

Probability Concepts and Distributions

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Subject code : FNT 631
Data Analysis

Lecture 4



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Chapter 3 Review Questions

Question 1: Explain the concepts of mean, median, and mode, and provide an example of how each is used in financial data analysis.

Answer:

- **Mean:** The mean, or average, is the sum of all values divided by the number of values.
 - **Example:** Calculating the average return on an investment portfolio to evaluate performance.
- **Median:** The median is the middle value of a dataset when arranged in ascending order.
 - **Example:** Analyzing median household income to understand income distribution without being skewed by extremely high incomes.
- **Mode:** The mode is the value that appears most frequently in a dataset.
 - **Example:** Identifying the most common transaction type in a bank to enhance related services.

Chapter 3 Review Questions

Question 2: What are the measures of dispersion, and why are they important in statistical analysis? Provide an example in finance.

Answer:

- **Measures of Dispersion:** Include range, variance, and standard deviation.
- **Range:** Difference between the maximum and minimum values.
- **Variance:** Average of the squared differences from the mean.
- **Standard Deviation:** Square root of the variance, showing how much data deviates from the mean.
- **Importance:** They help understand the spread and variability within a dataset.
- **Example in Finance:** Calculating the standard deviation of stock returns to assess investment risk.

Chapter 3 Review Questions

Question 3: Describe skewness and kurtosis and their significance in data analysis. Provide an example of how each is applied in financial analysis.

Answer:

- **Skewness: Measures the asymmetry of the data distribution.**
 - **Positive Skewness: Tail on the right side; indicates more high-value outliers.**
 - **Negative Skewness: Tail on the left side; indicates more low-value outliers.**
 - **Example: Assessing the skewness of return distributions to understand potential risk in investment portfolios.**
- **Kurtosis: Measures the "tailedness" of the data distribution.**
 - **High Kurtosis: Indicates heavy tails, meaning more outliers.**
 - **Low Kurtosis: Indicates light tails, meaning fewer outliers.**
 - **Example: Evaluating the kurtosis of insurance claims to predict the likelihood of extreme events and set appropriate premiums.**

Chapter
05

Subject's Objectives

1. Understand the Basics of Probability
2. Identify Experiments and Outcomes
3. Explore Random Variables and Probability Distributions
4. Apply Conditional Probability and Probability Laws
5. Analyze Continuous Probability Distributions

Probability

Probability – the likelihood that an outcome occurs

Probabilities are values between 0 and 1. The closer the probability is to 1, the more likely it is that the outcome will occur.

Some convert probabilities to percentages

The statement “there is a 10% chance that oil prices will rise next quarter” is another way of stating that “the probability of a rise in oil prices is 0.1.”

Experiments and Outcomes

Experiment – a process that results in some outcome

- Roll dice

- Observe and record weather conditions

- Conduct market survey

- Watch the stock market

Outcome – an observed result of an experiment

- Sum of the dice

- Description of the weather

- Proportion of respondents who favor a product

- Change in the Dow Jones Industrial Average

Experiments and Outcomes

Sample space - all possible outcomes of an experiment

Dice rolls: 2, 3, ..., 12

Weather outcomes: clear, partly cloudy, cloudy

Customer reaction: proportion who favor a product (a number between 0 and 1)

Change in DJIA: positive or negative real number

Three Views of Probability

Classical definition: based on theory

Relative frequency: based on empirical data

Subjective: based on judgment

Classical Definition

Probability = number of favorable outcomes divided by the total number of possible outcomes

Example: There are six ways of rolling a 7 with a pair of dice, and 36 possible rolls. Therefore, the probability of rolling a 7 is $6/36 = 0.167$.

Relative Frequency Definition

Probability = number of times an event has occurred in the past
divided by the total number of observations

Example: Of the last 10 days when certain weather conditions have
been observed, it has rained the next day 8 times. The probability
of rain the next day is 0.80

Subjective Definition

- What is the probability that the New York Yankees will win the World Series this year?
- What is the probability your school will win its conference championship this year?
- What is the probability the NASDAQ will go up 2% next week?

