

Mathematics

Lecture 1

MAT 112
Functions and Graphs

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- Lecture 1: Functions and Graphs
 - Lecture 2: Exponential Functions
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- **Functions**
 - **Special Functions**
 - **Combinations of functions**
 - **Linear Equation**
 - **Formatting the linear equation to graph it**
 - **Graphs in rectangular coordinates**
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1. Functions

- A **function** assigns each input number to one output number.
 - The set of all input numbers is the **domain** of the function.
 - The set of all output numbers is the **range**.
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Equality of Functions

- Two functions f and g are equal ($f = g$):
 1. Domain of $f =$ domain of g ;
 2. $f(x) = g(x)$.
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Example 1 – Determining Equality of Functions

Determine which of the following functions are equal.

a. $f(x) = \frac{(x+2)(x-1)}{(x-1)}$

b. $g(x) = x + 2$

c. $h(x) = \begin{cases} x + 2 & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$

d. $k(x) = \begin{cases} x + 2 & \text{if } x \neq 1 \\ 3 & \text{if } x = 1 \end{cases}$

Solution:

When $x = 1$,

$$f(1) \neq g(1),$$

$$f(1) \neq h(1),$$

$$f(1) \neq k(1)$$

By definition, $g(x) = h(x) = k(x)$ for all $x \neq 1$.

Since $g(1) = 3$, $h(1) = 0$ and $k(1) = 3$, we conclude that

$$g = k,$$

$$g \neq h,$$

$$h \neq k$$

Finding Domain and Function Values

Let $g(x) = 3x^2 - x + 5$. Any real number can be used for x , so the domain of g is all real numbers.

a. Find $g(z)$.

b. Find $g(r^2)$.

c. Find $g(x + h)$.

Finding Domain and Function Values

Let $g(x) = 3x^2 - x + 5$. Any real number can be used for x , so the domain of g is all real numbers.

a. Find $g(z)$.

Solution: $g(z) = 3z^2 - z + 5$

b. Find $g(r^2)$.

Solution: $g(r^2) = 3(r^2)^2 - r^2 + 5 = 3r^4 - r^2 + 5$

c. Find $g(x + h)$.

Solution: $g(x + h) = 3(x + h)^2 - (x + h) + 5$
 $= 3x^2 + 6hx + 3h^2 - x - h + 5$

- We begin with *constant function*.

Example 1 – Constant Function

Let $h(x) = 2$. The domain of h is all real numbers.

$$h(10) = 2 \quad h(-387) = 2 \quad h(x + 3) = 2$$

A function of the form $h(x) = c$, where $c = \text{constant}$, is a **constant function**.

Example – Rational Functions

a. $f(x) = \frac{x^2 - 6x}{x + 5}$ is a rational function, since the numerator and denominator are both polynomials.

b. $g(x) = 2x + 3$ is a rational function, since $2x + 3 = \frac{2x + 3}{1}$

Example – Absolute-Value Function

Absolute-value function is defined as $|x|$, e.g.

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

3. Combinations of functions

- We define the operations of function as:

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x) \cdot g(x)$$

$$\frac{f}{g}(x) = \frac{f(x)}{g(x)} \text{ for } g(x) \neq 0$$

Example 1 – Combining Functions

If $f(x) = 3x - 1$ and $g(x) = x^2 + 3x$, find

- a. $(f + g)(x)$
 - b. $(f - g)(x)$
 - c. $(fg)(x)$
 - d. $\frac{f}{g}(x)$
 - e. $(\frac{1}{2}f)(x)$
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Solution:

a. $(f + g)(x) = f(x) + g(x) = (3x - 1) + (x^2 + 3x) = x^2 + 6x - 1$

b. $(f - g)(x) = f(x) - g(x) = (3x - 1) - (x^2 + 3x) = -1 - x^2$

c. $(fg)(x) = f(x)g(x) = (3x - 1)(x^2 + 3x) = 3x^3 + 8x^2 - 3x$

d. $\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{3x - 1}{x^2 + 3x}$

e. $(\frac{1}{2}f)(x) = \frac{1}{2}(f(x)) = \frac{1}{2}(3x - 1) = \frac{3x - 1}{2}$

Definition:

A linear equation in the variable (X) is an equation that is equivalent to one that be written in the form.

$$ax + b = 0$$

Where a and b are constant and $a \neq 0$

→linear equation is also called a first-degree equation, since the highest power of the variable that occurs in Equation (1) is the first.

