

Probability and Statistics

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Lecture 7

جامعة الجميع الذكية

تسعى لتكون الكيان الأول الذي يتبنى مبدأ المجتمع التعليمي
التعاوني ويقدم العلم مقابل العمل لتنمية وبناء المجتمعات.

STA 201

Hypothesis Testing with One Sample



جامعة الجميع الذكية
EVERYONE'S SMART UNIVERSITY

7.1 Introduction to Hypothesis Testing

7.2 Hypothesis Testing for the Mean (Large Samples)

7.3 Hypothesis Testing for the Mean (Small Samples)

7.4 Hypothesis Testing for Proportions

7.1 Introduction to Hypothesis testing

Hypothesis test

A process that uses sample statistics to test a claim about the value of a population parameter.

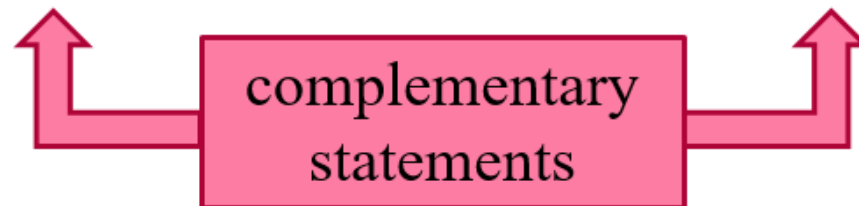
For example: An automobile manufacturer advertises that its new hybrid car has a mean mileage of 50 miles per gallon. To test this claim, a sample would be taken. If the sample mean differs enough from the advertised mean, you can decide the advertisement is wrong.

Null hypothesis

- A statistical hypothesis that contains a statement of equality such as $\leq$, $=$, or $\geq$.
- Denoted H_0 read “H subzero” or “H naught.”

Alternative hypothesis

- A statement of inequality such as $>$, $\neq$, or $<$.
- Must be true if H_0 is false.
- Denoted H_a read “H sub-a.”



Population Mean

First Case:

$$H_0: \mu =$$

$$H_a: \mu \neq$$

Second Case:

$$H_0: \mu \leq$$

$$H_a: \mu >$$

Third Case:

$$H_0: \mu \geq$$

$$H_a: \mu <$$

Population Proportion

First Case:

$$H_0: p =$$

$$H_a: p \neq$$

Second Case:

$$H_0: p \leq$$

$$H_a: p >$$

Third Case:

$$H_0: p \geq$$

$$H_a: p <$$

Write the claim as a mathematical sentence. State the null and alternative hypotheses and identify which represents the claim.

1. A university publicizes that the proportion of its students who graduate in 4 years is 82%.

Solution:

$$H_0: p = 0.82 \longleftarrow \text{Equality condition (Claim)}$$

$$H_a: p \neq 0.82 \longleftarrow \text{Complement of } H_0$$



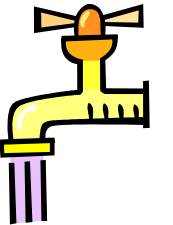
Write the claim as a mathematical sentence. State the null and alternative hypotheses and identify which represents the claim.

2. A water faucet manufacturer announces that the mean flow rate of a certain type of faucet is less than 2.5 gallons per minute.

Solution:

$H_0: \mu \geq 2.5$ gallons per minute ← Complement of H_a

$H_a: \mu < 2.5$ gallons per minute ← Inequality condition (Claim)



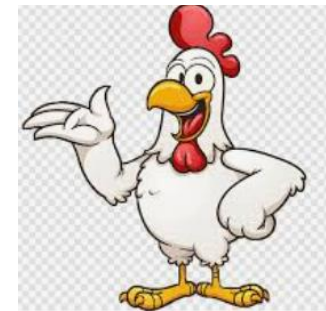
Decision	Actual Truth of H_0	
	H_0 is true	H_0 is false
Do not reject H_0	Correct Decision	Type II Error
Reject H_0	Type I Error	Correct Decision

- A **type I error** occurs if the null hypothesis is rejected when it is true.
- A **type II error** occurs if the null hypothesis is not rejected when it is false.

