interest (or discount) rate

PV = the present value or original amount invested at the beginning of the first year

m = the number of times compounding occurs during the year

Finding Present and Future Values with Nonannual Periods



In fact, all the formulas for moving money through time can be easily modified to accommodate nonannual periods. In each case, we begin with the formulas we introduced in this chapter, and make two adjustments—the first where n , the number of years, appears, and the second where r , the annual interest rate, appears.



Thus, the adjustment involves two steps:



n becomes the number of periods or n (the number of years) times m (the number of times compounding occurs per year).



Thus, if it is monthly compounding for 10 years, n becomes $10 * 12 = 120$ months in 10 years, and if it is daily compounding over 10 years, it becomes $10 * 365 = 3,650$ days in 10 years.

TABLE 5-4 The Value of \$100 Compounded at Various Intervals

FOR 1 YEAR AT r PERCENT				
$r =$	2%	5%	10%	15%
Compounded annually	\$102.00	\$105.00	\$110.00	\$115.00
Compounded semiannually	102.01	105.06	110.25	115.56
Compounded quarterly	102.02	105.09	110.38	115.87
Compounded monthly	102.02	105.12	110.47	116.08
Compounded weekly (52)	102.02	105.12	110.51	116.16
Compounded daily (365)	102.02	105.13	110.52	116.18
FOR 10 YEARS AT r PERCENT				
$r =$	2%	5%	10%	15%
Compounded annually	\$121.90	\$162.89	\$259.37	\$404.56
Compounded semiannually	122.02	163.86	265.33	424.79
Compounded quarterly	122.08	164.36	268.51	436.04
Compounded monthly	122.10	164.70	270.70	444.02
Compounded weekly (52)	122.14	164.83	271.57	447.20
Compounded daily (365)	122.14	164.87	271.79	448.03

Finding Present and Future Values with Nonannual Periods



r becomes the interest rate per period or r (the annual interest rate) divided by m (the number of times compounding occurs per year).



Thus, if it is a 6 percent annual rate with monthly compounding, r becomes $6\% / 12 = 0.5$ percent per month, and if it is a 6 percent annual rate compounded daily, then r becomes $(6\% / 365)$ per day.



We can see the value of intrayear compounding by examining Table 5-4. Because “interest is earned on interest” more frequently as the length of the compounding period declines, there is an inverse relationship between the length of the compounding period and the effective annual interest rate: The shorter the compounding period is, the higher the effective interest rate will be.



Conversely, the longer the compounding period is, the lower the effective interest rate will be.

EXAMPLE 5.11**Calculating the growth of an investment**

If we place \$100 in a savings account that yields 12 percent compounded quarterly, what will our investment grow to at the end of 5 years?

STEP 1: FORMULATE A SOLUTION STRATEGY

The future value of our savings account can be computed using equation (5-8) as follows:

$$\text{Future value} = \text{present value} \times \left(1 + \frac{r}{m}\right)^{m \cdot n}$$

STEP 2: CRUNCH THE NUMBERS

Substituting into equation (5-8), we compute the future value as follows:

$$\begin{aligned} FV &= \$100 \left(1 + \frac{0.12}{4}\right)^{4 \cdot 5} \\ &= \$100(1.8061) = \$180.61 \end{aligned}$$

STEP 3: ANALYZE YOUR RESULTS

Thus, we will have \$180.61 at the end of 5 years. In this problem, n becomes the number of quarters in 5 years while r/m becomes the interest rate per period—in effect, 3 percent per quarter for 20 quarters.

EXAMPLE 5.12**Calculating the future value of an investment**

In 2009 the average U.S. household owed about \$10,000 in credit card debt, and the average interest rate on credit card debt was 13.0 percent. On many credit cards the minimum monthly payment is 4 percent of the debt balance. If the average household paid off 4 percent of the initial amount it owed each month, that is, made payments of \$400 each month, how many months would it take to repay this credit card debt? Use a financial calculator to solve this problem.

STEP 1: FORMULATE A SOLUTION STRATEGY

The easiest approach here is to either use your financial calculator or use Excel to solve for N, the number of periods. Using Excel, you would simply use the `=NPER` function and input values for rate, pmt, pv, and fv. Using your financial calculator, you would simply solve for N; however, you'll need to make sure that I/Y is expressed as a monthly rate because you are solving for the number of months necessary to repay the credit card.

STEP 2: CRUNCH THE NUMBERS

The solution, using a financial calculator, appears in the margin. You'll notice in the solution that the value for PMT goes in as a negative.

STEP 3: ANALYZE YOUR RESULTS

The answer is over 29 months before you pay off your credit card if you pay off 4 percent of the initial amount owed each month—obviously, if you keep using the credit card, it will take longer.