Basic Probability Rules and Formulas

1. Probability associated with any outcome must be between 0 and 1

$$0 \leq P(O_i) \leq 1 \text{ for each outcome } O_i$$

2. Sum of probabilities over all possible outcomes must be 1.0

$$P(O_1) + P(O_2) + \dots + P(O_n) = 1$$

Example: Flip a coin three times

Outcomes: HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

Each has probability of $(1/2)^3 = 1/8$

Events

An **event** is a collection of one or more outcomes from

- Obtaining a 7 or 11 on a roll of dice

- Having a clear or partly cloudy day

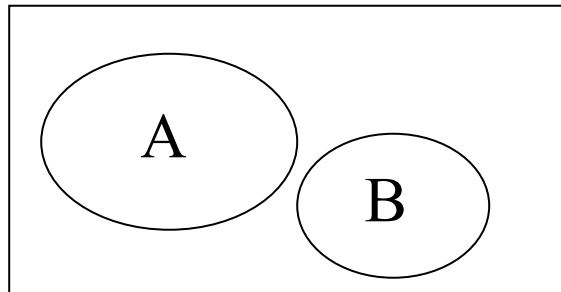
- The proportion of respondents that favor a product is at least 0.60

- Having a positive weekly change in the Dow

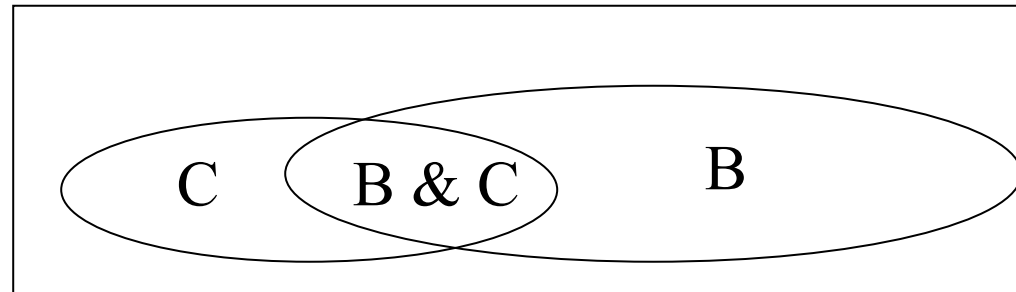
If A is any event, the complement of A , denoted A^c , consists of all outcomes in the sample space not in A .

Mutually Exclusive Events

Two events are **mutually exclusive** if they have no outcomes in common.



A and B are mutually exclusive



B and C are not

Rules

1. The probability of any event is the sum of the probabilities of the outcomes that compose that event.
2. The probability of the complement of any event A is $P(A^c) = 1 - P(A)$.

Rule 3

If events A and B are mutually exclusive, then

$$P(A \text{ or } B) = P(A) + P(B).$$

- Drawing Balls from a Bag:

- Event A: Drawing a red ball
- Event B: Drawing a blue ball
- Bag contains 5 red balls and 3 blue balls

Probabilities:

- $P(A) = \frac{5}{8}$
- $P(B) = \frac{3}{8}$

Calculation:

$$P(A \cup B) = P(A) + P(B) = \frac{5}{8} + \frac{3}{8} = 1$$

Rule 4

If two events A and B are not mutually exclusive, then
 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$

- Drawing a Card:

- Event A: Drawing a heart
- Event B: Drawing a face card

Probabilities:

- $P(A) = \frac{13}{52} = \frac{1}{4}$
- $P(B) = \frac{12}{52} = \frac{3}{13}$
- $P(A \text{ and } B) = \frac{3}{52}$

Calculation:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{1}{4} + \frac{3}{13} - \frac{3}{52}$$

Conclusion:

The probability of drawing either a heart or a face card is:

$$P(A \cup B) = \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52} = \frac{11}{26}$$

Conditional probability

the probability of the occurrence of one event, A, given that another event B is known to be true or have already occurred.

$$P(A|B) = P(A \text{ and } B)/P(B)$$

Example:

- Drawing Cards:
 - Event A: Drawing a King
 - Event B: Drawing a face card

Probabilities:

- $P(B) = \frac{12}{52} = \frac{3}{13}$
- $P(A \text{ and } B) = \frac{4}{52} = \frac{1}{13}$

Calculation:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$P(A|B) = \frac{\frac{1}{13}}{\frac{3}{13}} = \frac{1}{3}$$

Example

Cross-Tabulation	Brand 1	Brand 2	Brand 3	Total
Male	25	17	21	63
Female	9	6	22	37
Total	34	23	43	100

M = respondent is male

F = respondent is female

$B1$ = respondent prefers brand 1

$B2$ = respondent prefers brand 2

$B3$ = respondent prefers brand 3

$$P(B1 | M) = P(B1 \text{ and } M) / P(M) = (25/100) / (63/100) = 25/63 = 0.397.$$

$$P(B1 | F) = P(B1 \text{ and } F) / P(F) = (9/100) / (37/100) = 9/37 = 0.243.$$

$P(\text{Brand} \text{Gender})$	Brand 1	Brand 2	Brand 3
Male	0.397	0.270	0.333
Female	0.243	0.162	0.595

Multiplication Law of Probability

Used to calculate the probability of two events happening together (intersection of two events).

