

IT111 - Introduction to Information Technology Bachelor of Science in Information Technology

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Lecture 2 - Numbering Systems

Upon successful completion of this course, students should be able to:

- ☐ Understand the concept of number systems.
- ☐ Describe the decimal, binary, hexadecimal and octal system.
- ☐ Convert a number in binary, octal or hexadecimal to a number in the decimal system.
- ☐ Convert a number in the decimal, octal and hexadecimal system to a number in binary
- ☐ Find the number of digits needed in each system to represent a particular value.

1-The decimal system (base 10)

- The word decimal is derived from the Latin root **decimal**. In this system the base $b = 10$ and we use ten symbols.

$$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

- The symbols in this system are often referred to as **decimal digits** or just **digits**.

Integer Number

$$N = \pm S_{k-1} \times 10^{k-1} + S_{k-2} \times 10^{k-2} + \dots + S_2 \times 10^2 + S_1 \times 10^1 + S_0 \times 10^0$$

	10^{k-1}	10^{k-2}	$\dots$	10^2	10^1	10^0	Place values
$\pm$	S_{k-1}	S_{k-2}	$\dots$	S_2	S_1	S_0	Number
	↓	↓		↓	↓	↓	
$N = \pm$	$S_{k-1} \times 10^{k-1}$	$+ S_{k-2} \times 10^{k-2}$	$+ \dots +$	$S_2 \times 10^2$	$+ S_1 \times 10^1$	$+ S_0 \times 10^0$	Values

Integer Number Example

The following shows the place values for the integer +224 in the decimal system.

		10^2		10^1		10^0	Place values
		2		2		4	Number
N	$=$	$+$	2×10^2	$+$	2×10^1	$+$	4×10^0
							Values

Reals Number

$$R = \pm \underbrace{S_{k-1} \times 10^{k-1} + \dots + S_1 \times 10^1 + S_0 \times 10^0}_{\text{Integral part}} + \underbrace{S_{-1} \times 10^{-1} + \dots + S_{-l} \times 10^{-l}}_{\text{Fractional part}}$$

The following shows the place values for the real number +24.13.

	10^1	10^0		10^{-1}	10^{-2}	Place values
	2	4	•	1	3	Number
$R = +$	2×10	$+ 4 \times 1$	$+$	1×0.1	$+ 3 \times 0.01$	Values

2-The binary system (base 2)

In this system the base $b = 2$ and we use only two symbols,

$$S = \{0, 1\}$$

The symbols in this system are often referred to as binary digits or bits (binary digit).

binary system (Example)

The following shows that the number $(11001)_2$ in binary is the same as 25 in decimal. The subscript 2 shows that the base is 2.

	2^4	2^3	2^2	2^1	2^0	Place values
	1	1	0	0	1	Number
$N =$	1×2^4	$+ 1 \times 2^3$	$+ 0 \times 2^2$	$+ 0 \times 2^1$	$+ 1 \times 2^0$	Decimal

The equivalent decimal number is $N = 16 + 8 + 0 + 0 + 1 = 25$.

Reals Number (binary system)

The following shows that the number $(101.11)_2$ in binary is equal to the number 5.75 in decimal.

	2^2	2^1	2^0	2^{-1}	2^{-2}	Place values
	1	0	1	1	1	Number
R =	1×2^2	$+ 0 \times 2^1$	$+ 1 \times 2^0$	$+ 1 \times 2^{-1}$	$+ 1 \times 2^{-2}$	Values

3-The Hexadecimal system (base 16)

In this system the **base $b = 16$** and we use sixteen symbols to represent a number. The set of symbols is

$$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$$

Note that the symbols **A, B, C, D, E, F are equivalent to 10, 11, 12, 13, 14, and 15** respectively. The symbols in this system are often referred to as **hexadecimal digits**.

Hexadecimal system(Example)

The following shows that the number $(2AE)_{16}$ in hexadecimal is equivalent to 686 in decimal.

	16^2	16^1	16^0	Place values
	2	A	E	Number
N =	2×16^2	+ 10×16^1	+ 14×16^0	Values

The equivalent decimal number is $N = 512 + 160 + 14 = 686$.

4-The Octal system (base 8)

In this system the **base $b = 8$** and we use eight symbols to represent a number. The set of symbols is

$$S = \{0, 1, 2, 3, 4, 5, 6, 7\}$$

Octal system(Example)

The following shows that the number $(1256)_8$ in octal is the same as 686 in decimal.

