

Business Mathematics

Prepared by Dr. Alfaisal Hasan

MTH 112
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Lecture NO 8



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Chapter

Topics

- Chapter 1: Mathematical preliminaries
- Chapter 2: The straight line and applications
- Chapter 3: Functions and Graphs
- Chapter 4: Limits and Continuity
- Chapter 5: Financial Mathematics
- Chapter 6: Differentiation
- Chapter 7: Methods of Differentiation
- **Chapter 8: Function of several variables**
- Chapter 9: Integration and applications
- Chapter 10: Linear algebra and applications

Differentiation

Remember that

$f'(x)$ “ f prime of x ”

y' “ y prime”

$\frac{dy}{dx}$ “the derivative of y with respect to x ”

$\frac{d}{dx}$ is a verb. “Take the derivative with respect to x ...”

Differentiation

Remember that

- Differentiation is the process of computing the derivative of a function.

You may be asked to:

- Differentiate.
- Derive.
- Find the derivative of...

Differentiation

$$\frac{d}{dx}[c] = 0, \quad c \text{ is a constant}$$

Remember that

The derivative of a constant function is zero.

Differentiation

Remember that

$$\frac{d}{dx}[x^N] = Nx^{N-1}, \quad N \text{ is any real number}$$

$$\frac{d}{dx}[x] = 1$$

Differentiation

Remember that

$$\frac{d}{dx}[c(f(x))] = c(f'(x)), \quad c \text{ is a constant}$$

- The derivative of a constant times a function is equal to the constant times the derivative of the function.

Differentiation

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

Remember that

The derivative of a sum is the sum of the derivatives.

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

The derivative of a difference is the difference of the derivatives.

Differentiation

Remember that

The derivative of the product of two differentiable functions is given by

$$y = f(x) \cdot g(x)$$

$$\frac{dy}{dx} = y' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\frac{dy}{dx} = y' = f' \cdot g + f \cdot g'$$

Differentiation

Remember that

- The derivative of the quotient of two differentiable functions is given by

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \quad (g(x) \neq 0)$$

Methods of Differentiation

Remember that

- **The Rate of change** of the total Cost Function is the derivative of the total Cost Function

$$C'(x) = \frac{d}{dx} C(x)$$

Methods of Differentiation

Remember that

- **The Profit Function**

is difference between of **Revenue Function** and
Cost Function

$$P(x) = R(x) - C(x)$$

Methods of Differentiation

Remember that

- Now we will discuss the marginal analysis of various functions related to:
- Cost
- Revenue
- Profit

Methods of Differentiation

Remember that

- **The marginal Cost Function** $C'(x)$

is the derivative of the **Cost Function**

$$C'(x) = \frac{d}{dx} C(x)$$

Methods of Differentiation

Remember that

- The marginal Revenue Function $R'(x)$

is the derivative of the **Revenue Function**

$$R'(x) = \frac{d}{dx} R(x)$$

Methods of Differentiation

Remember that

- The marginal Revenue Function $R'(x)$

is the derivative of the **Revenue Function**

$$R'(x) = \frac{d}{dx} R(x)$$

Methods of Differentiation

Remember that

- The marginal Profit Function $P'(x)$

is the derivative of the Profit Function

$$P'(x) = \frac{d}{dx} P(x)$$

Functions of Several Variables

- The Chain Rule

$$h'(x) = \frac{d}{dx} g(f(x)) = g'[f(x)]f'(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Functions of Several Variables

- Consider the function
- To compute $h'(x)$, we can first expand $h(x)$

$$\begin{aligned}h(x) &= (x^2 + x + 1)^2 = (x^2 + x + 1)(x^2 + x + 1) \\ &= x^4 + 2x^3 + 3x^2 + 2x + 1\end{aligned}$$

and then derive the resulting polynomial

$$h'(x) = 4x^3 + 6x^2 + 6x + 2$$

- But how should we derive a function like $H(x)$?

$$H(x) = (x^2 + x + 1)^{100}$$

Functions of Several Variables

Note that $H(x) = (x^2 + x + 1)^{100}$ is a composite function:

- $H(x)$ is composed of two simpler functions

$$f(x) = x^2 + x + 1 \quad \text{and} \quad g(x) = x^{100}$$

- So that

$$H(x) = g[f(x)] = [f(x)]^{100} = (x^2 + x + 1)^{100}$$

- We can use this to find the derivative of $H(x)$.

