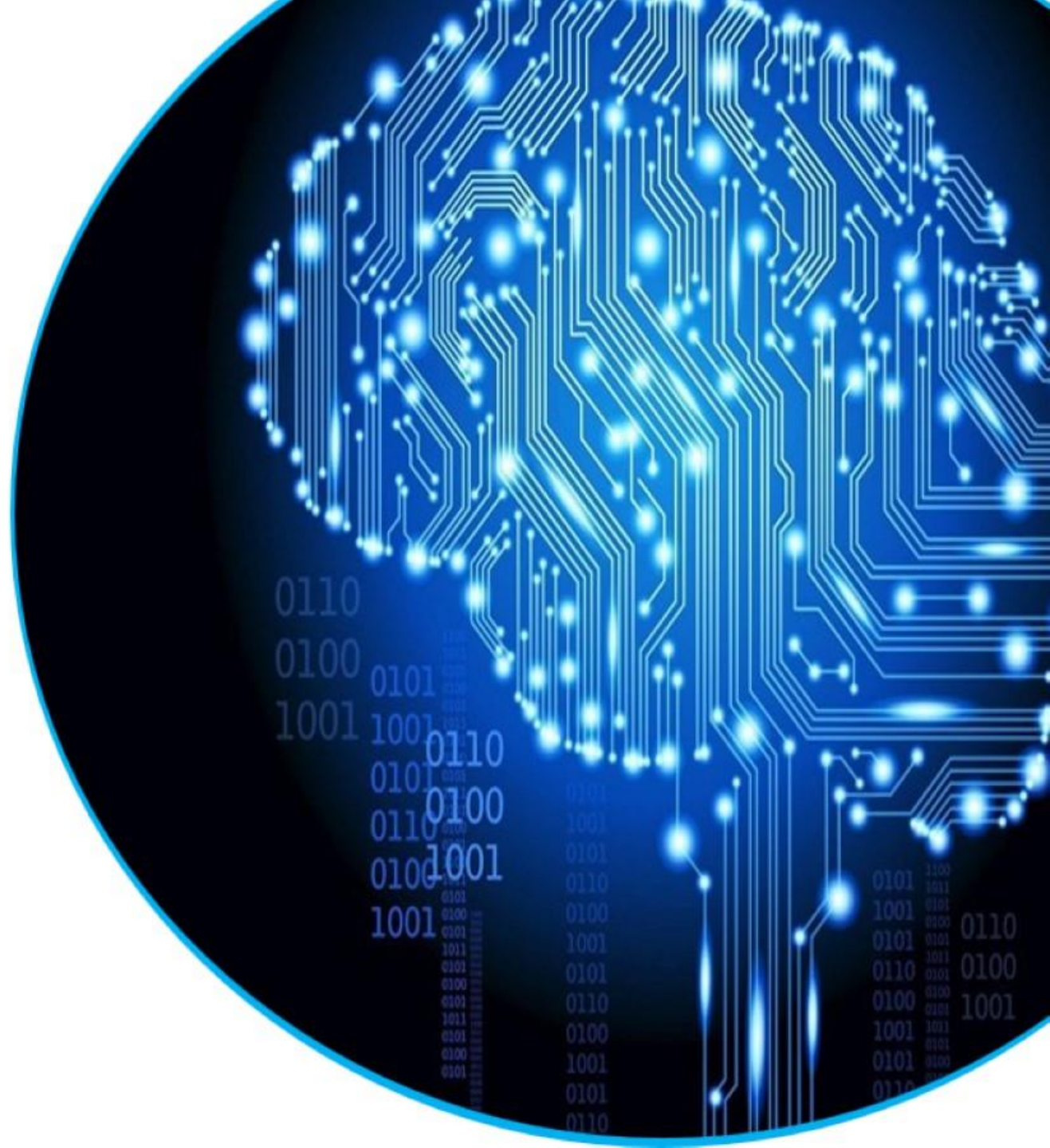


Discrete Mathematics

IT241

Dr. Shahira Ahmed

Lecture 1





Discrete Mathematics

Lecture 1

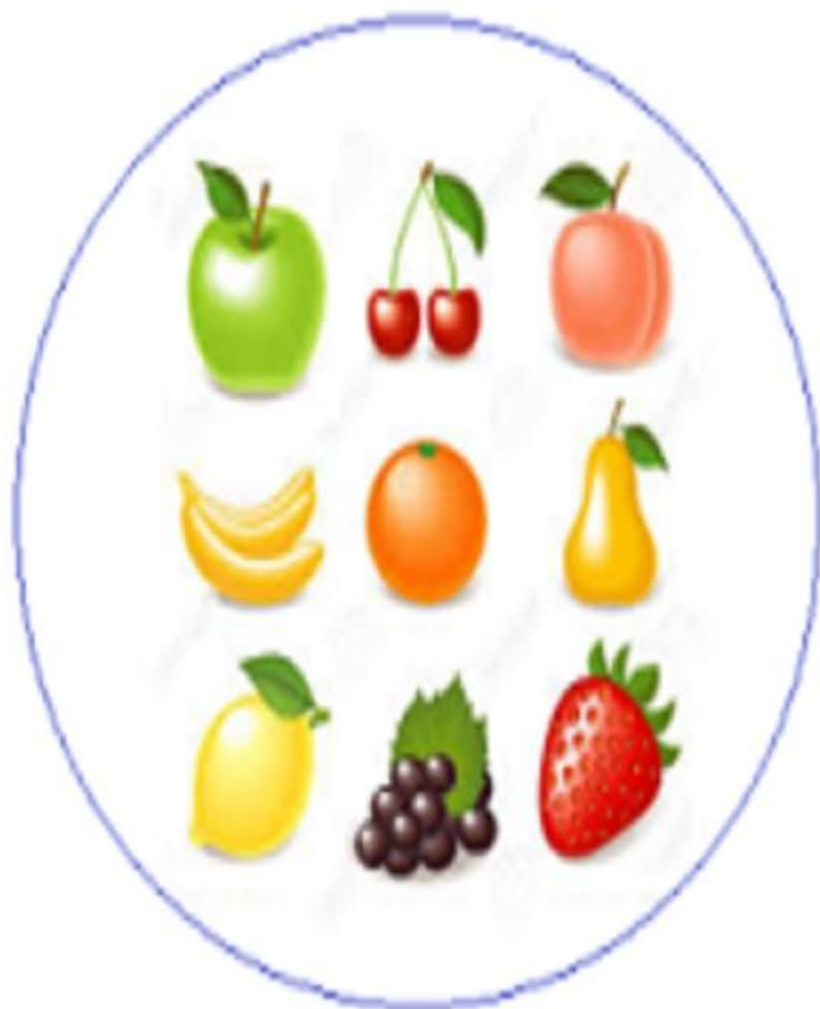
Basic Structures: Sets, Tuples

Dr. Shahira Azab

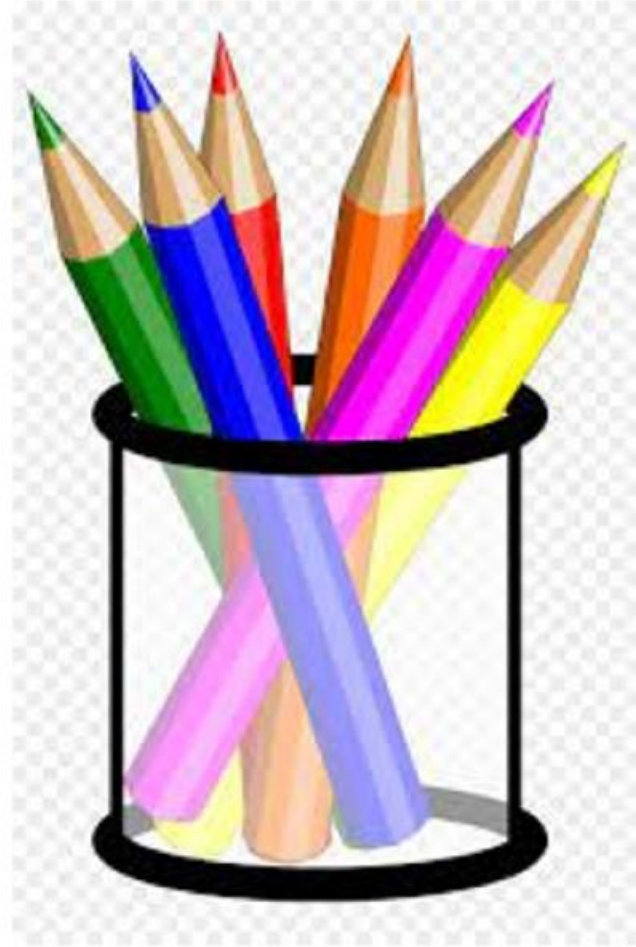
- Definition of sets
- Describing Sets
 - Roster Method
 - Set-Builder Notation
- Some Important Sets in Mathematics
- Empty Set and Universal Set
- Subsets and Set Equality
- Cardinality of Sets
- Set Operations
- Tuples
- Cartesian Product

- Important for Counting.
- Programming languages and Databases have set operations.
- Data structure.

Sets Examples



Sets Examples



- A *set* is an unordered collection of objects.
 - the students in this class
- The objects in a set are called the elements, or members of the set.
- The notation $a \in A$ denotes that a is an element of the set A .
- If a is not a member of A , write $a \notin A$

- $S = \{a, b, c, d\}$
- **Order not important**

$$S = \{a, b, c, d\} = \{b, c, a, d\}$$

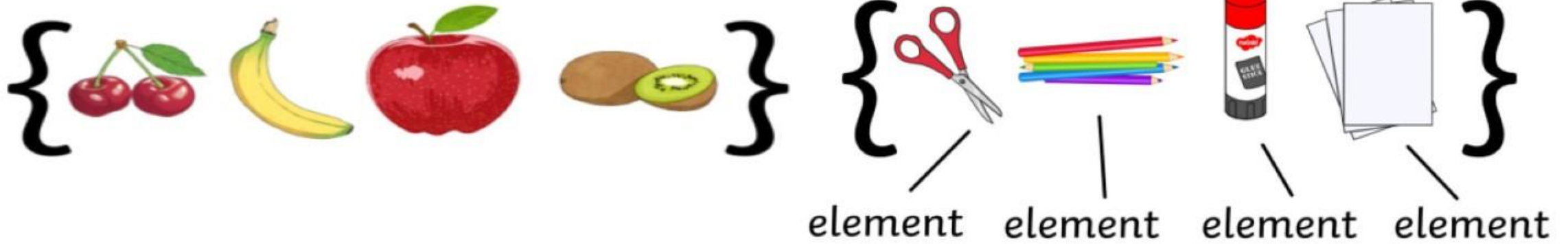
- **Ellipses (...) may be used to describe a set without listing all of the members when the pattern is clear.**

$$S = \{a, b, c, d, \dots, z\}$$

- **Repetition is not important**

$$S = \{a, b, c, d\} = \{a, b, c, b, c, d\}$$

Set



$\{1, 3, 5, 7, \dots\}$

set brackets or braces

- $L = \{a, b, c, d\}$
- $X = \{d, f, a, c\}$
- $W = \{c, a, b, d, d, d\}$
- $W = L?$
- $W = X?$
- $s \in X?$
- $a \in X?$
- $b \notin X?$
- $d \notin X?$

- $L = \{a, b, c, d\}$
- $X = \{d, f, a, c\}$
- $W = \{c, a, b, d, d, d\}$
- $W = L?$ True
- $W = X?$ False
- $s \in X?$ False
- $a \in X?$ True
- $b \notin X?$ True
