

Business Mathematics

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MTH 112
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Lecture NO 7



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Chapter

Topics

- Chapter 1: Mathematical preliminaries
- Chapter 2: The straight line and applications
- Chapter 3: Functions and Graphs
- Chapter 4: Limits and Continuity
- Chapter 5: Financial Mathematics
- Chapter 6: Differentiation
- **Chapter 7: Methods of Differentiation**
- Chapter 8: Function of several variables
- Chapter 9: Integration and applications
- Chapter 10: Linear algebra and applications

Differentiation

Remember that

$f'(x)$ “ f prime of x ”

y' “ y prime”

$\frac{dy}{dx}$ “the derivative of y with respect to x ”

$\frac{d}{dx}$ is a verb. “Take the derivative with respect to x ...”

Differentiation

Remember that

- Differentiation is the process of computing the derivative of a function.

You may be asked to:

- Differentiate.
- Derive.
- Find the derivative of...

Differentiation

$$\frac{d}{dx}[c] = 0, \quad c \text{ is a constant}$$

Remember that

The derivative of a constant function is zero.

Differentiation

Remember that

$$\frac{d}{dx}[x^N] = Nx^{N-1}, \quad N \text{ is any real number}$$

$$\frac{d}{dx}[x] = 1$$

Differentiation

Remember that

$$\frac{d}{dx}[c(f(x))] = c(f'(x)), \quad c \text{ is a constant}$$

- The derivative of a constant times a function is equal to the constant times the derivative of the function.

Differentiation

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

Remember that

The derivative of a sum is the sum of the derivatives.

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

The derivative of a difference is the difference of the derivatives.

Differentiation

Remember that

The derivative of the product of two differentiable functions is given by

$$y = f(x) \cdot g(x)$$

$$\frac{dy}{dx} = y' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\frac{dy}{dx} = y' = f' \cdot g + f \cdot g'$$

Differentiation

Remember that

- The derivative of the quotient of two differentiable functions is given by

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \quad (g(x) \neq 0)$$

Objectives

- **The Rate of change** of the total Cost Function is the derivative of the total Cost Function

$$C'(x) = \frac{d}{dx} C(x)$$

Objectives

- **The Profit Function**

is difference between of **Revenue Function** and
Cost Function

$$P(x) = R(x) - C(x)$$

Objectives

- Now we will discuss the marginal analysis of various functions related to:
- Cost
- Revenue
- Profit

Objectives

- **The marginal Cost Function** $C'(x)$

is the derivative of the **Cost Function**

$$C'(x) = \frac{d}{dx} C(x)$$

Objectives

- **The marginal Revenue Function** $R'(x)$

is the derivative of the **Revenue Function**

$$R'(x) = \frac{d}{dx} R(x)$$

Objectives

- **The marginal Profit Function** $P'(x)$

is the derivative of the **Profit Function**

$$P'(x) = \frac{d}{dx} P(x)$$

Methods of Differentiation

- Suppose the total cost in dollars incurred each week by company for manufacturing x refrigerators is given by the total cost function

$$C(x) = 8000 + 200x - 0.2x^2 \quad (0 \leq x \leq 400)$$

Methods of Differentiation

- What is the actual cost incurred for manufacturing the 251st refrigerator?
- Find the rate of change of the total cost function with respect to x when $x = 250$.

Methods of Differentiation

$$C(x) = 8000 + 200x - 0.2x^2$$

a. The cost incurred in producing the 251st refrigerator is

$$C(251) - C(250) =$$

$$= 45,599.8 - 45,500$$

$$= 99.80$$

or \$99.80.

Methods of Differentiation

b. The rate of change of the total cost function

$$C(x) = 8000 + 200x - 0.2x^2$$

with respect to x is given by

$$C'(x) = 200 - 0.4x$$

So, when production is 250 refrigerators, the rate of change of the total cost with respect to x is

$$\begin{aligned} C'(x) &= 200 - 0.4(250) \\ &= 100 \end{aligned}$$

or \$100.

Methods of Differentiation

A subsidiary of Electronics manufactures a portable music player. Management determined that the daily total cost of producing these players (in dollars) is

$$C(x) = 0.0001x^3 - 0.08x^2 + 40x + 5000$$

where x stands for the number of players produced.

- Find the marginal cost function.
- Find the marginal cost for $x = 200, 300, 400$, and 600 .