The general form of a linear equation is:

$$Ax + By = C$$

Example: find A, B & C

$$(1) x + 4y = 6 \quad \Rightarrow A = 1 \quad B = 4 \\ C = 6$$

$$(2) y = -\frac{1}{2}x - \frac{3}{4} \Rightarrow \frac{1}{2}x + y = -\frac{3}{4}$$

$$A = \frac{1}{2}$$

$$B = 1$$

$$C = -\frac{3}{4}$$

How to Graph a linear equation:

To graph a linear equation you need to substitute (X) by zero to get (Y) & substitute (Y) by zero to get (X), to get two points.

(1) $y = x - 4$

Step 1:

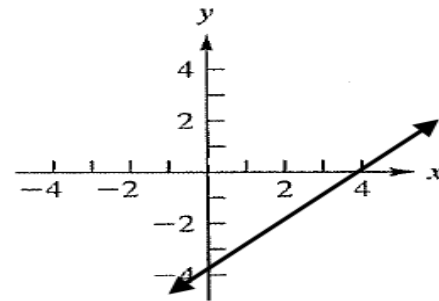
let $x = 0$ so the point is $(0, -4)$

$$Y = 0 - 4 \therefore y = -4$$

$$\text{Let } y = 0 \Rightarrow 0 = x - 4 \therefore x = 4$$

So the point is $(4, 0)$

Step 2:

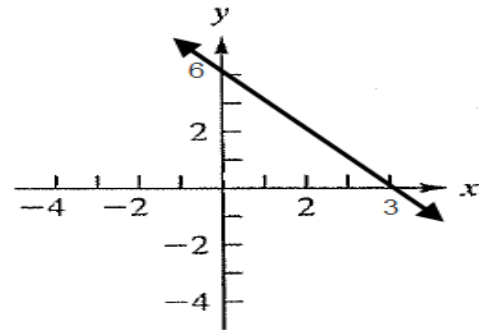


$$(2) y = -2x + 6$$

Step1: let $x = 0$ $(0, 6)$

Let $y = 0$ $(3, 0)$

Step2:

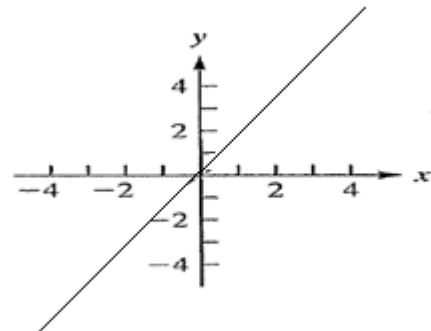


(3) $y = 3x$

Step1: let $x = 0$ $(0, 0)$

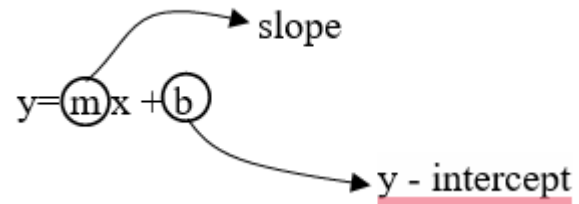
Let $x = 2$ $(2, 6)$

Step2:



How to get the slope and the equation:

Rules:



The diagram shows the equation $y = mx + b$. The variable m is circled, and an arrow points from it to the word "slope". The variable b is also circled, and an arrow points from it to the text "y - intercept".

$$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \text{to get the slope from two points.}$$

$$y - y_1 = m(x - x_1) \Rightarrow \text{to get the equation.}$$

Example:

Find the equation of a line passing through:

a) $(0, 4)$, $(2, 2)$

b) $(2, 6)$, $(1, 4)$

Solution:

(a) Step1:

find $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{2 - 4}{2 - 0} = \frac{-2}{2} = -1$$

Step2:

Substitute in $y - y_1 = m(x - x_1)$

$$y - 4 = -1(x - 0)$$

$$y - 4 = -x$$

$$\therefore y = -x + 4$$

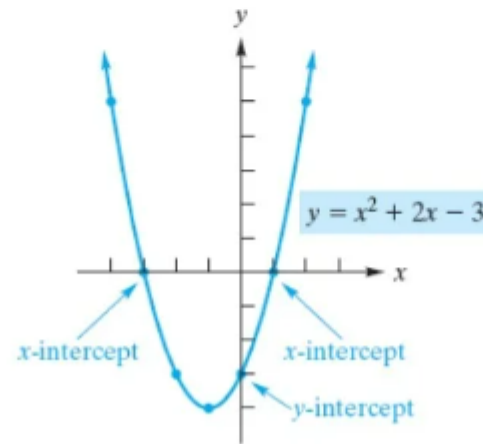
(b) Step1: find $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$M = \frac{4 - 6}{1 - 2} = \frac{-2}{-1} = 2$$

Step2: substitute in the equation:

$$y - y_1 = m (x - x_1)$$
$$y - 6 = 2 (x - 2)$$
$$y - 6 = 2x - 4$$
$$\therefore y = 2x + 2$$

- The **rectangular coordinate system** provides a geometric way to graph equations in two variables.
- An **x-intercept** is a point where the graph intersects the x-axis. **Y-intercept** is vice versa.



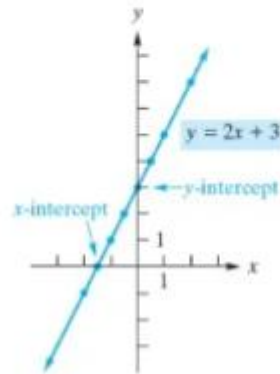
Example 1 – Intercepts and Graph

Find the x- and y-intercepts of the graph of $y = 2x + 3$, and sketch the graph.

Solution:

When $y = 0$, we have $0 = 2x + 3$ so that $x = -\frac{3}{2}$

When $x = 0$, $y = 2(0) + 3 = 3$



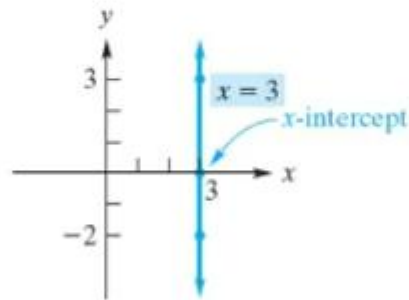
x	0	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	1	-1	2	-2
y	3	0	4	2	5	1	7	-1

Example – Intercepts and Graph

Determine the intercepts of the graph of $x = 3$, and sketch the graph.

Solution:

There is no y -intercept, because x cannot be 0.



x	3	3	3
y	0	3	-2

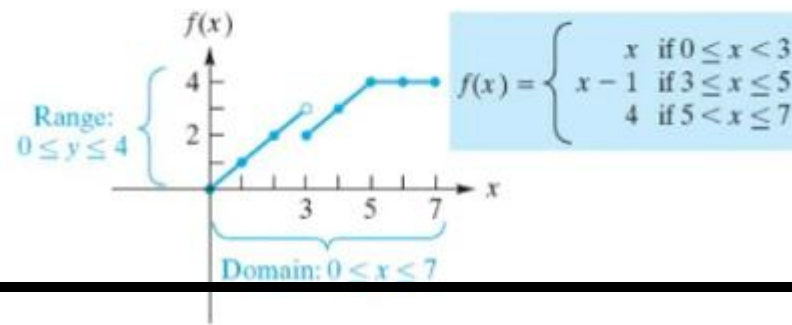
Example – Graph of a Case-Defined Function

Graph the case-defined function

$$f(x) = \begin{cases} x & \text{if } 0 \leq x < 3 \\ x - 1 & \text{if } 3 \leq x \leq 5 \\ 4 & \text{if } 5 < x \leq 7 \end{cases}$$

Solution:

x	0	1	2	3	4	5	6	7
$f(x)$	0	1	2	2	3	4	4	4



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