

The Straight Line

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Lecture 02



جامعة الجميع الذكية
EVERYONE'S SMART UNIVERSITY

MTH 112

Business Mathematics

- **At the end of this Lecture you should be able to:**
 - Define the slope and intercept of a straight line and plot its graph
 - Understand what 'the equation of a line' means
 - Plot any straight line given the equation in the form $y = mx + c$ or $ax + by + d = 0$.
 - Introducing Excel

□ The straight line

The straight line is one of the simplest mathematical functions: it is easy to manipulate and to graph. It is, therefore, essential to fully understand the straight line, what each part of the equation means and the effect of changes in each part on the graph of the line.

□ The straight line

Figure 2.2 shows the horizontal axis, x , and the vertical axis, y which intersect at the origin where $x = 0$ and $y = 0$. Any point can be plotted on this graph if the coordinates, (x, y) . The points $(2, 2)$, $(1, 1)$, $(2, -2)$, $(-4, -3)$, $(-3, 2)$ and $(0, 0)$ are plotted in **Figure 2.2**.

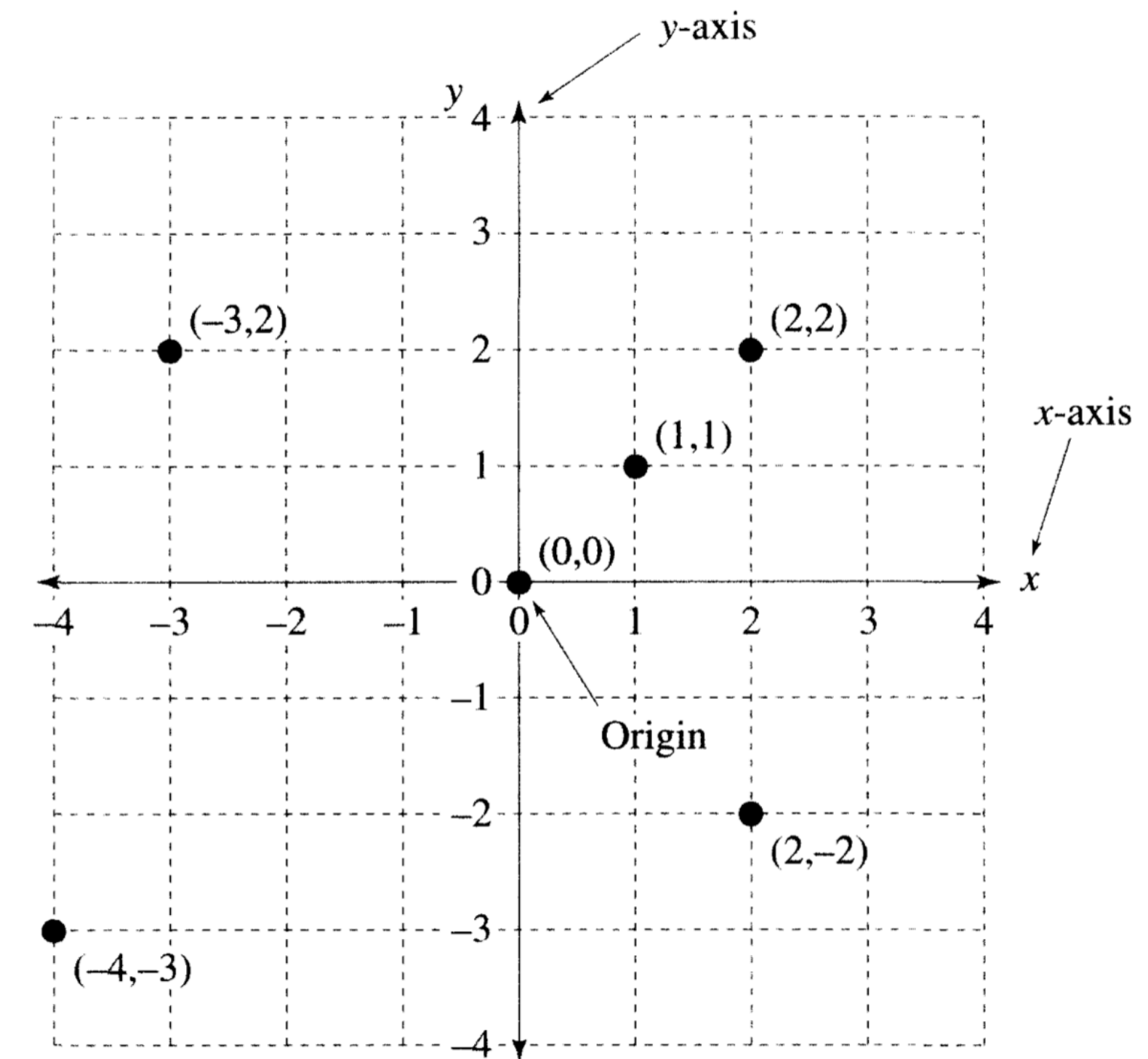


Figure 2.2 Plotting points on a graph

□ How to define a straight line

A straight line may be defined by two properties:

- **Slope**, usually represented by the symbol m .
- **Vertical intercept**, the point at which the line crosses the y -axis, usually represented by the symbol c .

Note: The vertical or y -intercept is often simply called ‘the intercept’. This is the convention adopted throughout this text.

Slope

- The slope of a line is simply the 'slant' of the line.
- The slope is negative if the line is falling from left to right.
- The slope is positive if the line is rising from left to right. In **Figure 2.3**, each line has a different slope and a different intercept. Note the horizontal line which has a zero slope and the vertical line which has an infinite slope.

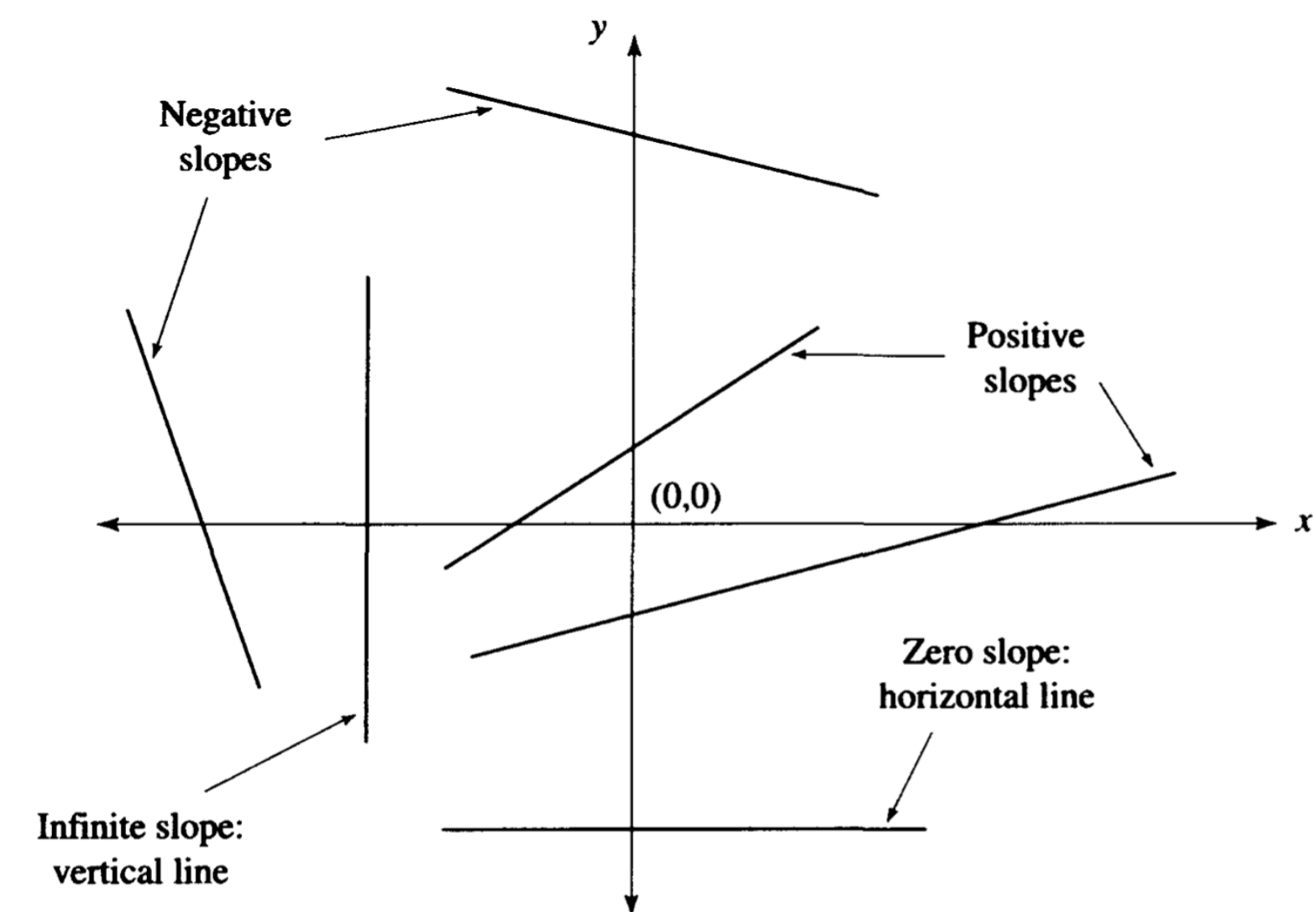


Figure 2.3 Lines with different slopes and different intercepts

□ Measuring the slope of a line

Figure 2.4 shows how the slope of a line is measured.

The line through the points A and B has a positive slope since it is rising from left to right.

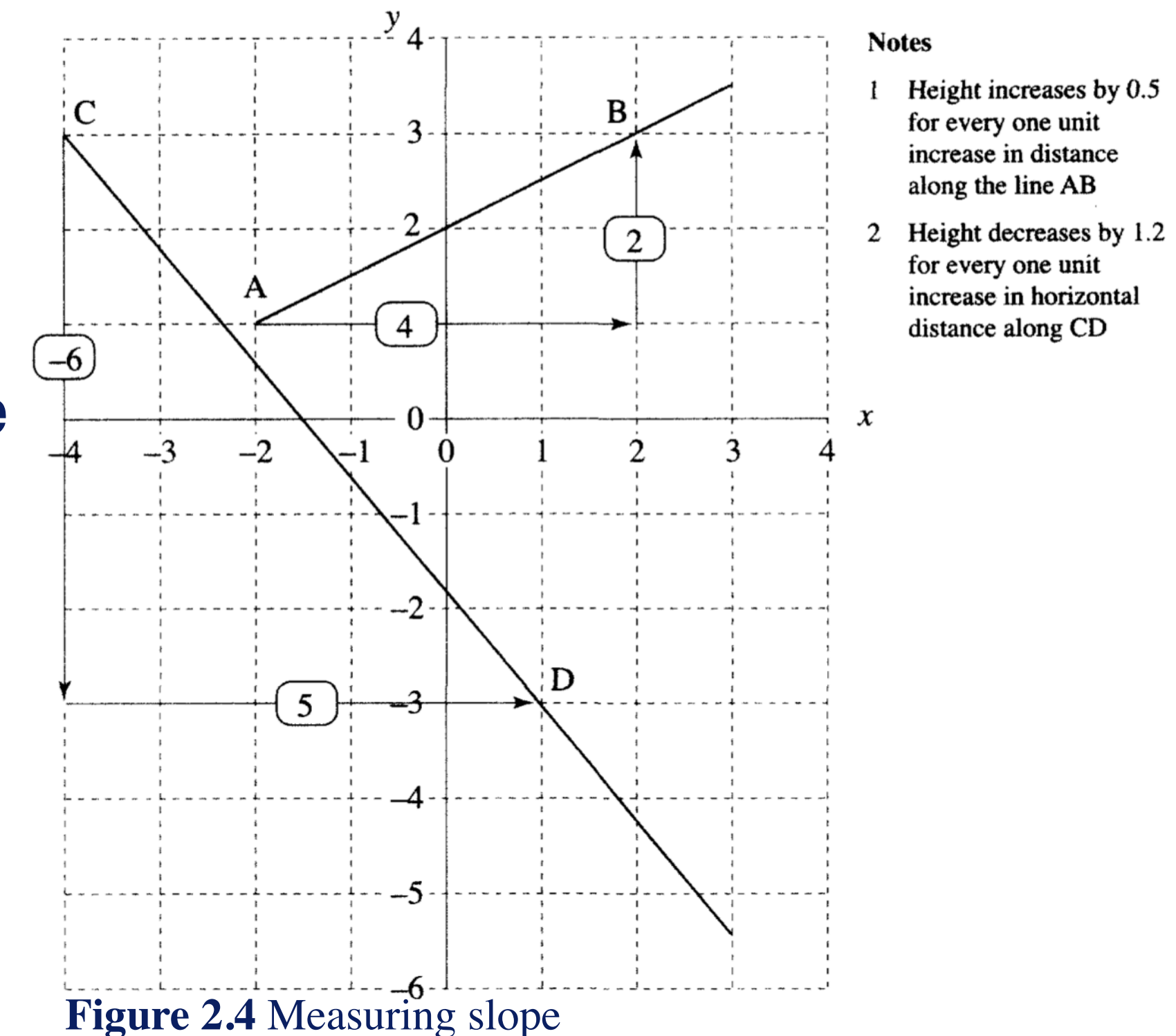


Figure 2.4 Measuring slope

□ Measuring the slope of a line

The slope is measured as follows:

$$\text{Slope} = \frac{\text{Change in height}}{\text{Change in distance}} = \frac{2}{4} = \frac{1}{2} = \frac{0.5}{1} = 0.5$$

That is, height increases by 0.5 units when the horizontal distance increases by 1 unit.