Methods of Differentiation

a. If the total cost function is:

$$C(x) = 0.0001x^3 - 0.08x^2 + 40x + 5000$$

then, its derivative is the marginal cost function:

$$C'(x) = 0.0003x^2 - 0.16x + 40$$

Methods of Differentiation

b. The marginal cost for $x = 200, 300, 400$, and 600 is:

$$C'(200) = 0.0003(200)^2 - 0.16(200) + 40 = 20$$

$$C'(300) = 0.0003(300)^2 - 0.16(300) + 40 = 19$$

$$C'(400) = 0.0003(400)^2 - 0.16(400) + 40 = 24$$

$$C'(600) = 0.0003(600)^2 - 0.16(600) + 40 = 52$$

or \$20/unit, \$19/unit, \$24/unit, and \$52/unit, respectively.

Methods of Differentiation

Given that the revenue function

$$R(x) = -0.02x^2 + 400x \quad (0 \leq x \leq 20,000)$$

- Find the marginal revenue function R' .
- Compute $R'(2000)$ and interpret your result.

Methods of Differentiation

a. Given the revenue function

$$R(x) = -0.02x^2 + 400x$$

We find its derivative to obtain the marginal revenue function:

$$R'(x) = -0.04x + 400$$

Methods of Differentiation

b. When quantity demanded is 2000, the marginal revenue will be:

$$\begin{aligned} R'(2000) &= -0.04(2000) + 400 \\ &= 320 \end{aligned}$$

Thus, the actual revenue realized from the sale of the 200 1st loudspeaker system is approximately \$320.

Methods of Differentiation

Continuing with the last example, suppose the total cost (in dollars) of producing x units of the Acrosonic model F loudspeaker system is

$$C(x) = 100x + 200,000$$

- Find the profit function P .
- Find the marginal profit function P' .
- Compute $P'(2000)$ and interpret the result.

Methods of Differentiation

a. From last example we know that the revenue function is

$$R(x) = -0.02x^2 + 400x$$

Profit is the difference between total revenue and total cost, so the profit function is

$$P(x) = R(x) - C(x)$$

$$P(x) = R(x) - C(x)$$

$$P(x) = [-0.02x^2 + 400x] - [100x + 200000]$$

$$P(x) = -0.02x^2 + 300x - 200000$$

Methods of Differentiation

b. Given the profit function

$$P(x) = -0.02x^2 + 300x - 200000$$

we find its derivative to obtain the marginal profit function:

$$P'(x) = -0.04x + 300$$

Methods of Differentiation

c. When producing $x = 2000$, the marginal profit is

$$\begin{aligned} P'(2000) &= -0.04(2000) + 300 \\ &= 220 \end{aligned}$$

Thus, the profit to be made from producing the 200 1st loudspeaker is \$220.

Methods of Differentiation

The derivative f' of a function f is also a function. As such, f' may also be differentiated. Thus, the function f' has a derivative f'' at a point x in the domain of f . The function f'' obtained in this manner is called the second derivative of the function f , just as the derivative f' of f is often called the first derivative of f . By the same token, you may consider the third, fourth, fifth, etc. derivatives of a function f .

Methods of Differentiation

Find the second derivative of the function

$$f(x) = x^4$$

Methods of Differentiation

$$f'(x) = 4x^3$$

$$f''(x) = 4(3x^2) = 12x^2$$

Methods of Differentiation

Find the second derivative of the function

$$f(x) = x^{\frac{3}{2}}$$

Methods of Differentiation

$$f'(x) = \frac{3}{2}x^{\frac{1}{2}}$$

$$f''(x) = \frac{3}{2} \left(\frac{1}{2} \right) x^{-1/2} = \frac{3}{4} x^{-1/2}$$

Methods of Differentiation

Find the third derivative of the function

$$f(x) = x^{\frac{2}{3}}$$

Methods of Differentiation

$$f'(x) = \frac{2}{3}x^{-1/3}$$

$$f''(x) = \frac{2}{3} \left(-\frac{1}{3} \right) x^{-4/3} = -\frac{2}{9}x^{-4/3}$$

$$f'''(x) = -\frac{2}{9} \left(-\frac{4}{3} \right) x^{-7/3} = \frac{8}{27}x^{-7/3} = \frac{8}{27x^{7/3}}$$