The Present Value of an Uneven Stream and Perpetuities

- Although some projects will involve a single cash flow and some will involve annuities, many projects will involve uneven cash flows over several years.

YEAR	CASH FLOW	YEAR	CASH FLOW
1	\$ 0	6	500
2	200	7	500
3	−400	8	500
4	500	9	500
5	500	10	500

Example

- For example, if we wished to find the present value of the following cash flows then by adding in the positive flows and subtracting the negative ones.
- However, this problem is complicated by the annuity of \$500 that runs from years 4 through 10.
- To accommodate this, we can first discount the annuity back to the beginning of period 4 (or end of period 3) and get its present value at that point in time.
- We then bring this single cash flow (which is the present value of the 7-year annuity) back to the present.
- In effect, we discount twice, first back to the end of period 3, then back to the present.
- This is shown graphically in Table 5-5 and numerically in Table 5-6. Thus, the present value of this uneven stream of cash flows is \$2,185.69.

Example

TABLE 5-5 The Present Value of an Uneven Stream Involving One Annuity Discounted to the Present at 6 Percent: An Example

YEAR	$r = 6\%$										
	0	1	2	3	4	5	6	7	8	9	10
Dollars received at end of year		0	200	-400	500	500	500	500	500	500	500
			\$ 178.00	-335.85							
				\$2,791.19							
			<u>2,343.54</u>								
Total present value			<u>\$2,185.69</u>								

Example

TABLE 5-6 Determining the Present Value of an Uneven Stream Involving One Annuity Discounted to the Present at 6 Percent: An Example

1. Present value of \$200 received at the end of 2 years at 6% =	\$ 178.00
2 Present value of a \$400 outflow at the end of 3 years at 6% =	−335.85
3. (a) Value at end of year 3 of a \$500 annuity, years 4–10 at 6% = \$2,791.19	
(b) Present value of \$2,791.19 received at the end of year 3 at 6% =	<u>2,343.54</u>
4. Total present value =	\$2,185.69

Remember, once the cash flows from an investment have been brought back to the present, they can be combined by adding and subtracting to determine the project's total present value.

Perpetuities

- A perpetuity is an annuity that continues forever; that is, every year following its establishment this investment pays the same dollar amount.
- An example of a perpetuity is preferred stock that pays a constant dollar dividend infinitely.
- Determining the present value of a perpetuity is delightfully simple; we merely need to divide the constant flow by the discount rate.
- For example, the present value of a \$100 perpetuity discounted back to the present at 5 percent is $\$100/0.05 = \$2,000$.
- Thus, the equation representing the present value of a perpetuity is

$$PV = \frac{PP}{r}$$

where: PV = the present value of the perpetuity

PP = the constant dollar amount provided by the perpetuity

r = the annual interest (or discount) rate

EXAMPLE 5.13**Calculating the future value of an investment**

What is the present value of a \$500 perpetuity discounted back to the present at 8 percent?

STEP 1: FORMULATE A SOLUTION STRATEGY

The present value of the perpetuity can be calculated using equation (5-9) as follows:

$$PV = \frac{PP}{r}$$

STEP 2: CRUNCH THE NUMBERS

Substituting $PP = \$500$ and $r = 0.08$ into equation (5-9), we find

$$PV = \frac{\$500}{0.08} = \$6,250$$

STEP 3: ANALYZE YOUR RESULTS

Thus, the present value of this perpetuity is \$6,250.

TABLE 5-7 Summary of Time Value of Money Equations

Calculation	Equation
Future value of a single payment	$FV_n = PV(1 + r)^n$
Present value of a single payment	$PV = FV_n \left[\frac{1}{(1 + r)^n} \right]$
Future value of an annuity	$FV \text{ of an annuity} = PMT \left[\frac{(1 + r)^n - 1}{r} \right]$
Present value of an annuity	$PV \text{ of an annuity} = PMT \left[\frac{1 - (1 + r)^{-n}}{r} \right]$
Future value of an annuity due	$FV_n(\text{annuity due}) = \text{future value of an annuity} \times (1 + r)$
Present value of an annuity due	$PV(\text{annuity due}) = \text{present value of an annuity} \times (1 + r)$
Effective annual return (EAR) =	$\left[1 + \frac{\text{quoted rate}}{m} \right]^m - 1$
Future value of a single payment with nonannual compounding	$FV_n = PV \left(1 + \frac{r}{m} \right)^{mn}$
Present value of a perpetuity	$PV = \frac{PP}{r}$
Notations: FV_n = the future value of the investment at the end of n years n = the number of years until payment will be received or during which compounding occurs r = the annual interest or discount rate PV = the present value of the future sum of money m = the number of times compounding occurs during the year PMT = the annuity payment deposited or received at the end of each year PP = the constant dollar amount provided by the perpetuity	