Decision	Actual Truth of H_0	
	H_0 is true	H_0 is false
Do not reject H_0	Correct Decision	Type II Error
Reject H_0	Type I Error	Correct Decision

- A **type I error** occurs if the null hypothesis is rejected when it is true.
- A **type II error** occurs if the null hypothesis is not rejected when it is false.

The USDA limit for salmonella contamination for chicken is 20%. A meat inspector reports that the chicken produced by a company exceeds the USDA limit. You perform a hypothesis test to determine whether the meat inspector's claim is true. When will a type I or type II error occur? Which is more serious? *(Source: United States Department of Agriculture)*

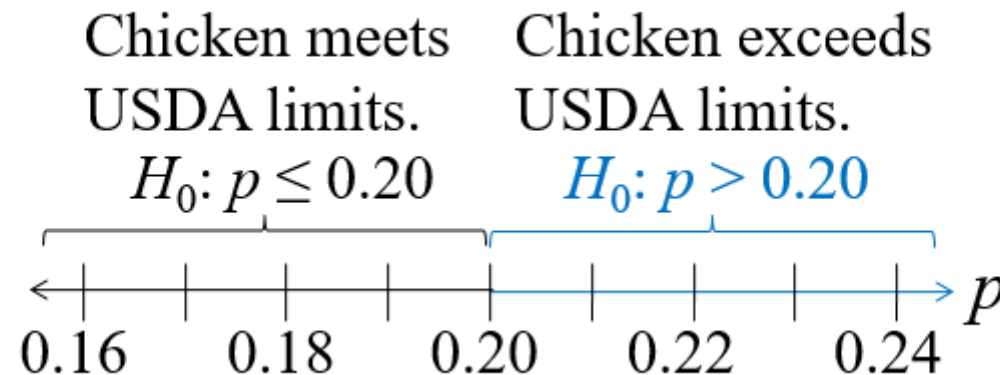


Solution: Identifying Type I and Type II Errors

Let p represent the proportion of chicken that is contaminated.

Hypotheses: $H_0: p \leq 0.2$

$H_a: p > 0.2$ (Claim)



Hypotheses: $H_0: p \leq 0.2$

$H_a: p > 0.2$ (Claim)

A type I error is rejecting H_0 when it is true.

The actual proportion of contaminated chicken is less than or equal to 0.2, but you decide to reject H_0 .

A type II error is failing to reject H_0 when it is false.

The actual proportion of contaminated chicken is greater than 0.2, but you do not reject H_0 .

Hypotheses: $H_0: p \leq 0.2$

$H_a: p > 0.2$ (Claim)

- With a type I error, you might create a health scare and hurt the sales of chicken producers who were actually meeting the USDA limits.
- With a type II error, you could be allowing chicken that exceeded the USDA contamination limit to be sold to consumers.
- A type II error could result in sickness or even death.

Level of significance

Your maximum allowable probability of making a type I error.
Denoted by α , the lowercase Greek letter alpha.

By setting the level of significance at a small value, you are saying that you want the probability of rejecting a true null hypothesis to be small.

Commonly used levels of significance:

$$\alpha = 0.10 \quad \alpha = 0.05 \quad \alpha = 0.01$$

$$P(\text{type II error}) = \beta \text{ (beta)}$$

Taking decision by Traditional way - First way

1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.