Methods of Differentiation

Find the fourth derivative of the function

$$f(x) = 5x^3$$

Methods of Differentiation

$$f(x) = 5x^3$$

$$\frac{df}{dx} = f'(x) = 15x^2$$

$$\frac{d^2f}{dx^2} = f''(x) = 30x$$

$$\frac{d^3f}{dx^3} = f'''(x) = 30$$

$$\frac{d^4f}{dx^4} = f^{(4)}(x) = 0$$

Methods of Differentiation

Find the third derivative of the function

$$f(x) = 5x^2 + 7x - 6$$

Methods of Differentiation

$$f(x) = 5x^2 + 7x - 6$$

$$\frac{df}{dx} = f'(x) = 10x + 7$$

$$\frac{d^2f}{dx^2} = f''(x) = 10$$

$$\frac{d^3f}{dx^3} = f'''(x) = 0$$

Methods of Differentiation

Given the function

$$g(x) = 4x^6 - 3x^5 - 10x^2 + 5x + 16$$

Find g' and g''

Methods of Differentiation

$$g(x) = 4x^6 - 3x^5 - 10x^2 + 5x + 16$$

$$g'(x) = 24x^5 - 15x^4 - 20x + 5$$

$$g''(x) = 120x^4 - 60x^3 - 20$$

Methods of Differentiation

- Given that $f(x) = x^3$, then find the value of $f''(4)$

$$f'(x) = 3x^2$$

$$f''(x) = 6x$$

$$f''(4) = 6(4)$$

$$f''(4) = 24$$

Methods of Differentiation

Find the second derivative of

$$f(x) = x^9 + 2x - 3$$

Methods of Differentiation

Find the second derivative of

$$f(x) = x^9 + 2x - 3$$

$$f'(x) = 9x^8 + 2$$

$$f''(x) = 72x^7$$

Methods of Differentiation

Find the third derivative of

$$f(x) = x^5 + 2x^3 - 13$$

Methods of Differentiation

Find the third derivative of

$$f(x) = x^5 + 2x^3 - 13$$

$$f'(x) = \frac{df}{dx} = 5x^4 + 6x^2$$

$$f''(x) = \frac{d^2f}{dx^2} = 20x^3 + 12x$$

$$f'''(x) = \frac{d^3f}{dx^3} = 60x^2 + 12$$

Methods of Differentiation

Given the function $f(x) = x^4 + 12$

Find the value of $f''(1)$

Methods of Differentiation

Given the function $f(x) = x^4 + 12$

Find the value of $f''(1)$

$$f'(x) = 4x^3$$

$$f''(x) = 12x^2$$

$$f''(1) = 12(1)^2 = 12$$

Methods of Differentiation

Given the function

$$f(x) = \frac{3}{x^5}$$

Find the value of

$$f''(1)$$

Given the function $f(x) = \frac{3}{x^5}$

Find the value of $f''(1)$

$$f(x) = \frac{3}{x^5} = 3x^{-5}$$

$$f'(x) = -15x^{-6}$$

$$f''(x) = 90x^{-7} = \frac{90}{x^7}$$

$$f''(1) = \frac{90}{(1)^7} = 90$$

Methods of Differentiation

The distance s (in feet) covered by a particle moving along a straight track t seconds after starting from rest is given by the function $s = 4t^2$ ($0 \leq t \leq 10$)

What is the particle acceleration after 30 seconds?

Methods of Differentiation

- **The Velocity Function** $v(t)$

is the derivative of the **distance Function** $s(t)$

$$v(t) = \frac{d}{dt} s(t)$$

Methods of Differentiation

- **The acceleration Function** $a(t)$

is the derivative of the **Velocity Function** $v(t)$

$$a(t) = \frac{d}{dt}v(t)$$

Methods of Differentiation

The velocity of the particle t seconds from rest is given by

$$v = \frac{ds}{dt} = \frac{d}{dt}(4t^2) = 8t$$

Methods of Differentiation

The acceleration of the maglev t seconds from rest is given by the rate of change of the velocity of t , given by

$$a = \frac{d}{dt}v = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2} = \frac{d}{dt}(8t) = 8$$

or 8 feet per second per second (ft/sec²).

Methods of Differentiation

The distance s (in feet) covered by a particle moving along a straight track t seconds after starting from rest is given by the function $s = 10t^4$ ($0 \leq t \leq 10$)

What is the particle acceleration after 5 seconds?