For Independent Events: If events A and B are independent:

$$P(A \cap B) = P(A) \times P(B)$$

For Dependent Events:

If events A and B are dependent:

$$P(A \cap B) = P(A) \times P(B|A)$$

where $P(B|A)$ is the conditional probability of B given A .

Random Variables

Random variable – a numerical description of the outcome of an experiment. Random variables are denoted by capital letters, X , Y , ...; specific values by lower case letters, x , y , ...

Discrete random variable – the number of possible outcomes can be counted

Continuous random variable – outcomes over one or more continuous intervals of real numbers

Examples of Random Variables

Experiment: flip a coin 3 times.

Outcomes: TTT, TTH, THT, THH, HTT, HTH, HHT, HHH

Random variable: X = number of heads.

X can be either 0, 1, 2, or 3.

Experiment: observe end-of-week closing stock price.

Random variable: Y = closing stock price.

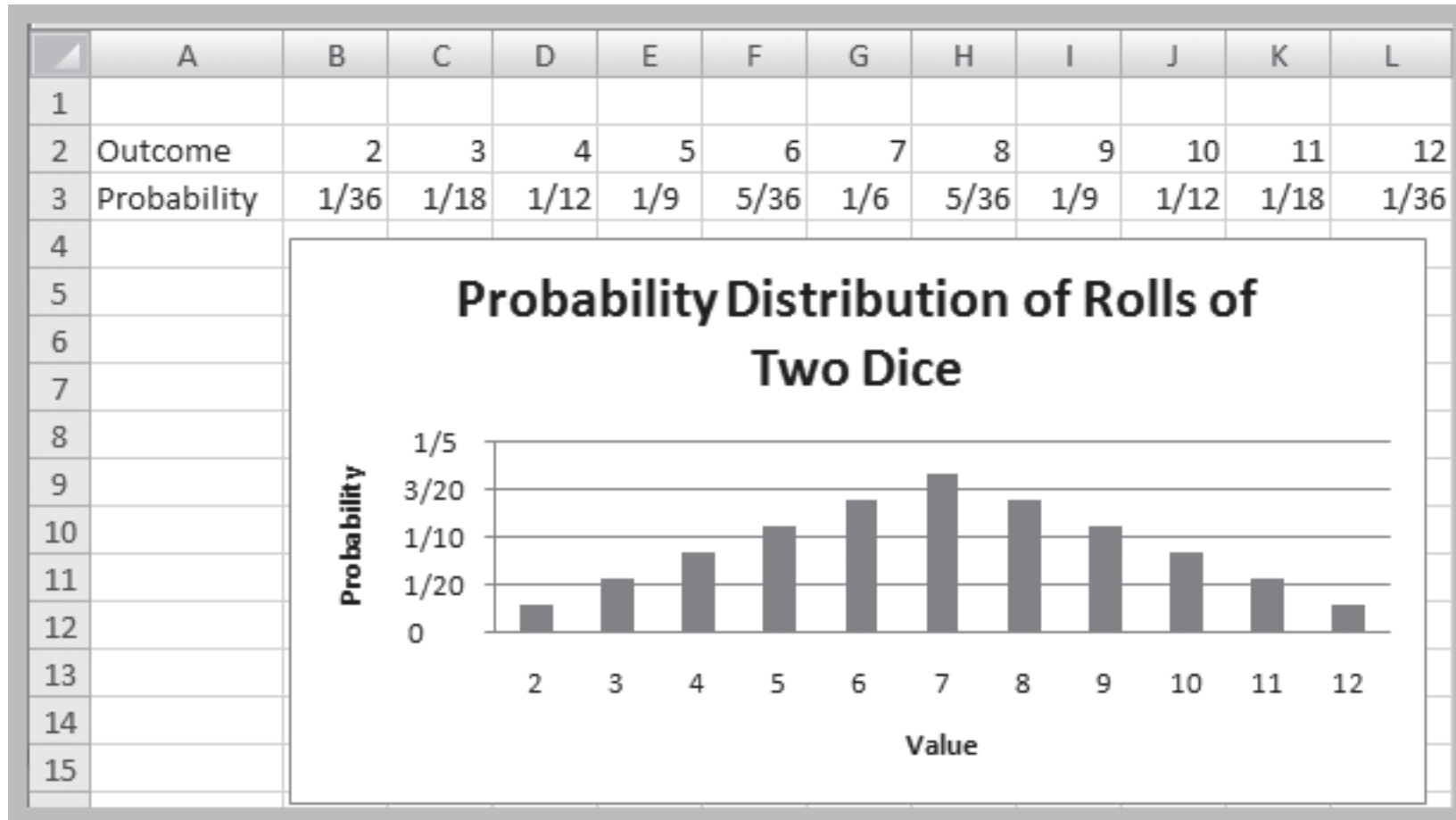
Y can be any nonnegative real number.