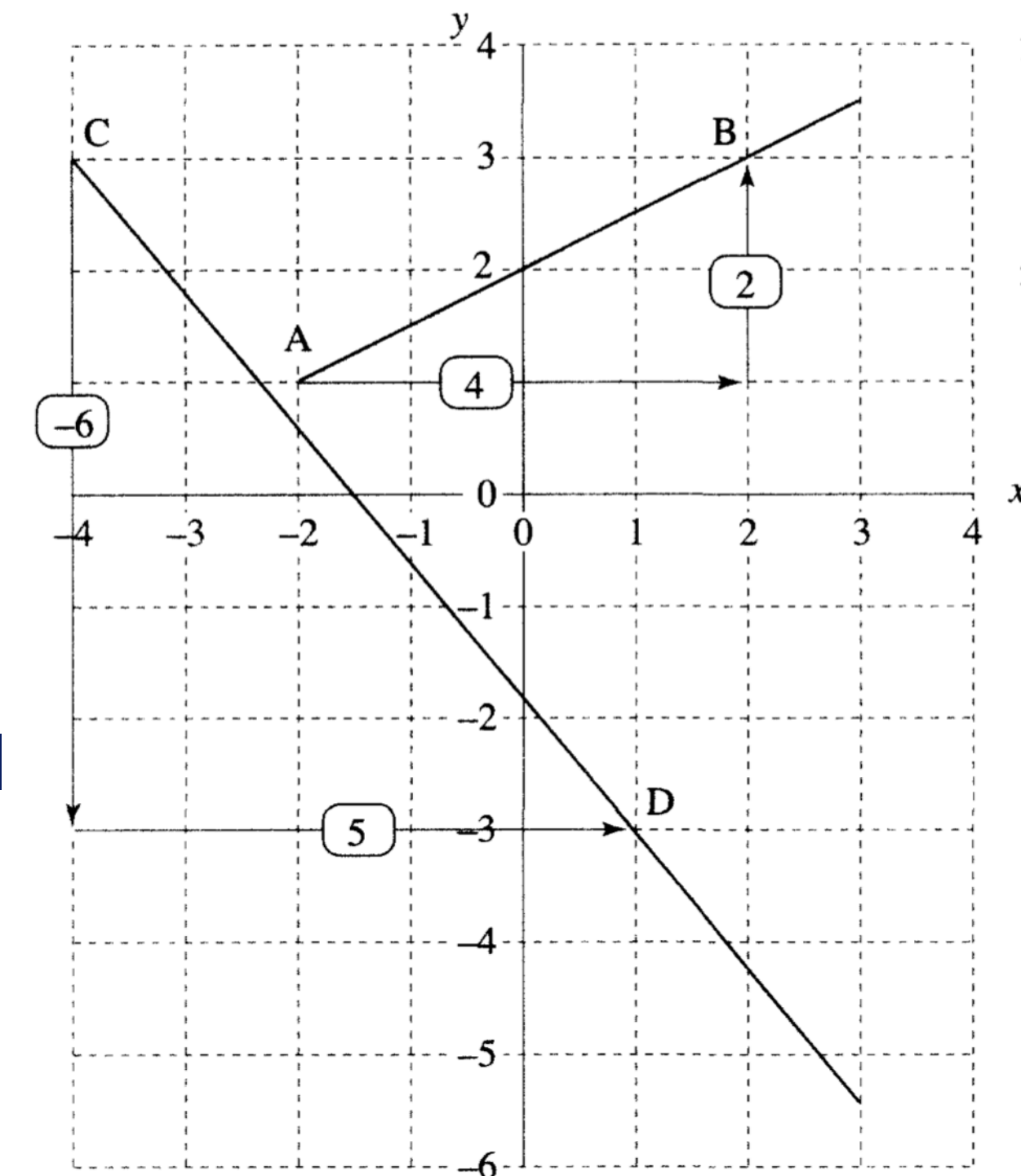


Figure 2.4 Measuring slope

Notes

- 1 Height increases by 0.5 for every one unit increase in distance along the line AB
- 2 Height decreases by 1.2 for every one unit increase in horizontal distance along CD