	8^3	8^2	8^1	8^0	Place values			
	1	2	5	6	Number			
$N =$	1×8^3	$+$	2×8^2	$+$	5×8^1	$+$	6×8^0	Values

Note that the decimal number is $N = 512 + 128 + 40 + 6 = 686$.

summary of the four positional number systems

summary of the four positional number systems discussed in this chapter.

<i>System</i>	<i>Base</i>	<i>Symbols</i>	<i>Examples</i>
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	2345.56
Binary	2	0, 1	$(1001.11)_2$
Octal	8	0, 1, 2, 3, 4, 5, 6, 7	$(156.23)_8$
Hexadecimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F	$(A2C.A1)_{16}$

Any Base to Decimal Conversion

Binary  Decimal

- The column weights of binary numbers are powers of two that increase from right to left beginning with $2^0 = 1$:

$$\dots 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0.$$

- For fractional binary numbers, the column weights are negative powers of two that decrease from left to right:

$$2^2 \ 2^1 \ 2^0. \ 2^{-1} \ 2^{-2} \ 2^{-3} \ 2^{-4} \dots$$

Any Base to Decimal Conversion (Example)

Binary  Decimal

- The decimal equivalent of a binary number can be determined by multiplying each bit by its corresponding weight and then add the values of all.

Example

Convert the binary number **100101.01** to decimal.

Solution

Start by writing the column weights; then add the weights that correspond to each 1 in the number. **37.25**

2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}
32	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$
*	*	*	*	*	*	*	*
1	0	0	1	0	1	0	1
↓	↓	↓	↓	↓	↓	↓	↓
32	0	0	4	0	1	0	$\frac{1}{4}$

$32 + 0 + 0 + 4 + 0 + 1 + 0 + \frac{1}{4} = 37.25$

Any Base to Decimal Conversion (Example)

Binary  Decimal

Example

Convert the binary number **1101.11** to decimal.

Solution

Start by writing the column weights; then add the weights that correspond to each 1 in the number.

$$2^3 \ 2^2 \ 2^1 \ 2^0 . 2^{-1} \ 2^{-2}$$

$$8 \ 4 \ 2 \ 1 . \frac{1}{2} \ \frac{1}{4}$$

$$1 \ 1 \ 0 \ 1 . \ 1 \ 1$$

$$8 + 4 + 0 + 1 + \frac{1}{2} + \frac{1}{4} = \mathbf{13.75}$$

Any Base to Decimal Conversion (Example)

Binary  Decimal

• $(111001.011)_2 = (57.375)_{10}$

Binary			1	1	1	0	0	1	0	1	1	
weight	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
Value	128	64	32	16	8	4	2	1	0.5	0.25	0.125	0.0625

$$(1*2^5) + (1*2^4) + (1*2^3) + (1*2^0) + (1*2^{-2}) + (1*2^{-3}) = 51.375$$

• $(10100.0101)_2 = (20.3125)_{10} \longrightarrow (1*2^4) + (1*2^2) + (1*2^{-2}) + (1*2^{-4}) = 20.3125$

Binary				1	0	1	0	0	0	1	0	1
weight	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
Value	128	64	32	16	8	4	2	1	0.5	0.25	0.125	0.0625

Any Base to Decimal Conversion (Example)

Octal  Decimal

Example

Convert the **octal number 174.1** to decimal.

Solution

Start by writing the column weights; then add the weights that correspond to each number.

$$\begin{array}{cccc}
 8^2 & 8^1 & 8^0 & 8^{-1} \\
 64 & 8 & 1 & 1/8 \\
 1 & 7 & 4 & 1 \\
 64+56+4 & +1/8 = & \mathbf{124.125}
 \end{array}$$

Examples

Convert the following Octal to Decimal by using the *sum-of-weighted method*

Octal  Decimal

$8^2 \ 8^1 \ 8^0$

Ex1) 272

$$(2 * 8^2) + (7 * 8^1) + (2 * 8^0) = 186$$

$8^1 \ 8^0$

Ex2) 61

$$(6 * 8^1) + (1 * 8^0) = 49$$

$8^2 \ 8^1 \ 8^0$

Ex3) 126

$$(1 * 8^2) + (2 * 8^1) + (6 * 8^0) = 86$$

Any Base to Decimal Conversion (Example)

Hexadecimal  Decimal

Example

Convert the hexadecimal number A01.3 to decimal.