- $d \notin X?$ False

- **List using Roster Method**
 1. **Set of all vowels in the English alphabet:**
 2. **Set of all odd positive integers less than 10:**
 3. **Set of all positive integers less than 100:**
 4. **Set of all integers less than 0:**

- **Set of all vowels in the English alphabet:**

$$V = \{a, e, i, o, u\}$$

- **Set of all odd positive integers less than 10:**

$$O = \{1, 3, 5, 7, 9\}$$

- **Set of all positive integers less than 100:**

$$S = \{1, 2, 3, \dots, 99\}$$

- **Set of all integers less than 0:**

$$S = \{\dots, -3, -2, -1\}$$

Some Important Sets

- $\mathbf{N} = \text{natural numbers} = \{0, 1, 2, 3, \dots\}$
- $\mathbf{Z} = \text{integers} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- $\mathbf{Z}^+ = \text{positive integers} = \{1, 2, 3, \dots\}$
- $\mathbf{R} = \text{set of real numbers}$
- $\mathbf{R}^+ = \text{set of positive real numbers}$

- Specify the property or properties that all members must satisfy:

$$S = \{x \mid x \text{ is a positive integer less than } 100\}$$

$$O = \{x \mid x \text{ is an odd positive integer less than } 10\}$$

$$O = \{x \in \mathbf{Z}^+ \mid x \text{ is odd and } x < 10\}$$

$$[a,b] = \{x \mid a \leq x \leq b\}$$

$$[a,b) = \{x \mid a \leq x < b\}$$

$$(a,b] = \{x \mid a < x \leq b\}$$

$$(a,b) = \{x \mid a < x < b\}$$

$$[a,b] = \{x \mid a \leq x \leq b\}$$

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closed interval $[a,b]$

open interval (a,b)

$$[a,b] = \{x \mid a \leq x \leq b\}$$

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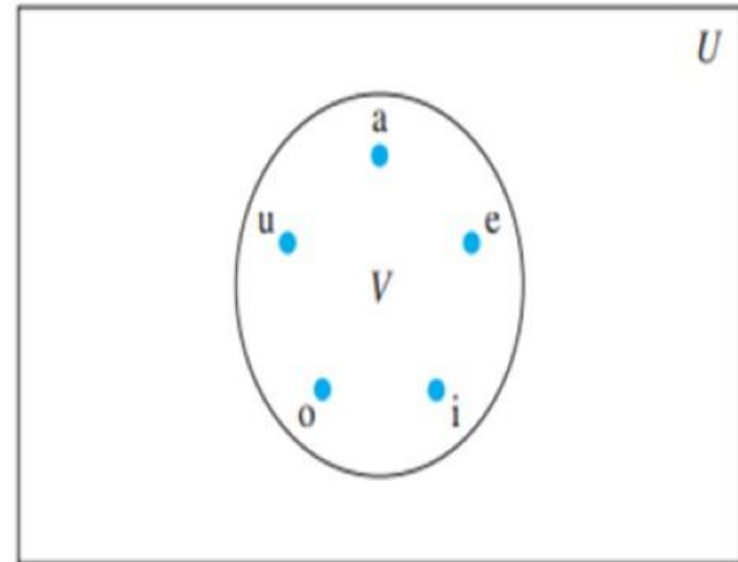
$$]a,b[= \{x \mid a < x < b\}$$

closed interval $[a,b]$

open interval (a,b)