Methods of Differentiation

$$s = 10t^4$$

$$v = \frac{ds}{dt} = 40t^3$$

$$a = \frac{d}{dt}v = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2} = 120t^2$$

$$a(5) = 120(5)^2 = 3000$$

Un Solved Examples

Suppose the relationship between the unit price p in dollars and the quantity demanded x of the model F loudspeaker system is given by the equation

$$p(x) = -0.01x^2 + 500x \quad (0 \leq x \leq 20,000)$$

$$C(x) = 100x + 200,000$$

- Find the revenue function R .
- Find the marginal revenue function R' .
- Compute $R'(1000)$ and interpret your result.

Un Solved Examples

$$P(x) = R(x) - C(x)$$

We find the Revenue function as

$$R(x) = P(x) + C(x)$$

$$R(x) = (-0.01x^2 + 500x) + (100x + 200000)$$

$$R(x) = -0.01x^2 + 600x + 200000$$

Un Solved Examples

$$R(x) = -0.01x^2 + 600x + 200000$$

$$R'(x) = -0.02x + 600$$

$$R'(1000) = -0.02(1000) + 600$$

$$R'(1000) = 580$$

Un Solved Examples

- Suppose the total cost in dollars incurred each week by company for manufacturing x refrigerators is given by the total cost function

- $$C(x) = 6000 + 200x - 0.1x^2 \quad (0 \leq x \leq 400)$$

Find the rate of change of the total cost function with respect to x when $x = 150$.

Un Solved Examples

$$C(x) = 6000 + 200x - 0.1x^2$$

$$C'(x) = 200 - 0.2x$$

$$C'(150) = 200 - 0.2(150)$$

$$C'(150) = 170$$

Un Solved Examples

Find the second derivative of

$$f(x) = x^4 + 2$$

Un Solved Examples

Find the second derivative of

$$f(x) = x^4 + 2$$

$$f'(x) = 4x^3$$

$$f''(x) = 12x^2$$

Un Solved Examples

Find the third derivative of

$$f(x) = x^7 - 5$$

Un Solved Examples

Find the third derivative of

$$f(x) = x^7 - 5$$

$$f'(x) = 7x^6$$

$$f''(x) = 42x^5$$

$$f'''(x) = 210x^4$$

Un Solved Examples

Find the second derivative of

$$f(x) = \sqrt{x}$$

Un Solved Examples

Find the second derivative of

$$f(x) = \sqrt{x}$$

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$$

$$f''(x) = -\frac{1}{4} x^{-\frac{3}{2}}$$

Un Solved Examples

Given the function $f(x) = x^3 + 2$

Find the value of $f''(1)$

Un Solved Examples

Given the function $f(x) = x^3 + 2$

Find the value of $f''(1)$

$$f(x) = x^3 + 2$$

$$f'(x) = 3x^2$$

$$f''(x) = 6x$$

$$f''(1) = 6(1) = 6$$

Un Solved Examples

Given the function $f(x) = 5x^2 + 7x - 6$

Find the value of $f'''(0)$

Un Solved Examples

Given the function $f(x) = 5x^2 + 7x - 6$

Find the value of $f'''(0)$

$$f'(x) = 10x + 7$$

$$f''(x) = 10$$

$$f'''(x) = 0$$

$$f'''(0) = 0$$

Un Solved Examples

Find the second derivative of

$$f(x) = \frac{4}{x^6}$$

Un Solved Examples

Find the second derivative of

$$f(x) = \frac{4}{x^6}$$

$$f(x) = 4x^{-6}$$

$$f'(x) = -24x^{-7}$$

$$f''(x) = 168x^{-8}$$

$$f''(x) = \frac{168}{x^8}$$

Un Solved Examples

Find the second derivative of

$$f(x) = \frac{2}{x^{-5}}$$

Un Solved Examples

Find the second derivative of

$$f(x) = \frac{2}{x^{-5}}$$

$$f(x) = 2x^5$$

$$f'(x) = 10x^4$$

$$f''(x) = 40x^3$$

Un Solved Examples

The distance s (in feet) covered by a particle moving along a straight track t seconds after starting from rest is given by the function $s = 5t^3$ ($0 \leq t \leq 10$)

What is the particle acceleration after 10 seconds?

Un Solved Examples

$$s = 5t^3$$

$$v = \frac{ds}{dt} = 15t^2$$

$$a = \frac{d}{dt}v = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2} = 30t$$

$$a(10) = 30(10) = 300$$



Thank You