Solution

Start by writing the column weights; then add the weights that correspond to each number.

$$16^2 \quad 16^1 \quad 16^0 \quad 16^{-1}$$

$$256 \quad 16 \quad 1 \quad 1/16$$

$$10 \quad 0 \quad 1 \quad 1$$

$$2560 + 0 + 1 + 1/16 = 2561.0625$$

Examples

Convert the following Octal to Decimal by using the *sum-of-weighted method*

Hexadecimal  Decimal

$16^2 \ 16^1 \ 16^0$

Ex1) B02

$$(11 * 16^2) + (0 * 16^1) + (2 * 16^0) = 2818$$

$16^1 \ 16^0$

Ex2) EC

$$(14 * 16^1) + (12 * 16^0) = 236$$

$16^2 \ 16^1 \ 16^0$

Ex3) 296

$$(2 * 16^2) + (9 * 16^1) + (6 * 16^0) = 662$$

Decimal to Binary Conversion

Decimal to Binary Conversion

sum-of-weighted

1. Choose largest weight $\leq$ the given number to convert
2. Subtract the given number from the weight
3. Repeat step 1 & 2 till the subtraction results $=0$

repeated division by 2.

1. Divide the whole number by 2
2. Check if there is a remainder put 1
3. Repeat the division processes for the whole number till the division results reaches 0.5

Examples

Convert the *following decimal numbers to binary* by using the *sum-of-weighted method*

	Decimal 								Binary
weight	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
Value	128	64	32	16	8	4	2	1	
Output				1	0	0	0	1	

Choose the largest weight $\leq 17 \rightarrow 16$

Subtract $17 - 16 = 1$

Put 1 under the weight that has been chosen (16)

Choose the largest weight $\leq 1 \rightarrow 1$

Put 1 under the weight that has been chosen (1)

Subtract $1 - 1 = 0$

Put 0s under the other weights in between till weight (2^0)

Done

Output = 10001

Choose the largest weight $\leq 186 \rightarrow 128$

Subtract $186 - 128 = 58$

Put 1 under the weight that has been chosen (128)

Choose the largest weight $\leq 58 \rightarrow 32$

Put 1 under the weight that has been chosen (32)

Subtract $58 - 32 = 26$

Choose the largest weight $\leq 26 \rightarrow 16$

Put 1 under the weight that has been chosen (16)

Subtract $26 - 16 = 10$

Choose the largest weight $\leq 10 \rightarrow 8$

Put 1 under the weight that has been chosen (8)

Subtract $10 - 8 = 2$

Choose the largest weight $\leq 2 \rightarrow 2$

Put 1 under the weight that has been chosen (2)

Subtract $2 - 2 = 0$

Put 0s under the other weights in between till weight (2^0)

Done

Output = 10111010

Examples

Convert the following *decimal numbers to binary* by using the *sum-of-weighted method*

➤ Conversion from decimal to binary :

convert from binary to decimal: Sum of weights method

1)Sum of weights method :

Examples :

$$(234)_{10} = (11101010)_2$$

128	64	32	16	8	4	2	1
1	1	1	0	1	0	1	0

→

$$(148)_{10} = (10010100)_2$$

128	64	32	16	8	4	2	1
1	0	0	1	0	1	0	0

→

Value	Weight
1	2^0
2	2^1
4	2^2
8	2^3
16	2^4
32	2^5
64	2^6
128	2^7

Examples

Convert the following *decimal numbers to binary* by using the *sum-of-weighted method*