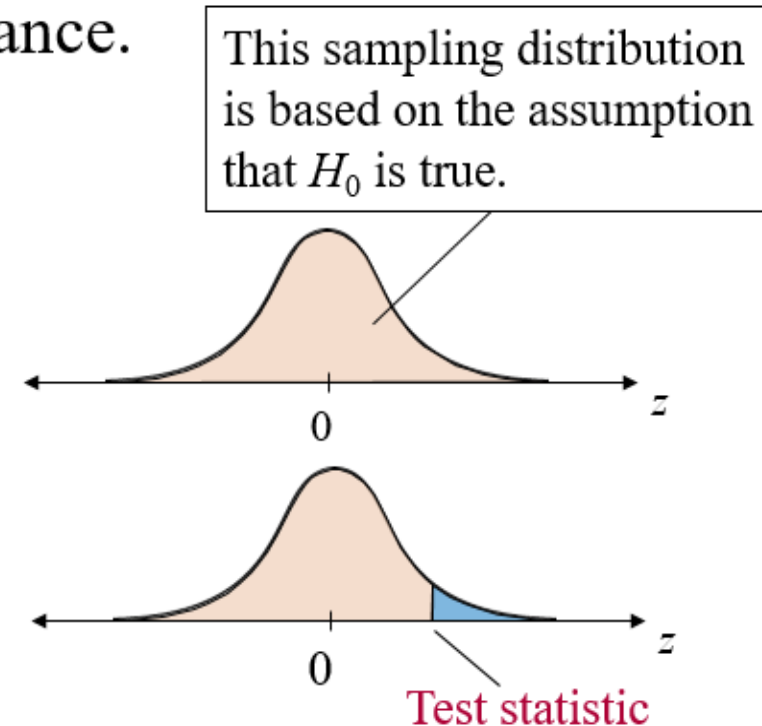
$$H_0: ? \quad H_a: ?$$

2. Specify the level of significance.

$$\alpha = ?$$

3. Determine the standardized sampling distribution and draw its graph.

4. Calculate the test statistic and its standardized value. Add it to your sketch.



P-value (or probability value)

The probability, if the null hypothesis is true, of obtaining a sample statistic with a value as extreme or more extreme than the one determined from the sample data.

Depends on the nature of the test.

Decision Rule Based on P -value

- Compare the P -value with α .
 - If $P \leq \alpha$, then reject H_0 .
 - If $P > \alpha$, then fail to reject H_0 .

Decision	Claim	
	Claim is H_0	Claim is H_a
Reject H_0	There is enough evidence to reject the claim	There is enough evidence to support the claim
Fail to reject H_0	There is not enough evidence to reject the claim	There is not enough evidence to support the claim

Example: Interpreting a Decision

You perform a hypothesis test for the following claim.
How should you interpret your decision if you reject H_0 ? If you fail to reject H_0 ?

2. H_a (Claim): *Consumer Reports* states that the mean stopping distance (on a dry surface) for a Honda Civic is less than 136 feet.

Solution:

- The claim is represented by H_a .
- H_0 is “the mean stopping distance...is greater than or equal to 136 feet.”



- If you reject H_0 you should conclude “there is enough evidence to support *Consumer Reports*’ claim that the stopping distance for a Honda Civic is less than 136 feet.”
- If you fail to reject H_0 , you should conclude “there is not enough evidence to support *Consumer Reports*’ claim that the stopping distance for a Honda Civic is less than 136 feet.”

7.2 Hypothesis Testing for the Mean (Large Samples)

In Words

State the claim mathematically and verbally. Identify the null and alternative hypotheses.

Specify the level of significance.

Determine the critical value(s).

Find the standardized test statistic.

In Symbols

State H_0 and H_a .

Identify .

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \quad \text{if } n \geq 30$$

When $n \geq 30$, the sample standard deviation s can be substituted for σ .

Population Mean

First Case:

$$H_0: \mu =$$
$$H_a: \mu \neq$$

Second Case:

$$H_0: \mu \leq$$
$$H_a: \mu >$$

Third Case:

$$H_0: \mu \geq$$
$$H_a: \mu <$$

Population Proportion

First Case:

$$H_0: p =$$
$$H_a: p \neq$$

Second Case:

$$H_0: p \leq$$
$$H_a: p >$$

Third Case:

$$H_0: p \geq$$
$$H_a: p <$$

 **Two Tail Test**

 **Right Tail Test**

 **Left Tail Test**

Employees in a large accounting firm claim that the mean salary of the firm's accountants is less than that of its competitor's, which is \$45,000. A random sample of 30 of the firm's accountants has a mean salary of \$43,500 with a standard deviation of \$5200. At $\alpha = 0.05$, test the employees' claim.

Note: Critical value = -1.645

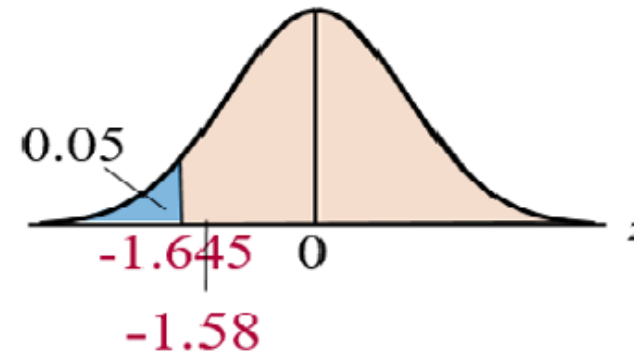


- $H_0: \mu \geq \$45,000$
- $H_a: \mu < \$45,000$
- $\alpha = 0.05$
- critical value = -1.645

- **Test Statistic**

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{43,500 - 45,000}{5200/\sqrt{30}} = -1.58$$

- **Rejection Region:**



- **Decision: Fail to reject H_0**

At the 5% level of significance, there is not sufficient evidence to support the employees' claim that the mean salary is less than \$45,000.