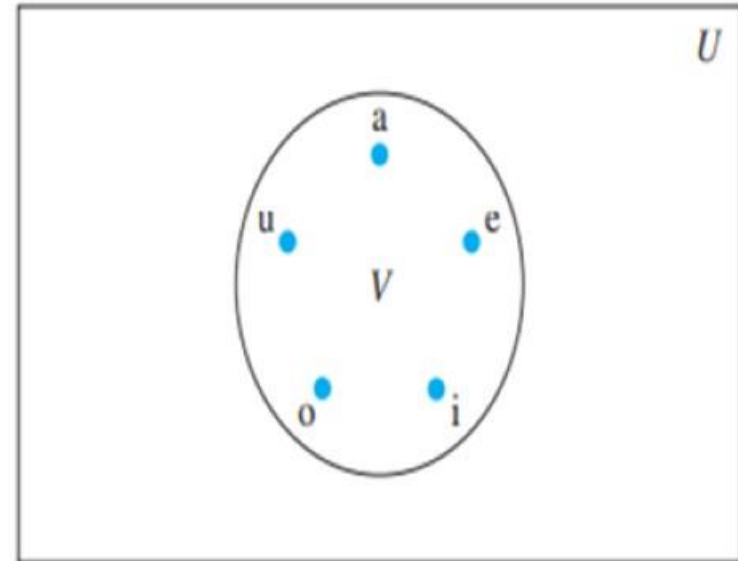
Universal Set and Empty Set

- The *universal set* U is the set containing everything currently under consideration.
- The empty set(null set) is the set with no elements. $\emptyset$, $\{\}$



Venn diagram for the set of vowels.

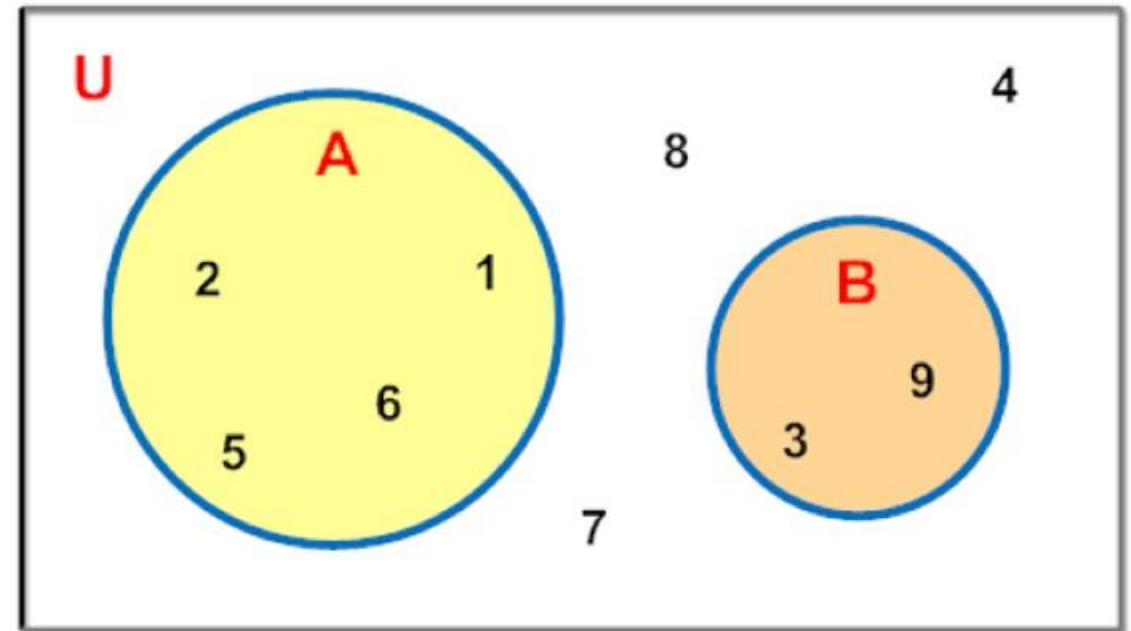
- Sets can be represented graphically using Venn diagrams, named after the English mathematician John Venn, who introduced their use in 1881



Venn diagram for the set of vowels.

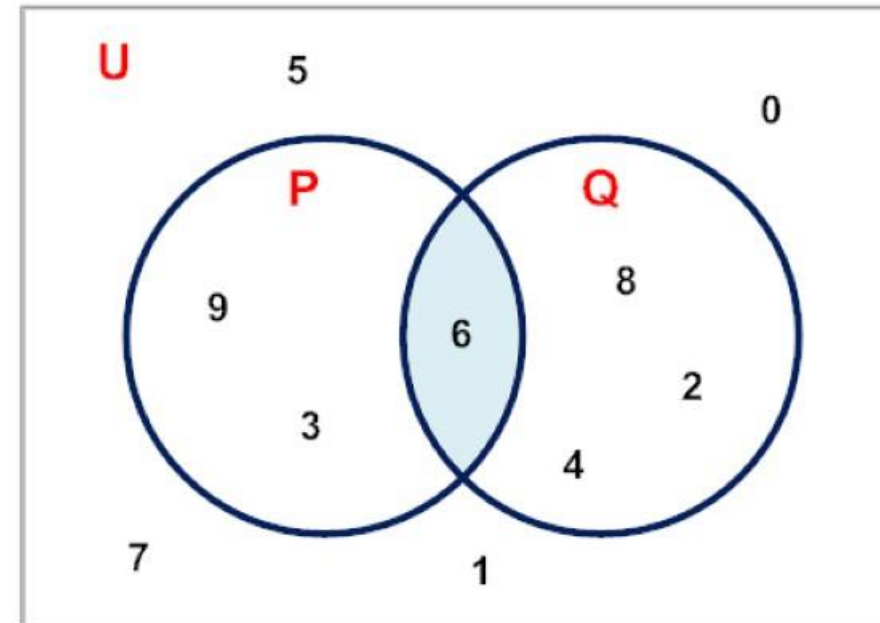
Quiz

- $A = \{1, 2, 5, 6\}$ and
- $B = \{3, 9\}$
- $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$,