$$(467)_{10} = (111010011)_2$$

256	128	64	32	16	8	4	2	1
1	1	1	0	1	0	0	1	1

→

$$(23)_{10} = (10111)_2$$

128	64	32	16	8	4	2	1
0	0	0	1	0	1	1	1

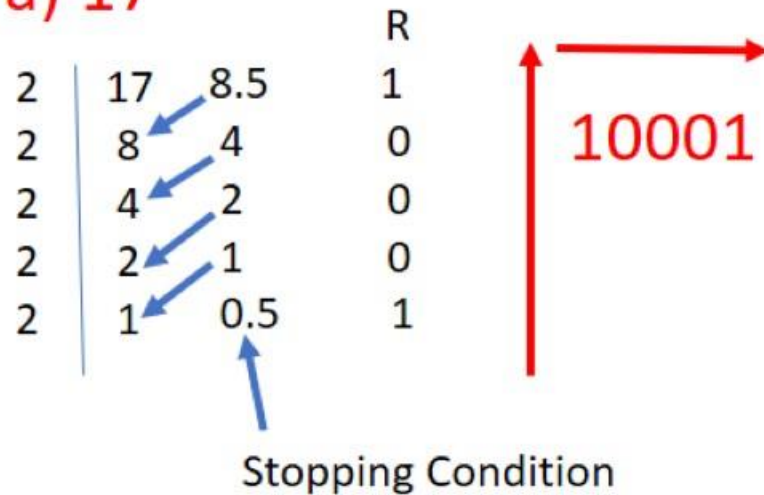
→

Examples

Convert the following *decimal numbers to binary* by using the *division by 2*

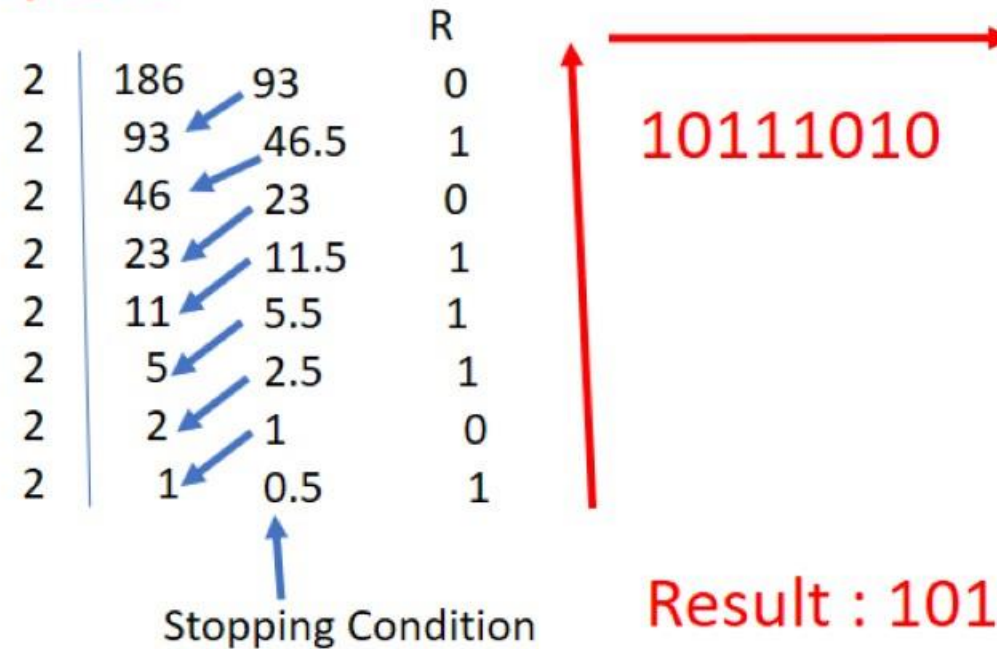
➤ Conversion from decimal to binary using the *division by 2*:

a) 17



Result : 10001

c) 186



Result : 10111010

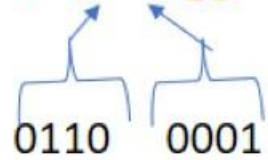
Examples

Convert the following *Hexadecimal numbers to binary*

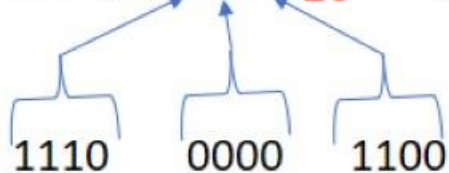
Hexadecimal  Binary

- Note: Every digit in Hexa-decimal has to be represented by **4 bits** in **Binary** as the maximum digit in Hexa-decimal (F=15) is represented by (1111) in binary

Ex1) $(61)_{16} = ()_2$



Ex2) $(E0C)_{16} = ()_2$



Hexa-decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Convert the following *Hexadecimal numbers to binary*

Hexadecimal



Binary

What is the binary equivalent of $(24C)_{16}$?

Solution

Each hexadecimal digit is converted to 4-bit patterns:

$2 \rightarrow 0010$, $4 \rightarrow 0100$, and $C \rightarrow 1100$

The result is $(001001001100)_2$.

Examples

Convert the following *Octal numbers to binary*

Octal  Binary

- Octal number can easily be converted to binary by expanding each digit into **3 bits at a time**

Example

Express 107_8 in binary:

Solution

001 000 111₂

Examples

Convert the following *Octal numbers to binary*

Octal  Binary

- Octal number can easily be converted to binary by expanding each digit into **3 bits at a time**

What is the binary equivalent of for $(24)_8$?

Solution

Write each octal digit as its equivalent bit pattern to get

$2 \rightarrow 010$ and $4 \rightarrow 100$

The result is $(010100)_2$.

Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000		8
9	1001		9
10	1010		A
11	1011		B
12	1100		C
13	1101		D
14	1110		E
15	1111		F

Numbering System	Base	Digits Range
Binary	2	0,1
Octal	8	0-7
Decimal	10	0-9
HexaDecimal	16	0-9 ,A,B,C,D,E,F



Thank you