Probability Distributions

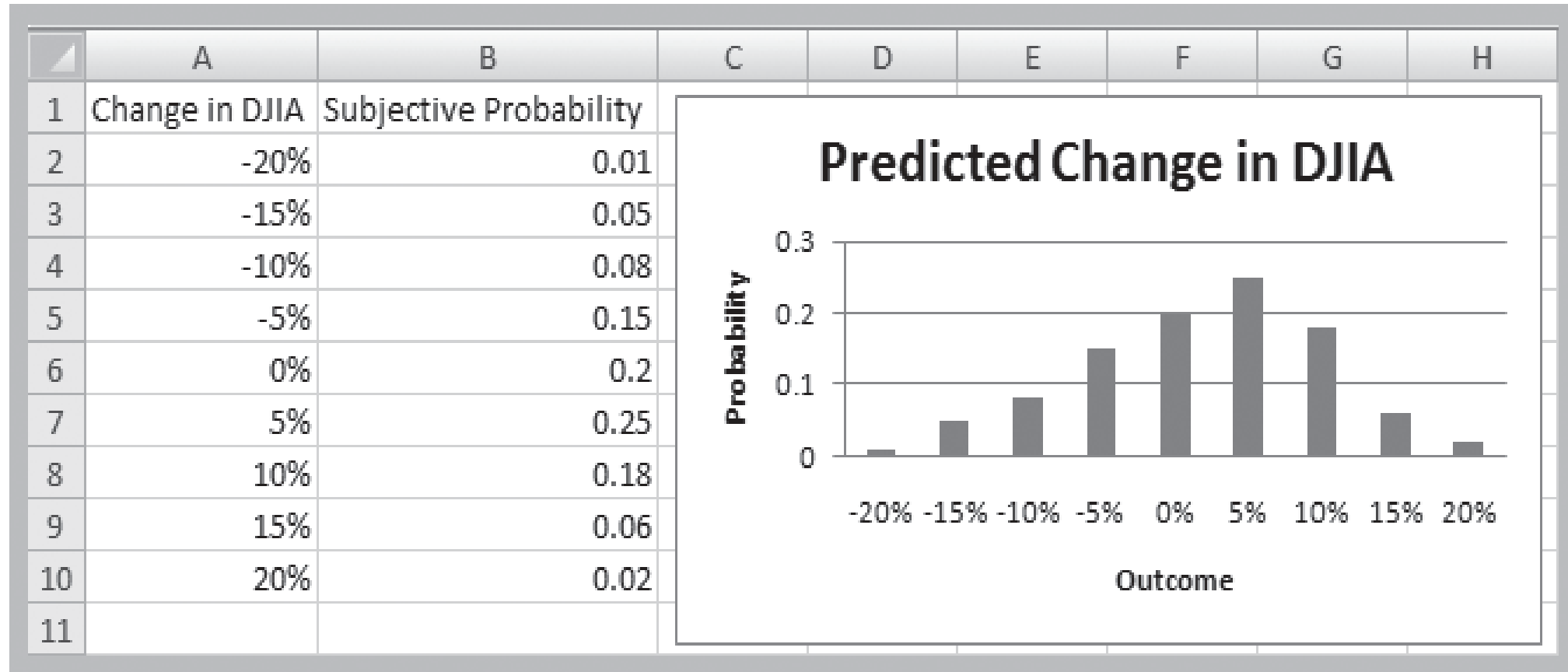
Probability distribution – a characterization of the possible values a random variable may assume along with the probability of assuming these values.

Probability distributions may be defined for both discrete and continuous random variables.

Example: Theoretical Probability Distribution



Example: Subjective Probability Distribution



Probability Mass Function (PMF):

- **Definition:** A function that gives the probability of each possible value of a discrete random variable.
- **Formula:** $P(X = x)$, where X is the discrete random variable and x is a specific value it can take.
- **Example:** If X is the number of heads in two coin flips, PMF gives probabilities for $X = 0, 1, 2$.