DISJOINT SETS

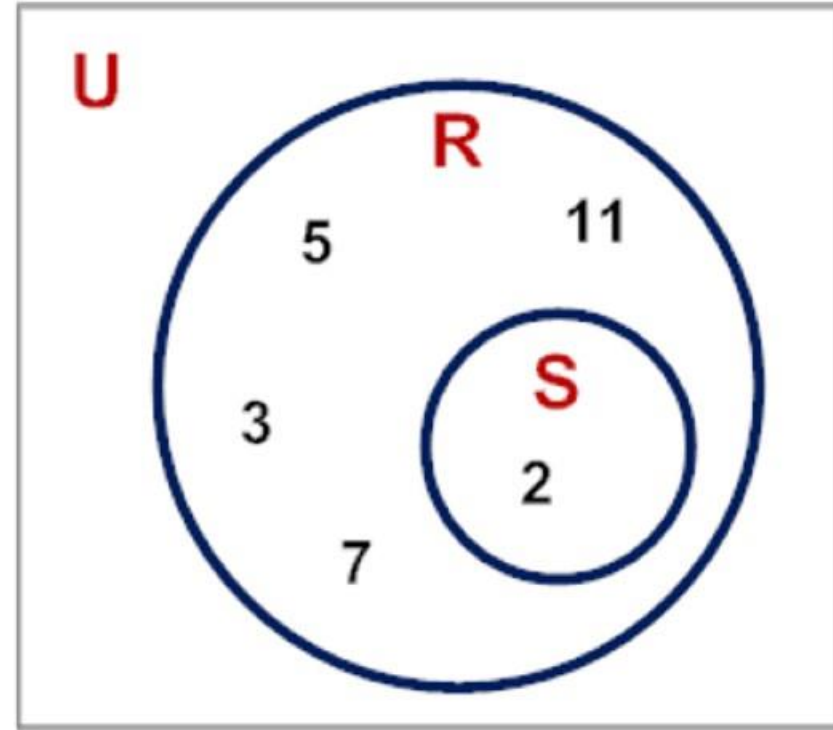
- $P = \{3, 6, 9\}$ and
- $Q = \{2, 4, 6, 8\}$
- $U = \{3, 6, 9, 2, 4, 8, 0, 1, 5, 7\}$



JOINT SETS

Quiz

- $R = \{2, 3, 5, 7, 11\}$ and
- $S = \{2\}$
- $U = \{2, 3, 5, 7, 11\}$



SUBSETS

- **Sets can be elements of sets.**

$$A = \{\{1,2,3\}, a, \{b,c\}\}$$

- **The empty set is different from a set containing the empty set.**

$$\emptyset \neq \{\emptyset\}$$

Two sets are *equal* if and only if they have the same elements.

- We write $A = B$ if A and B are equal sets.

$$\{1, 3, 5\} = \{3, 5, 1\}$$

$$\{1, 5, 5, 5, 3, 3, 1\} = \{1, 3, 5\}$$

- if A and B are sets, then A and B are equal if and only if

$$\forall x (x \in A \leftrightarrow x \in B)$$

- List the members of these sets.
 - a) $\{x \mid x \text{ is a real number such that } x^2 = 1\}$
 - b) $\{x \mid x \text{ is a positive integer less than } 12\}$
 - c) $\{x \mid x \text{ is an integer such that } x^2 = 2\}$
- Solutions
 - a) $\{-1, 1\}$
 - b) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$
 - d) $\emptyset$

- Which of the intervals
 $(0, 5)$, $(0, 5]$, $[0, 5)$, $[0, 5]$, $(1, 4]$, $[2, 3]$, $(2, 3)$
contains 0?
- Solution
 $[0, 5)$, $[0, 5]$

- Which of the intervals
 $(0, 5)$, $(0, 5]$, $[0, 5)$, $[0, 5]$, $(1, 4]$, $[2, 3]$, $(2, 3)$
contains 1?
- Solution
 $(0, 5)$, $(0, 5]$, $[0, 5)$, $[0, 5]$

Definition: The set A is a *subset* of B , if and only if every element of A is also an element of B .

- The notation $A \subseteq B$ is used to indicate that A is a subset of the set B .
- $A \subseteq B$ holds if and only if $\forall x(x \in A \rightarrow x \in B)$ is true.
 - $a \in \emptyset$ is always false, $\emptyset \subseteq S$, for every set S .

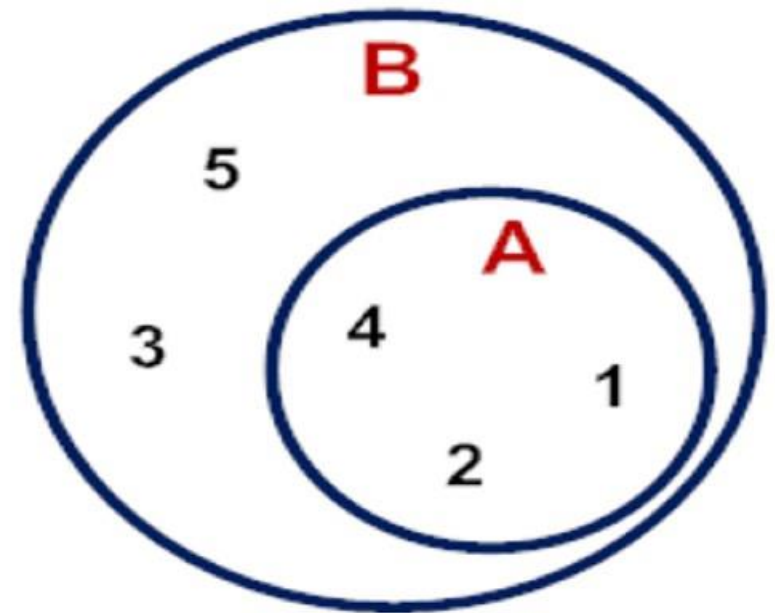
- $\{1,2,3\} \subseteq \{1,2,3,4,5, \dots, 99\}$
- $\{\text{Orange}, \text{Apple}\} \subseteq \text{set of fruits}$
- $\text{Student in IT241} \subseteq \text{Student in the university}$

- Remember
 - $1 \in \{1,2,3,4,5, \dots, 99\}$
 - $\text{Orange} \in \text{set of fruits}$