Decision Rule Based on P -value

To use a P -value to make a conclusion in a hypothesis test, compare the P -value with α .

1. If $P \leq \alpha$, then reject H_0 .
2. If $P > \alpha$, then fail to reject H_0 .

In an advertisement, a pizza shop claims that its mean delivery time is less than 30 minutes. A random selection of 36 delivery times has a sample mean of 28.5 minutes and a standard deviation of 3.5 minutes. Is there enough evidence to support the claim at $\alpha = 0.01$? Use a P -value. $P\text{-value} = 0.0051$



- $H_0: \mu \geq 30 \text{ min}$
- $H_a: \mu < 30 \text{ min}$
- $\alpha = 0.01$
- $P\text{-value} = 0.0051$
- **Decision:** $0.0051 < 0.01$

Reject H_0

At the 1% level of significance, you have sufficient evidence to conclude the mean delivery time is less than 30 minutes.

7.3 Hypothesis Testing for the Mean (Small Samples)

In Words

State the claim mathematically and verbally. Identify the null and alternative hypotheses.

Specify the level of significance and the degrees of freedom

Determine any critical value(s).

Find the standardized test statistic.

In Symbols

State H_0 and H_a .

Identify α .
d.f. = $n - 1$.

Use Table T distribution

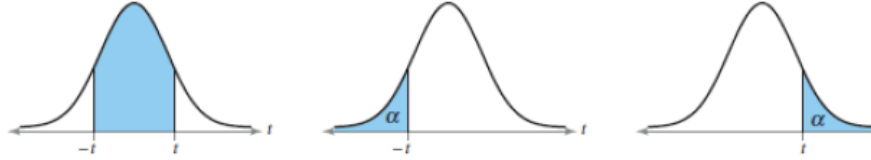
$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Find the critical value t_0 for a left-tailed test given $\alpha = 0.05$ and $n = 21$.

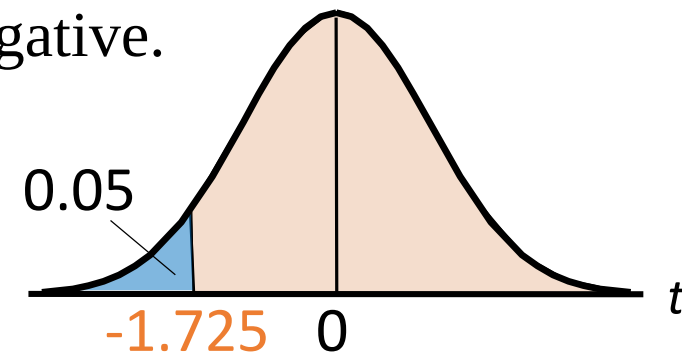
Solution:

- The degrees of freedom are $d.f. = n - 1 = 21 - 1 = 20$.
- Look at $\alpha = 0.05$ in the “One Tail, α ” column.
- Because the test is left-tailed, the critical value is negative.

Table 5 — t -Distribution



	Level of confidence, c					
	0.50	0.80	0.90	0.95	0.98	0.99
One tail, α	0.25	0.10	0.05	0.025	0.01	0.005
Two tails, α	0.50	0.20	0.10	0.05	0.02	0.01
d.f.						
1	1.000	3.078	6.314	12.706	31.821	63.657
2	.816	1.886	2.920	4.303	6.965	9.925
3	.765	1.638	2.353	3.182	4.541	5.841
4	.741	1.533	2.132	2.776	3.747	4.604
5	.727	1.476	2.015	2.571	3.365	4.032
18	.688	1.330	1.734	2.101	2.552	2.878
19	.688	1.328	1.729	2.093	2.539	2.861
20	.687	1.325	1.725	2.086	2.528	2.845
21	.686	1.323	1.721	2.080	2.518	2.831
22	.686	1.321	1.717	2.074	2.508	2.819
23	.685	1.319	1.714	2.069	2.500	2.807