Relationship:

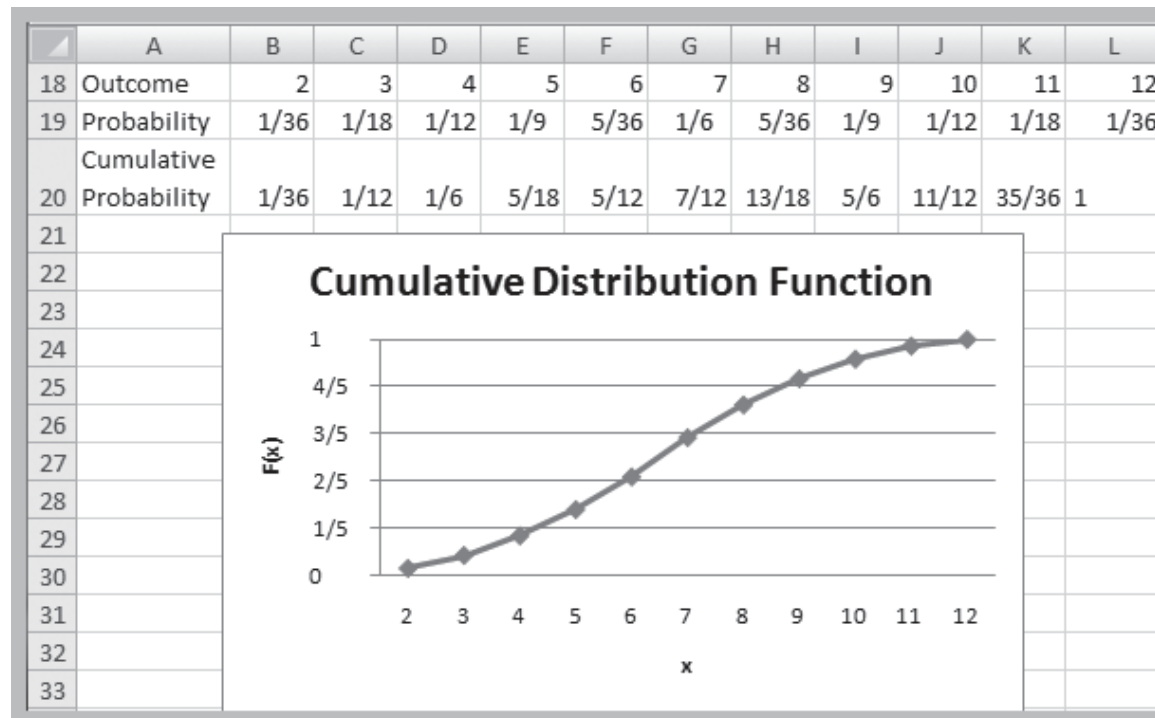
- The PMF provides the probabilities for all possible values of a discrete random variable.
- It ensures the sum of all probabilities equals 1.

Example:

- **Random Variable (X):** Number of heads in 2 coin flips (0, 1, 2).
- **PMF:**
 - $P(X = 0) = 0.25$
 - $P(X = 1) = 0.5$
 - $P(X = 2) = 0.25$
- The PMF gives the likelihood of each outcome of the discrete random variable X .

Cumulative Distribution Function, $F(x)$

Specifies the probability that the random variable X will be *less than or equal to* x , denoted as $P(X \leq x)$.



Expected Value and Variance of Discrete Random Variables

Expected Value (Mean):

- **Meaning:** The average outcome you expect to see if you were to repeat an experiment many times.
- **Example in Finance:** If you invest in a stock that can gain \$10, \$20, or \$30 with different probabilities, the expected value helps you understand the average return you might get.

Variance:

- **Meaning:** It measures how much the outcomes deviate from the expected value, showing the spread or risk associated with the investment.
- **Example in Finance:** In the same stock investment, the variance tells you how much the returns fluctuate from the average return, helping you understand the risk.

Standard Deviation:

- **Meaning:** It is the square root of the variance and provides a measure of risk in the same units as the returns, showing how much the returns typically deviate from the average.
- **Example in Finance:** For the stock investment, the standard deviation gives you a sense of the typical range of returns around the expected value, helping you compare the riskiness of different investments.