- $A = \{1, 2, 4\}$ and $B = \{1, 2, 3, 4, 5\}$

- $A \subseteq B?$

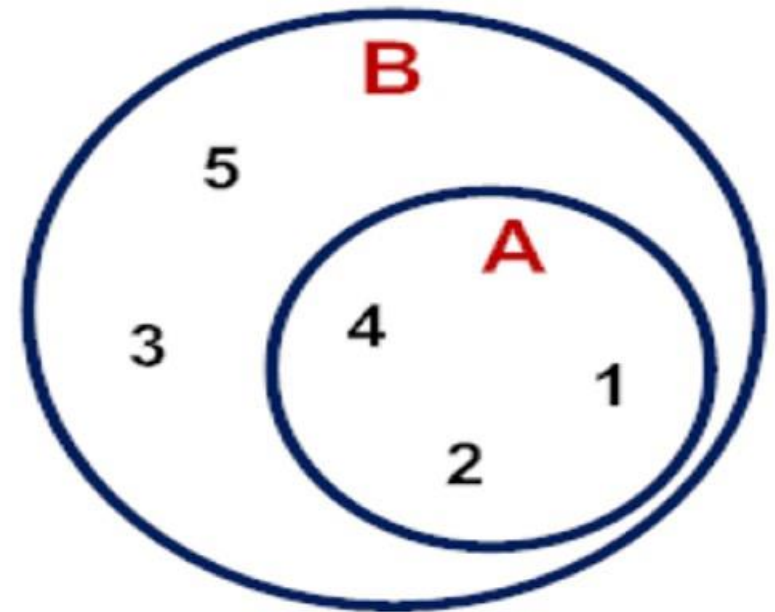
- *True*



- $A = \{1, 2, 4\}$ and $B = \{1, 2, 3, 4, 5\}$

- $A \subseteq B?$

- *True*

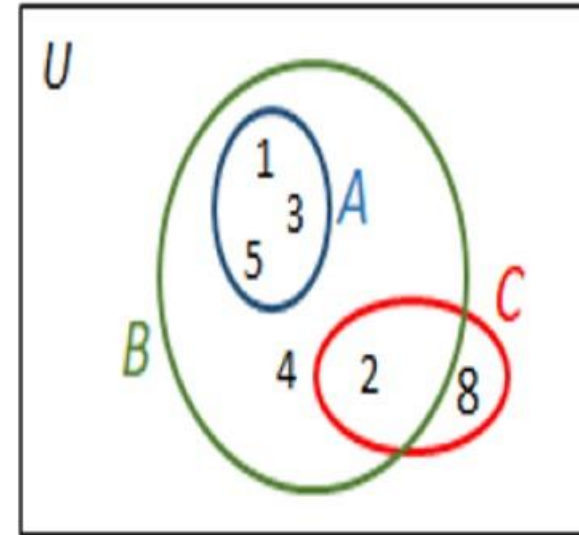


$$A = \{1, 3, 5\} \quad B = \{1, 2, 3, 4, 5\}$$

$$C = \{2, 8\}$$

$$A \subseteq B? \quad \text{True}$$

$$C \subseteq B? \quad \text{False}$$

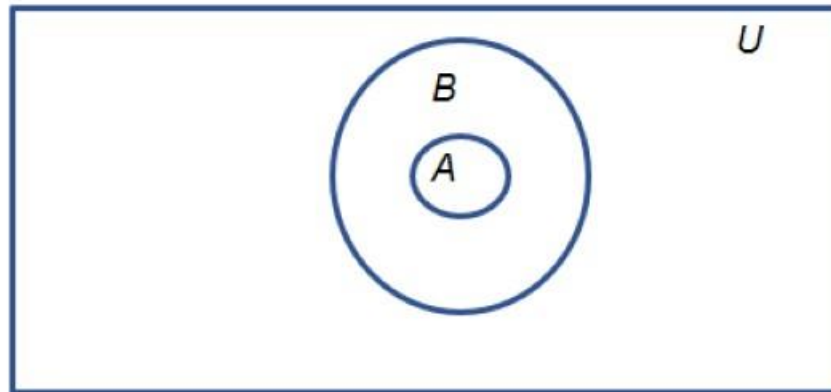


- **Showing that A is a Subset of B:** To show that $A \subseteq B$, show that if x belongs to A , then x also belongs to B .
- **Showing that A is not a Subset of B:** To show that A is not a subset of B , $A \not\subseteq B$, find an element $x \in A$ with $x \notin B$. (Such an x is a counterexample to the claim that $x \in A$ implies $x \in B$.)

Definition: If $A \subseteq B$, but $A \neq B$, then we say A is a *proper subset* of B , denoted by $A \subset B$. If $A \subset B$, then

is true. $\forall x(x \in A \rightarrow x \in B) \wedge \exists x(x \in B \wedge x \notin A)$

Venn Diagram



Definition: If there are exactly n distinct elements in S where n is a nonnegative integer, we say that S is *finite*. Otherwise it is *infinite*.

Definition: The *cardinality* of a finite set A , denoted by $|A|$, is the number of (distinct) elements of A .

Examples:

1. $|\emptyset| = 0$
2. Let S be the letters of the English alphabet. Then $|S| = 26$
3. $|\{1,2,3\}| = 3$
4. $|\{\emptyset\}| = 1$
5. The set of integers is infinite.

Definition: The set of all subsets of a set A , denoted $P(A)$, is called the *power set* of A .

Example: If $A = \{a, b\}$ then

$$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$

Example: If $A = \{a, b, c\}$ then

$$P(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

- If a set has n elements, then the cardinality of the power set is 2^n .

- The *ordered n-tuple* $(a_1, a_2, \dots, a_n)$ is the ordered collection that has a_1 as its first element and a_2 as its second element and so on until a_n as its last element.
- Two n-tuples are equal if and only if their **corresponding** elements are equal.
- 2-tuples are called *ordered pairs*.
- The ordered pairs (a, b) and (c, d) are equal if and only if $a = c$ and $b = d$.

Definition: The *Cartesian Product* of two sets A and B , denoted by

$A \times B$ is the set of ordered pairs (a,b) where $a \in A$ and $b \in B$.

$$A \times B = \{(a, b) | a \in A \wedge b \in B\}$$

Example:

$$A = \{a, b\} \quad B = \{1, 2, 3\}$$

$$A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$$

Definition: The *Cartesian Product* of two sets A and B , denoted by

$A \times B$ is the set of ordered pairs (a,b) where $a \in A$ and $b \in B$.

$$A \times B = \{(a, b) | a \in A \wedge b \in B\}$$

Example:

$$A = \{a, b\} \quad B = \{1, 2, 3\}$$

$$A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$$

$$B \times A = \{(1, a), (2, a), (3, a), (1, b), (2, b), (3, b)\}$$

Quiz

Suppose that $A = \{1, 2\}$

$A \times A$

Solution

$\{1, 2\} \times \{1, 2\}$

$\{(1,1), (1,2), (2,1), (2,2)\}$



Suppose that $A = \{1, 2\}$

Find A^3 ?

Solution

$A \times A \times A$

$\{1, 2\} \times \{1, 2\} \times \{1, 2\}$

$\{(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)\}.$

Determine whether each of these pairs of sets are equal.

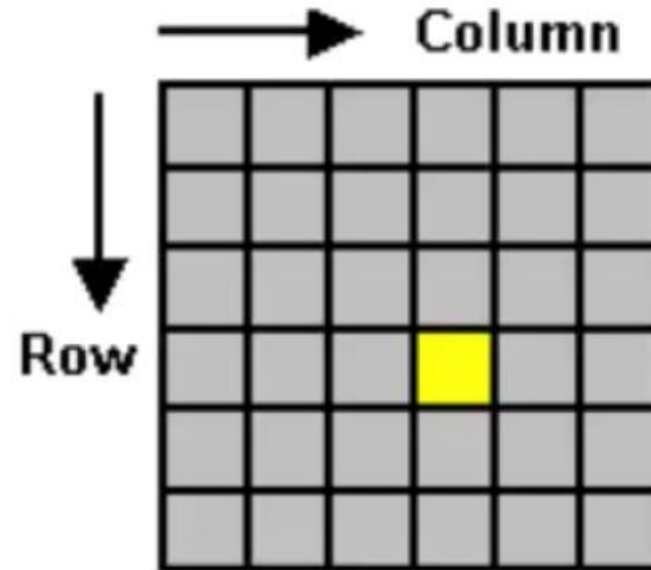
- a) $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}, \{5, 3, 1\}$
- b) $\{\{1\}\}, \{1, \{1\}\}$
- c) $\emptyset, \{\emptyset\}$

Solution

- a) Yes
- b) No
- c) No

- Pixel Coordinates Application

- A digital image is made up of rows and columns of pixels. A pixel in such an image can be specified by saying which column and which row contains it. In terms of coordinates, a pixel can be identified by a pair of integers giving the column number and the row number.