Example: Testing μ with a Small Sample – First Way

A used car dealer says that the mean price of a 2005 Honda Pilot LX is at least \$23,900. You suspect this claim is incorrect and find that a random sample of 14 similar vehicles has a mean price of \$23,000 and a standard deviation of \$1113. Is there enough evidence to reject the dealer's claim at $\alpha = 0.05$? Assume the population is normally distributed. *(Adapted from Kelley Blue Book)*

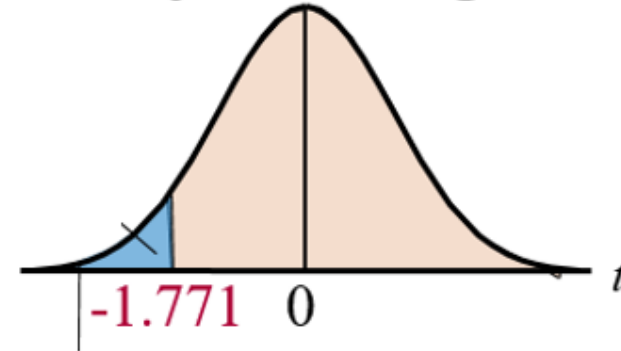


Solution: Testing μ with a Small Sample

- $H_0: \mu \geq \$23,900$
- $H_a: \mu < \$23,900$
- $\alpha = 0.05$
- $df = 14 - 1 = 13$
- critical value = -1.771
- **Test Statistic:**

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{23,000 - 23,900}{1113/\sqrt{14}} \approx -3.026$$

- **Rejection Region:**



-3.026

- **Decision: Reject H_0**
At the 0.05 level of significance, there is enough evidence to reject the claim that the mean price of a 2005 Honda Pilot LX is at least \$23,900

7.4 Hypothesis Testing for the Proportion p

Verify that $np \geq 5$ and $nq \geq 5$

In Words

State the claim mathematically and verbally. Identify the null and alternative hypotheses.

Specify the level of significance.

Determine any critical value(s).

Find the standardized test statistic.

In Symbols

State H_0 and H_a .

Identify α .

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

Zogby International claims that 45% of people in the United States support making cigarettes illegal within the next 5 to 10 years. You decide to test this claim and ask a random sample of 200 people in the United States whether they support making cigarettes illegal within the next 5 to 10 years. Of the 200 people, 49% support this law. At $\alpha = 0.05$ is there enough evidence to reject the claim?

Note: Critical values = -1.96 and 1.96

Solution:

- Verify that $np \geq 5$ and $nq \geq 5$.

$$np = 200(0.45) = 90 \text{ and } nq = 200(0.55) = 110$$

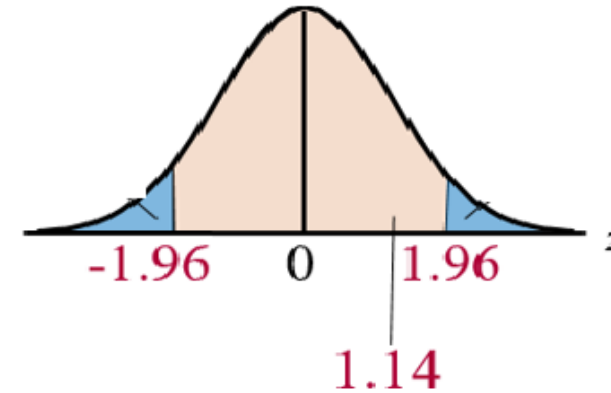


- $H_0: p = 0.45$
- $H_a: p \neq 0.45$
- $\alpha = 0.05$
- critical value = -1.96 and 1.96

- **Test Statistic**

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.49 - 0.45}{\sqrt{(0.45)(0.55)/200}} \approx 1.14$$

- **Rejection Region:**



- **Decision: Fail to reject H_0**

At the 5% level of significance, there is not enough evidence to reject the claim that 45% of people in the U.S. support making cigarettes illegal within the next 5 to 10 years.

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7.4 Hypothesis Testing for Proportions