Example

- You play a lottery in which you buy a ticket for \$50 and are told you have a 1 in 1000 chance of winning \$25,000. The random variable X is your net winnings, and its probability distribution is

x	$f(x)$
-\$50	0.999
\$24,950	0.001

$$50- = {}_1x$$

$$0.999 = f(x_1)$$

$$950,24 = {}_2x$$

$$0.001 = f(x_2)$$

$$0.001 \cdot 950,24 + 0.999 \cdot (50-) = E[X]$$

$$25- = 24.95 + 49.95- = E[X]$$

Calculations

$$E[X] = -\$50(0.999) + \$24,950(0.001) = -\$25.00$$

$$\text{Var}[X] = (-50 - [-25.00])^2(0.999) + (24,950 - [-25.00])^2(0.001) =$$

$$624,375$$

Discrete Probability Distributions

1. Bernoulli Distribution:

The Bernoulli distribution describes a single trial experiment that has only two possible outcomes: success or failure.

2. Binomial Distribution:

The Binomial distribution describes the number of successes in a fixed number of independent and identical trials, where each trial has only two possible outcomes (success or failure).

3. Poisson Distribution:

The Poisson distribution describes the number of events occurring within a fixed interval of time or space, provided that these events occur with a known constant mean rate and independently of the time since the last event.

Bernoulli Distribution

A random variable with two possible outcomes: “success” ($x = 1$) and “failure” ($x = 0$)

p = probability of “success”

Expected value = p ; variance = $p(1 - p)$

Formula:

If X is a Bernoulli random variable:

- $P(X = 1) = p$: probability of success
- $P(X = 0) = 1 - p$: probability of failure

Example:

- Experiment: Flipping a coin.
 - Success (Heads) $X = 1$: $P(X = 1) = 0.5$
 - Failure (Tails) $X = 0$: $P(X = 0) = 0.5$

Binomial Distribution

The Binomial distribution describes the number of successes in a fixed number of independent and identical trials, where each trial has only two possible outcomes (success or failure).

Formula:

If X is the number of successes in n trials:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

- n : number of trials
- k : number of successes
- p : probability of success on each trial
- $\binom{n}{k}$: binomial coefficient (number of ways to choose k successes out of n trials)

Example:

- Experiment: Flipping a coin 3 times.
 - Number of trials $n = 3$
 - Probability of success on each trial (Heads) $p = 0.5$
 - Calculate the probability of getting Heads exactly 2 times $k = 2$

$$P(X = 2) = \binom{3}{2} (0.5)^2 (1 - 0.5)^{3-2} = 3 \times 0.25 \times 0.5 = 0.375$$

Excel Function

BINOM.DIST(*number_s*, *trials*, *probability_s*, *cumulative*)

	A	B	C	D	E	F
1	Binomial Probabilities					
2			=BINOMDIST(A7,\$B\$3,\$B\$4,FALSE)			
3	n	10				
4	p	0.2	=BINOMDIST(A7,\$B\$3,\$B\$4,TRUE)			
5						
6	x	f(x)	F(x)			
7	0	0.107374	0.107374			
8	1	0.268435	0.375810			
9	2	0.301990	0.677800			
10	3	0.201327	0.879126			
11	4	0.088080	0.967207			
12	5	0.026424	0.993631			
13	6	0.005505	0.999136			
14	7	0.000786	0.999922			
15	8	0.000074	0.999996			
16	9	0.000004	1.000000			
17	10	0.000000	1.000000			