Functions of Several Variables

- If $h(x) = g[f(x)]$, then

$$h'(x) = \frac{d}{dx} g(f(x)) = g'(f(x))f'(x)$$

- Equivalently, if we write $y = h(x) = g(u)$, where $u = f(x)$, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Functions of Several Variables

- If the function f is differentiable and

$$h(x) = [f(x)]^n \quad (n, \text{ a real number}),$$

then

$$h'(x) = \frac{d}{dx} [f(x)]^n = n [f(x)]^{n-1} f'(x)$$

Functions of Several Variables

- Find the derivative of

$$f(x) = (2x + 3)^5$$

Functions of Several Variables

$$f(x) = (2x + 3)^5$$

$$f'(x) = (5)(2x + 3)^4 \frac{d}{dx}(2x + 3)$$

$$= (5)(2x + 3)^4 (2)$$

$$= 10(2x + 3)^4$$

Functions of Several Variables

- Find the derivative of

$$G(x) = \sqrt{x^2 + 1}$$

Functions of Several Variables

$$G(x) = \sqrt{x^2 + 1}$$

- Rewrite as a power function: $G(x) = (x^2 + 1)^{1/2}$
- Apply the general power rule:

$$\begin{aligned} G'(x) &= \frac{1}{2} (x^2 + 1)^{-1/2} \frac{d}{dx} (x^2 + 1) \\ &= \frac{1}{2} (x^2 + 1)^{-1/2} (2x) \\ &= \frac{x}{\sqrt{x^2 + 1}} \end{aligned}$$

Functions of Several Variables

Find the derivative of $f(x) = \frac{1}{(4x^2 - 7)^2}$

- Rewrite as a power function: $f(x) = (4x^2 - 7)^{-2}$
- Apply the general power rule:

$$f'(x) = -2(4x^2 - 7)^{-3}(8x)$$

$$= -\frac{16x}{(4x^2 - 7)^3}$$

Functions of Several Variables

- Differentiate:

$$f(x) = (5x^2 + 7x - 6)^3$$

Functions of Several Variables

- Differentiate:

$$f(x) = (5x^2 + 7x - 6)^3$$

$$f'(x) = 3(5x^2 + 7x - 6)^2(10x + 7)$$

$$f'(x) = (30x + 21)(5x^2 + 7x - 6)^2$$

Functions of Several Variables

- Differentiate:

$$g(x) = (4x^6 - 3x^5 + 16)^5$$

Functions of Several Variables

$$g(x) = (4x^6 - 3x^5 + 16)^5$$

$$g'(x) = 5(4x^6 - 3x^5 + 16)^4 (24x^5 - 15x^4)$$

Functions of Several Variables

- Given that $f(x) = (x^3 + 12)^4$,

then find the value of $f'(1)$

Functions of Several Variables

$$f(x) = (x^3 + 12)^4$$

$$f'(x) = 4(x^3 + 12)^3 (3x^2)$$

$$f'(1) = 4((1)^3 + 12)^3 (3(1)^2)$$

$$f'(1) = 26364$$

Functions of Several Variables

- Given that $f(x) = (4x^2 + 5x - 2)^2$
then find the value of $f'(2)$

Functions of Several Variables

$$f(x) = (4x^2 + 5x - 2)^2$$

$$f'(x) = 2(4x^2 + 5x - 2)(8x + 5)$$

$$f'(x) = (16x + 10)(4x^2 + 5x - 2)$$

$$f'(2) = (16(2) + 10)(4(2)^2 + 5(2) - 2)$$

$$f'(2) = (32 + 10)(16 + 10 - 2)$$

$$f'(2) = 1008$$

Functions of Several Variables

- Differentiate: $f(x) = 5x^2 + 7x - 6$

$$f'(0)$$

Functions of Several Variables

- Differentiate: $f(x) = 5x^2 + 7x - 6$
, then find the value of $f'(0)$

Take care

$$f'(x) = 10x + 7$$

$$f'(0) = 10(0) + 7$$

$$f'(0) = 7$$

Functions of Several Variables

- Find the second derivative of

$$f(x) = (6x + 3)^5$$

Functions of Several Variables

$$f(x) = (6x + 3)^5$$

$$f'(x) = 5(6x + 3)^4 (6)$$

$$f'(x) = 30(6x + 3)^4$$

$$f''(x) = 120(6x + 3)^3 (6)$$

$$f''(x) = 720(6x + 3)^3$$

Functions of Several Variables

- Find the third derivative of

$$f(x) = (4x + 1)^3$$

Functions of Several Variables

$$f(x) = (4x + 1)^3$$

$$f'(x) = 3(4x + 1)^2(4)$$

$$f'(x) = 12(4x + 1)^2$$

$$f''(x) = 24(4x + 1)(4)$$

$$f''(x) = 96(4x + 1)$$

$$f''(x) = 384x + 96$$

$$f'''(x) = 384$$

Functions of Several Variables

- Find the derivative of

$$f(x) = x^2(2x + 3)^5$$

Functions of Several Variables

$$f(x) = x^2(2x + 3)^5$$

- Apply the product rule and the general power rule:

$$\begin{aligned} f'(x) &= x^2 \frac{d}{dx} (2x + 3)^5 + (2x + 3)^5 \frac{d}{dx} x^2 \\ &= x^2 (5)(2x + 3)^4 (2) + (2x + 3)^5 (2)x \\ &= 10x^2 (2x + 3)^4 + 2x (2x + 3)^5 \\ &= 2x (2x + 3)^4 (5x + 2x + 3) \\ &= 2x (2x + 3)^4 (7x + 3) \end{aligned}$$