Database Application

R

FName	LName	Age
John	Smith	35
Mary	Shelley	29
John	Doe	45
Nicky	Little	25

S

Car	Color
BMW	Black
Audi	White

**Cartesian
Product**
↓
R x S =

FName	LName	Age	Car	Color
John	Smith	35	BMW	Black
John	Smith	35	Audi	White
Mary	Shelley	29	BMW	Black
Mary	Shelley	29	Audi	White
John	Doe	45	BMW	Black
John	Doe	45	Audi	White
Nicky	Little	25	BMW	Black
Nicky	Little	25	Audi	White

Set Operations

Dr. Shahira Ahmed

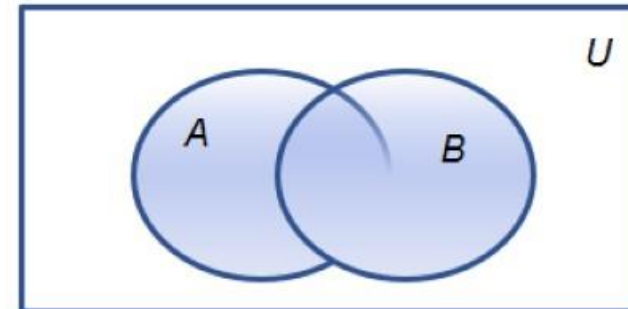
- **Definition:** Let A and B be sets. The *union* of the sets A and B , denoted by $A \cup B$, is the set:

$$\{x | x \in A \vee x \in B\}$$

- **Example:** What is $\{1,2,3\} \cup \{3,4,5\}$?

Solution: $\{1,2,3,4,5\}$

Venn Diagram for $A \cup B$



- **Definition:** The *intersection* of sets A and B , denoted by $A \cap B$, is

$$\{x | x \in A \wedge x \in B\}$$

- Note if the intersection is empty, then A and B are said to be *disjoint*.
- **Example:** What is? $\{1,2,3\} \cap \{3,4,5\}$?

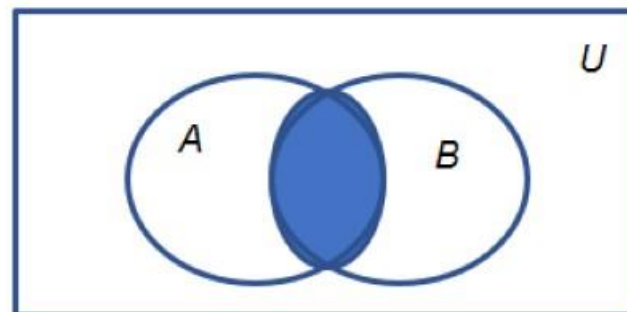
Solution: $\{3\}$

- **Example:** What is?

$$\{1,2,3\} \cap \{4,5,6\} ?$$

Solution: $\emptyset$

Venn Diagram for $A \cap B$



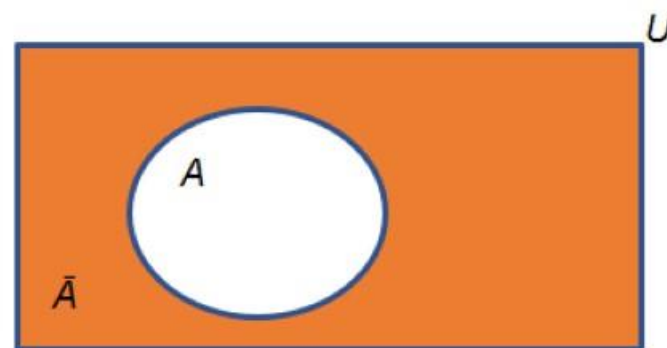
Definition: If A is a set, then the complement of the A (with respect to U), denoted by $\bar{A}$ or A^c is the set $U - A$

$$\bar{A} = \{x \in U \mid x \notin A\}$$

Example: If U is the positive integers less than 100, what is the complement of $\{x \mid x > 70\}$

Solution: $\{x \mid x \leq 70\}$

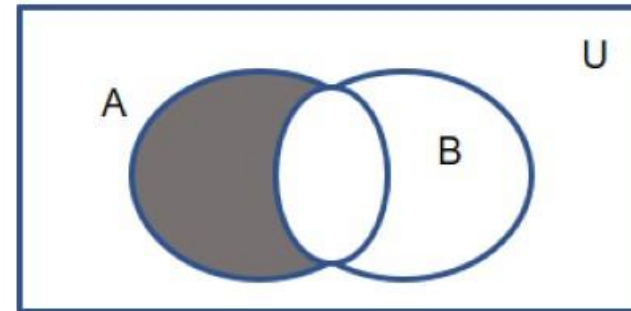
Venn Diagram for Complement



Definition: Let A and B be sets. The *difference* of A and B , denoted by $A - B$, is the set containing the elements of A that are not in B .
The difference of A and B .

$$A - B = \{x \mid x \in A \wedge x \notin B\} = A \cap \bar{B}$$

Venn Diagram for $A - B$



Example: $U = \{0,1,2,3,4,5,6,7,8,9,10\}$ $A = \{1,2,3,4,5\}$, $B = \{4,5,6,7,8\}$

➤ $A \cup B$

Solution: $\{1,2,3,4,5,6,7,8\}$

➤ $A \cap B$

Solution: $\{4,5\}$

➤ $\bar{A}$

Solution: $\{0,6,7,8,9,10\}$

Example: $U = \{0,1,2,3,4,5,6,7,8,9,10\}$ $A = \{1,2,3,4,5\}$, $B = \{4,5,6,7,8\}$

➤ $\bar{B}$

Solution: $\{0,1,2,3,9,10\}$

➤ $A - B$

Solution: $\{1,2,3\}$

➤ $B - A$

Solution: $\{6,7,8\}$

Definition: The *symmetric difference* of **A** and **B**, denoted by
is the set

$$A \oplus B$$
$$(A - B) \cup (B - A)$$

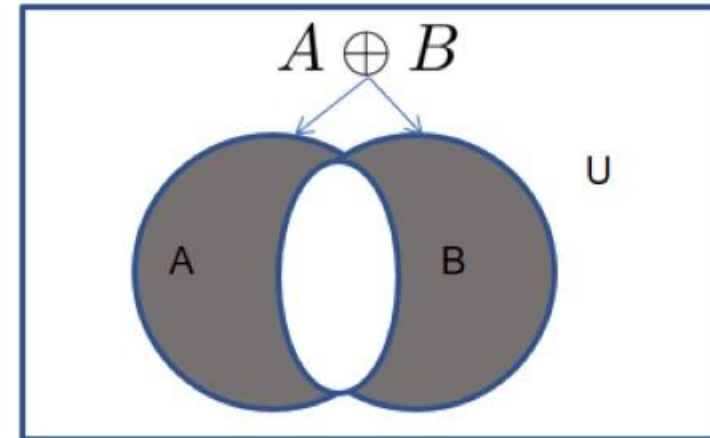
Example:

$$U = \{0,1,2,3,4,5,6,7,8,9,10\}$$

$$A = \{1,2,3,4,5\} \quad B = \{4,5,6,7,8\}$$

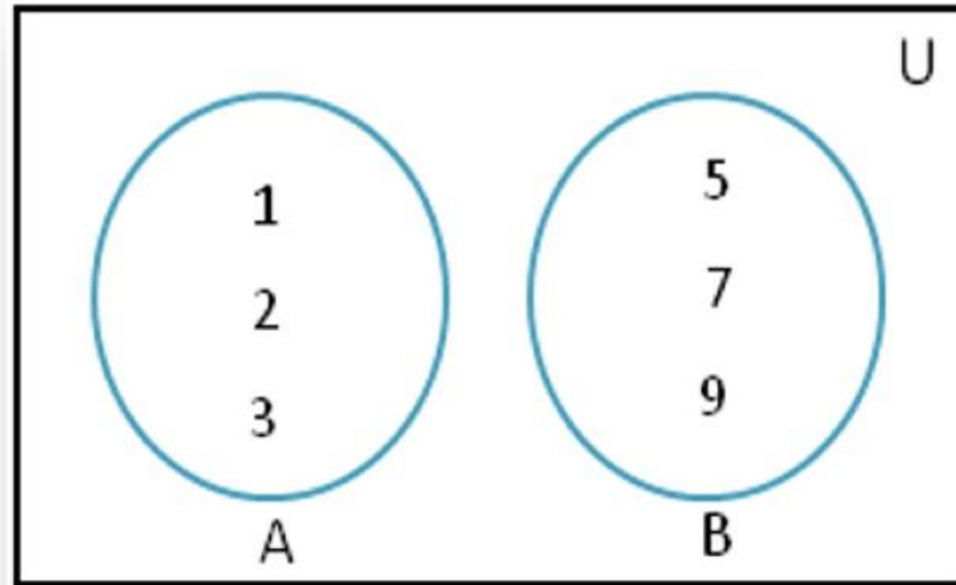
What is $A \oplus B$

- **Solution:** $\{1,2,3,6,7,8\}$



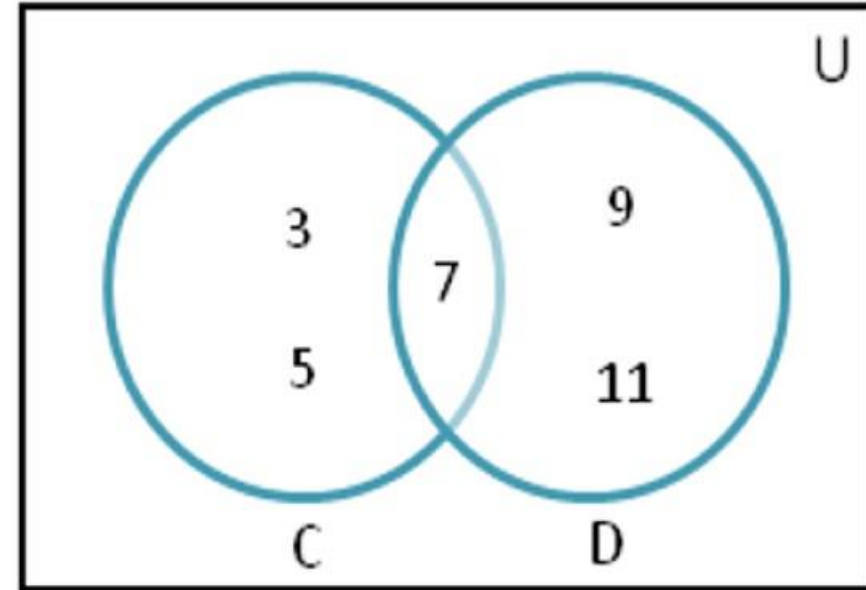
Venn Diagram

Cardinality of union of two set



Disjoint Sets

$$|A \cup B| = |A| + |B|$$



Not Disjoint Sets

- Inclusion-Exclusion

$$|A \cup B| = |A| + |B| - |A \cap B|$$

- Identity laws

$$A \cup \emptyset = A \quad A \cap U = A$$

- Domination laws

$$A \cup U = U \quad A \cap \emptyset = \emptyset$$

- Idempotent laws

$$A \cup A = A \quad A \cap A = A$$

- Complementation law $\overline{\overline{A}} = A$

- Commutative laws

$$A \cup B = B \cup A \quad A \cap B = B \cap A$$

- Associative laws

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

- Distributive laws

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

- De Morgan's laws

$$\overline{A \cup B} = \overline{A} \cap \overline{B} \quad \overline{A \cap B} = \overline{A} \cup \overline{B}$$

- Absorption laws

$$A \cup (A \cap B) = A \quad A \cap (A \cup B) = A$$

- Complement laws

$$A \cup \overline{A} = U \quad A \cap \overline{A} = \emptyset$$

Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find

- a) $A \cup B$.
- b) $A \cap B$.
- c) $A - B$.
- d) $B - A$.

Solution

- a) $\{0, 1, 2, 3, 4, 5, 6\}$
- b) $\{3\}$
- c) $\{1, 2, 4, 5\}$
- d) $\{0, 6\}$

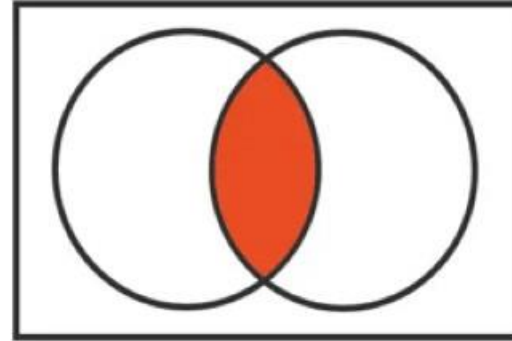
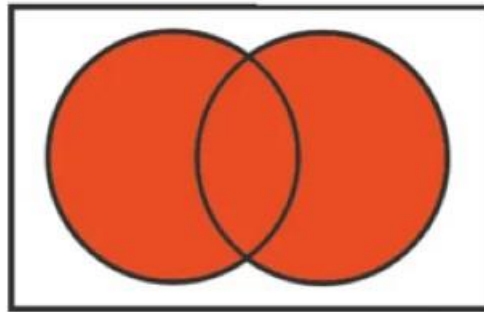
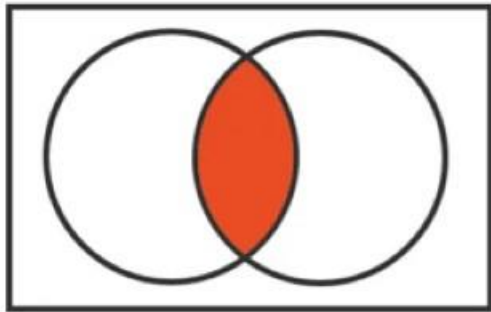
Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Find

- a) $A \cap B \cap C$.
- b) $A \cup B \cup C$.
- c) $(A \cup B) \cap C$.
- d) $(A \cap B) \cup C$.