Poisson Distribution

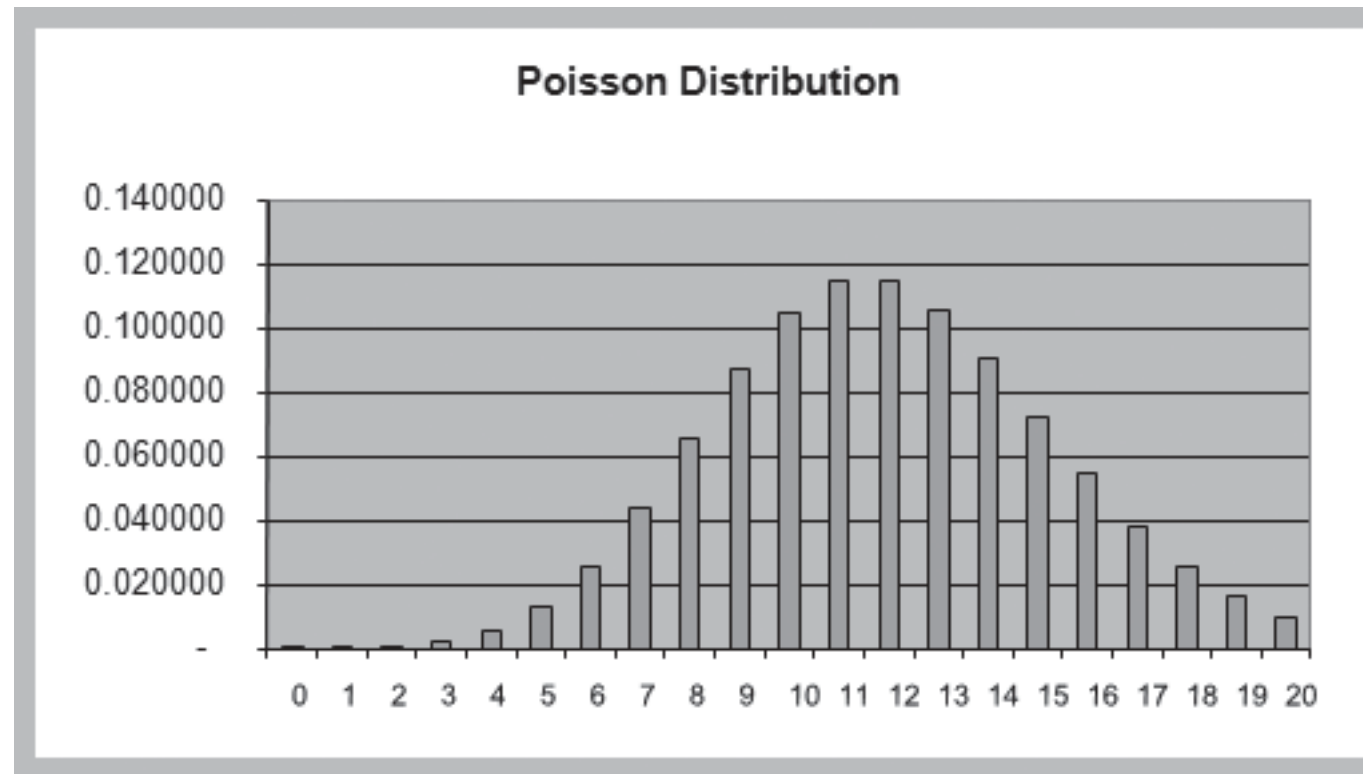
Models the number of occurrences in some unit of measure, e.g., events per unit time, number of items per order

X = number of events that occur; $x = 0, 1, 2,$

Expected value = λ ; variance = λ

Poisson approximates binomial when n is large and p small

Example of the Poisson Distribution ($\lambda = 12$)



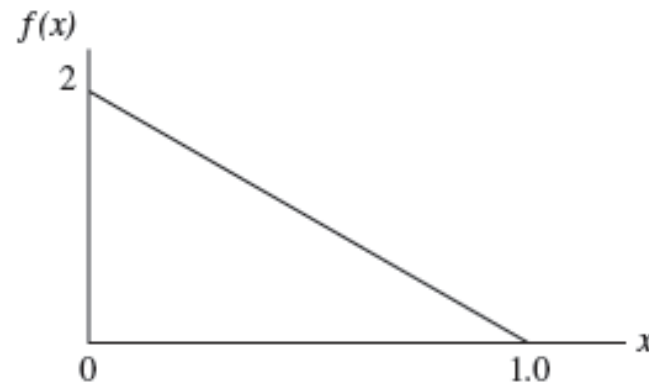
Excel Function

POISSON.DIST(x, mean, cumulative)