Functions of Several Variables

- Find the derivative of

$$f(x) = x^3(4x + 1)^4$$

Functions of Several Variables

$$f(x) = x^3 (4x + 1)^4$$

$$f'(x) = (3x^2)(4x + 1)^4 + x^3(4)(4x + 1)^3(4)$$

$$f'(x) = 3x^2(4x + 1)^4 + 16x^3(4x + 1)^3$$

Functions of Several Variables

- Find the derivative of

$$f(x) = \left(\frac{2x + 1}{3x + 2} \right)^3$$

Functions of Several Variables

$$f(x) = \left(\frac{2x+1}{3x+2} \right)^3$$

- Apply the general power rule and the quotient rule:

$$f'(x) = 3 \left(\frac{2x+1}{3x+2} \right)^2 \frac{d}{dx} \left(\frac{2x+1}{3x+2} \right)$$

$$= 3 \left(\frac{2x+1}{3x+2} \right)^2 \left[\frac{(3x+2)(2) - (2x+1)(3)}{(3x+2)^2} \right]$$

$$= 3 \left(\frac{2x+1}{3x+2} \right)^2 \left[\frac{6x+4-6x-3}{(3x+2)^2} \right] = \frac{3(2x+1)^2}{(3x+2)^4}$$

Functions of Several Variables

- Find the derivative of

$$f(x) = \left(\frac{5x + 10}{6x + 1} \right)^4$$

$$f(x) = \left(\frac{5x + 10}{6x + 1} \right)^4$$

$$f'(x) = 4 \left(\frac{5x + 10}{6x + 1} \right)^3 \left(\frac{(5)(6x + 1) - (5x + 10)(6)}{(6x + 1)^2} \right)$$

$$f'(x) = 4 \left(\frac{5x + 10}{6x + 1} \right)^3 \left(\frac{(30x + 5) - (30x + 60)}{(6x + 1)^2} \right)$$

$$f'(x) = 4 \left(\frac{5x + 10}{6x + 1} \right)^3 \left(\frac{30x + 5 - 30x - 60}{(6x + 1)^2} \right)$$

$$f'(x) = 4 \left(\frac{5x + 10}{6x + 1} \right)^3 \left(\frac{-55}{(6x + 1)^2} \right)$$

Functions of Several Variables

- Given that

$$f(x) = (x^3 + 12)^4$$

then find the second value of $f''(x)$

Functions of Several Variables

$$f(x) = (x^3 + 12)^4$$

$$f'(x) = 4(x^3 + 12)^3 (3x^2)$$

$$f'(x) = (12x^2)(x^3 + 12)^3$$

$$f''(x) = (24x)(x^3 + 12)^3 + (12x^2)(3)(x^3 + 12)^2 (3x^2)$$

$$f''(x) = (24x)(x^3 + 12)^3 + (108x^4)(x^3 + 12)^2$$

Functions of Several Variables

- Given that

$$f(x) = (2x^4 + 5)^3$$

then find the second value of $f''(x)$

Functions of Several Variables

$$f(x) = (2x^4 + 5)^3$$

$$f'(x) = 3(2x^4 + 5)^2 (8x^3)$$

$$f'(x) = (24x^3)(2x^4 + 5)^2$$

$$f''(x) = (72x^2)(2x^4 + 5)^2 + (24x^3)(2)(2x^4 + 5)(8x^3)$$

$$f''(x) = 72x^2(2x^4 + 5)^2 + 384x^6(2x^4 + 5)$$

Functions of Several Variables

- Given that

$$f(x) = \sqrt{4x^2 + 5x - 2}$$

then find the first derivative of $f'(x)$

Functions of Several Variables

$$f(x) = (4x^2 + 5x - 2)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(4x^2 + 5x - 2)^{-\frac{1}{2}}(8x + 5)$$

$$f'(x) = \left(4x + \frac{5}{2}\right)(4x^2 + 5x - 2)^{-\frac{1}{2}}$$

Functions of Several Variables

Remember that

- The Chain Rule

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- But how should we derive a function like $H(x)$?

$$H(x) = (x^2 + x + 1)^{100}$$

Remember that

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- $H(x)$ is composed of two simpler functions

$$f(x) = x^2 + x + 1 \quad \text{and} \quad g(x) = x^{100}$$

- So that

$$H(x) = g[f(x)] = [f(x)]^{100} = (x^2 + x + 1)^{100}$$

- We can use this to find the derivative of $H(x)$.

Un Solved Examples

- Find the derivative of

$$f(x) = (4x + 3)^7$$

Un Solved Examples

- Find the derivative of

$$G(x) = \sqrt{x^3 + 9}$$

Un Solved Examples

- Find the derivative of

$$f(x) = \frac{1}{(4x^2 - 7)^2}$$

Un Solved Examples

- Find the derivative of

$$f(x) = (2x^4 + 6x^2 - 6)^9$$

Un Solved Examples

Given the function

$$f(x) = (2x^2 - 6)^3$$

Find the value of

$$f'(1)$$

Un Solved Examples

- Find the derivative of

$$f(x) = x^3(4x + 3)^7$$

Un Solved Examples

- Find the derivative of

$$f(x) = \left(\frac{4x + 1}{7x + 2} \right)^4$$



Thank You