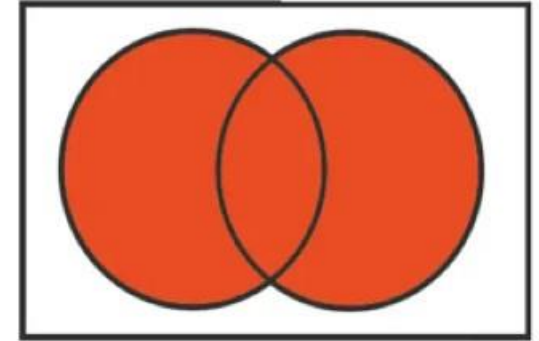
Solution

- a) $\{4, 6\}$
- b) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- c) $\{4, 5, 6, 8, 10\}$
- d) $\{0, 2, 4, 5, 6, 7, 8, 9, 10\}$

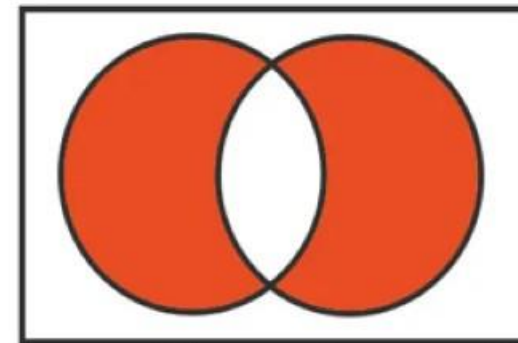
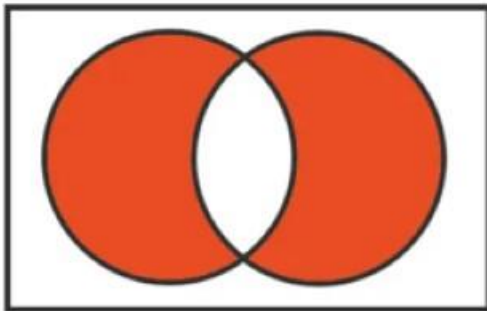
- Express set operation in the following venn diagram



Intersection of two sets ($A \cap B$)



Union of two sets ($A \cup B$)



Symmetric difference of two sets ($A \oplus B$)

- If $A = \{x, y\}$ and $B = \{1, 2, 3, 4, 5\}$. Find cartesian product of $A \times B$ and $B \times A$
- Here, $A = \{x, y\}$ and $B = \{1, 2, 3, 4, 5\}$
- $A \times B = \{(x, 1), (x, 2), (x, 3), (x, 4), (x, 5), (y, 1), (y, 2), (y, 3), (y, 4), (y, 5)\}$
- $B \times A = \{(1, x), (2, x), (3, x), (4, x), (5, x), (1, y), (2, y), (3, y), (4, y), (5, y)\}$
- Here, we note that cartesian product of $A \times B \neq B \times A$,

If A and B are two sets such that $|A| = 17$ and $|B| = 23$ and $|A \cup B| = 38$ then find $|A \cap B|$.

Solution:

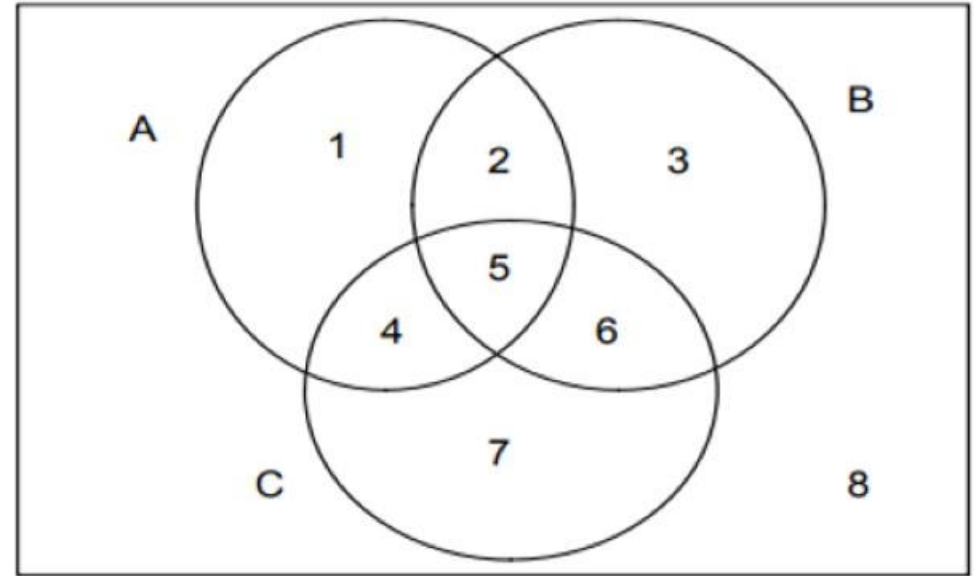
- We know that $|A \cup B| = |A| + |B| - |A \cap B|$
- $\Rightarrow 38 = 17 + 23 - |A \cap B|$
- $\Rightarrow |A \cap B| = 40 - 38 = 2$

Quiz

Given the following Venn diagram

Find

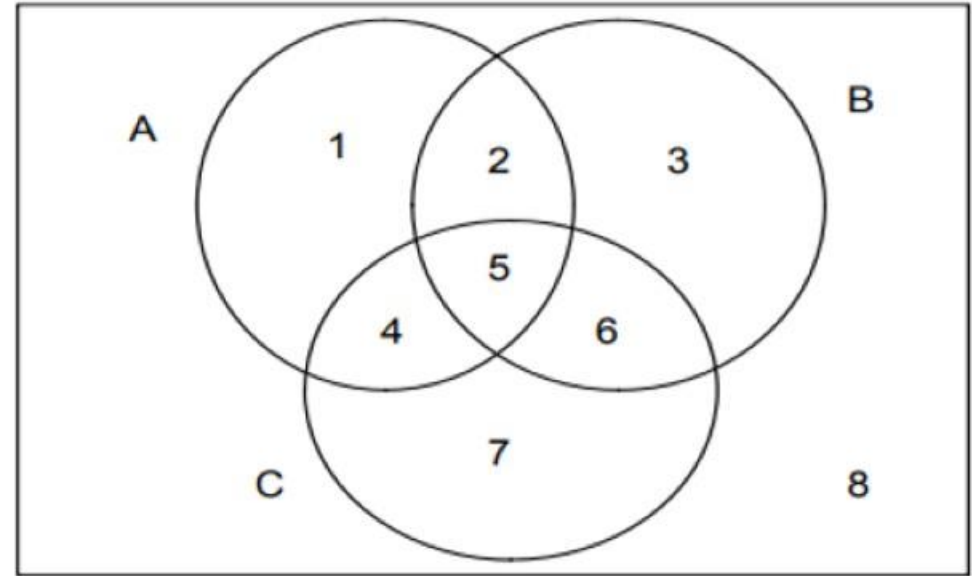
- Set A
- Set B
- Set C
- $A \cap B \cap C$
- $A \cap C$
- $A \cap B$
- $B \cap C$



Given the following Venn diagram

Find

- Set $A = \{1, 2, 5, 4\}$
- Set $B = \{2, 3, 5, 6\}$
- Set $C = \{4, 5, 6, 7\}$
- The set $A \cap B \cap C = \{5\}$
- The set $A \cap C = \{4, 5\}$
- The set $A \cap B = \{2, 5\}$
- The set $B \cap C = \{5, 6\}$



- Definition of sets
- Describing Sets
- Subsets and Set Equality
- Cardinality of Sets
- Tuples
- Cartesian Product
- Set Operations

*Thank
you*