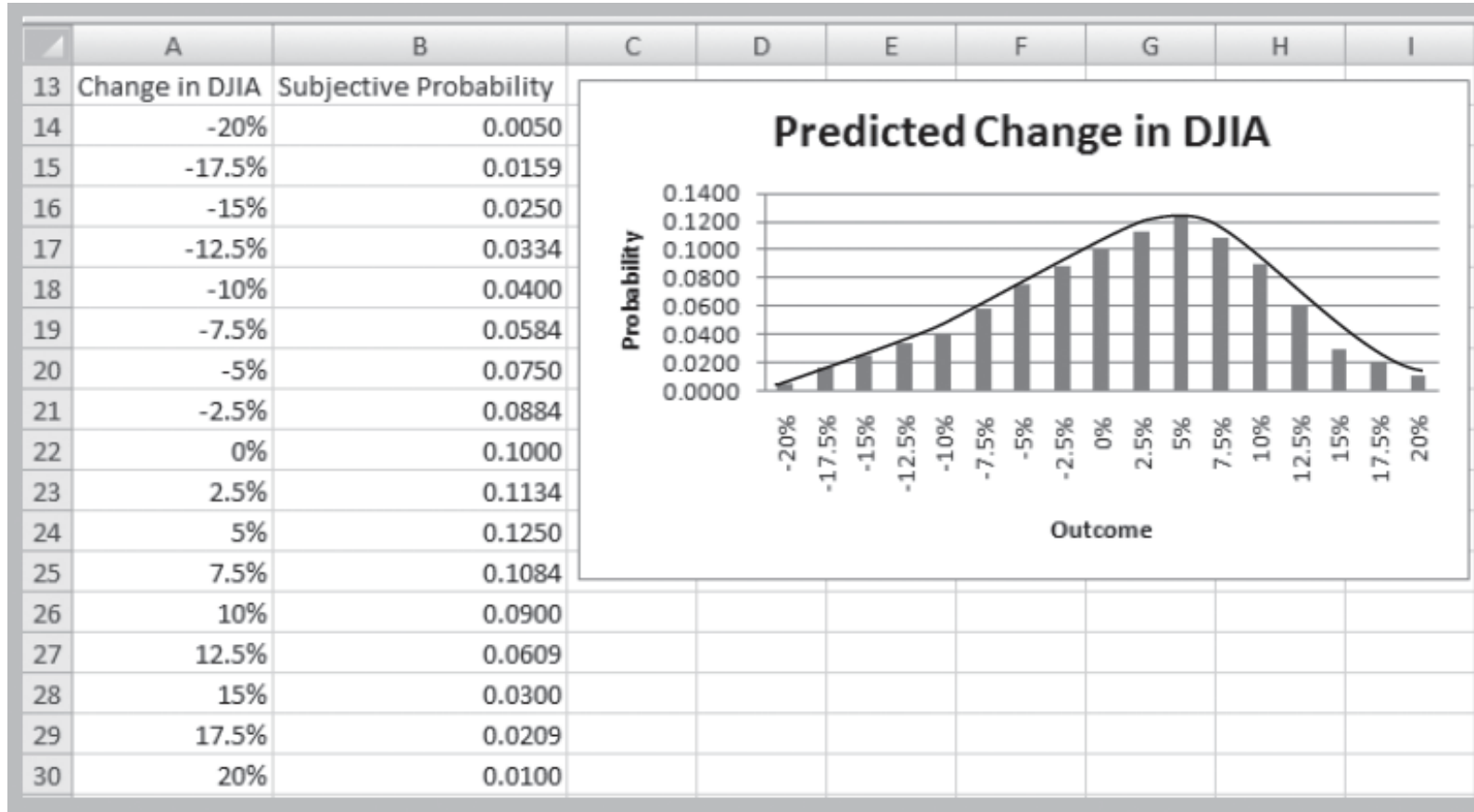
	A	B	C	D	E	F
1	Poisson Probabilities					
2			=POISSON(A7,\$B\$3,FALSE)			
3	Mean	12				
4			=POISSON(A7,\$B\$3,TRUE)			
5						
6	x	f(x)	F(x)			
7	0	0.000006	0.000006			
8	1	0.000074	0.000080			
9	2	0.000442	0.000522			
10	3	0.001770	0.002292			
11	4	0.005309	0.007600			
12	5	0.012741	0.020341			
13	6	0.025481	0.045822			
14	7	0.043682	0.089504			
15	8	0.065523	0.155028			
16	9	0.087364	0.242392			
17	10	0.104837	0.347229			
18	11	0.114368	0.461597			
19	12	0.114368	0.575965			
20	13	0.105570	0.681536			
21	14	0.090489	0.772025			
22	15	0.072391	0.844416			
23	16	0.054293	0.898709			
24	17	0.038325	0.937034			
25	18	0.025550	0.962584			
26	19	0.016137	0.978720			
27	20	0.009682	0.988402			

Continuous Probability Distributions

Probability density function, $f(x)$, a continuous function that describes the probability of outcomes for a continuous random variable X . A histogram of sample data approximates the shape of the underlying density function.



Refining Subjective Probabilities Toward a Continuous Distribution



Conclusion:

- Probability Basics: Understanding the likelihood of events occurring, ranging from 0 to 1.
- Probability Distributions: Describes the possible values of a random variable and their probabilities.
- Discrete vs Continuous: Discrete variables have countable outcomes, while continuous variables have outcomes over intervals.
- Normal Distribution: A key distribution in finance, characterized by the bell curve.
 - Applications in Finance:
 - - Risk Analysis: Estimating investment risks.
 - - Market Predictions: Forecasting stock market movements.
- Expected Value and Variance: Measures of the central tendency and spread of a probability distribution.
- Conditional Probability: The probability of an event given that another event has occurred.

Review Questions:

- Q1: What is the difference between discrete and continuous random variables?**
- Q2: How is the expected value of a random variable calculated?**
- Q3: What is the significance of the normal distribution in finance?**



Thank You