□ **The slope of a line may be described in several ways:**

1. The change in height (Δy) divided by the corresponding increase in horizontal distance (Δx):

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

2. The change in height per unit increase in horizontal distance

3. The number of units by which y changes when x increases by 1 unit.

□ Plotting a line

Ex: Plotting lines given slope and intercept

Plot the lines given the following information on slope and intercept:

(a) Slope = 1, intercept = 0 ($m = 1, c = 0$)

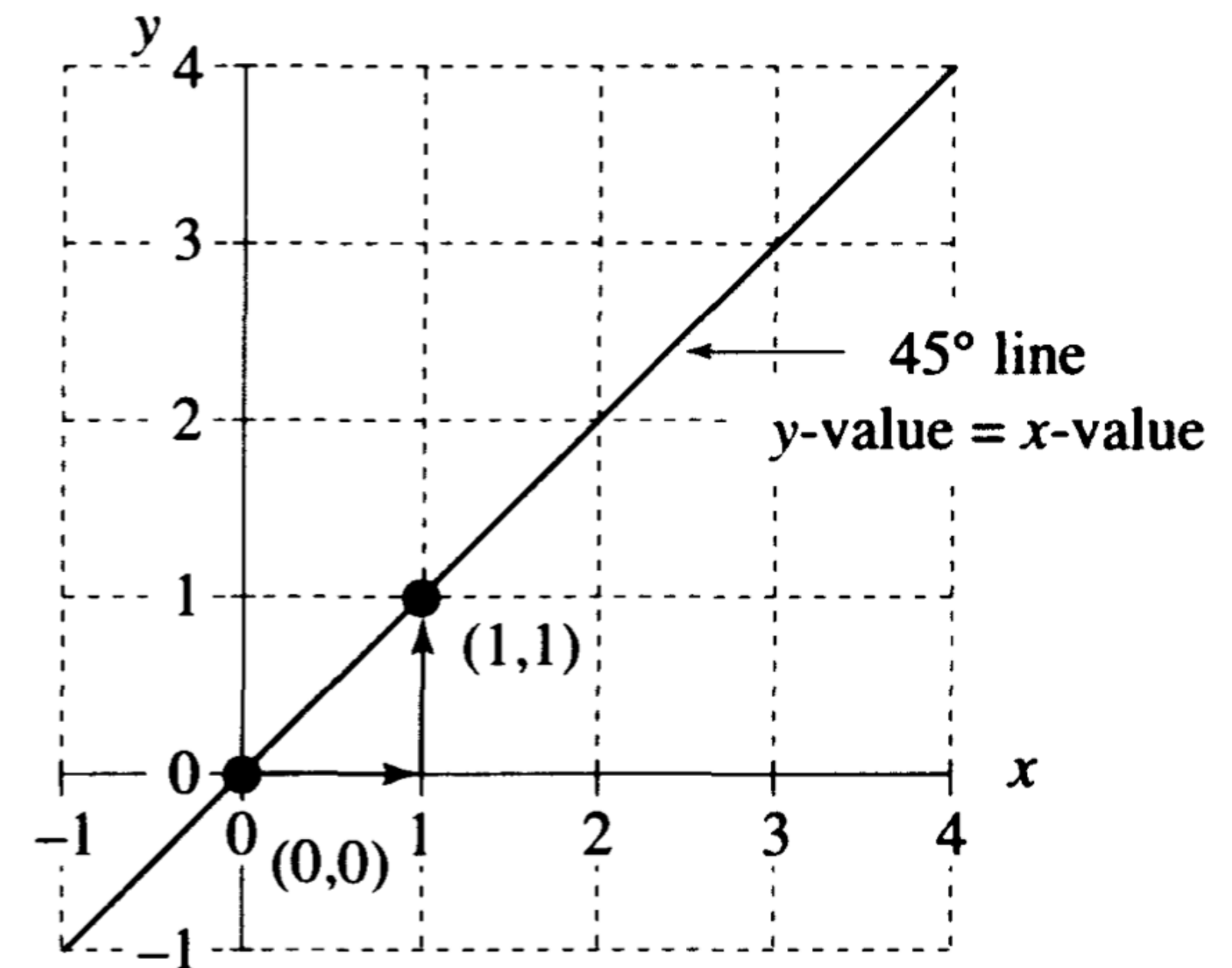


Figure 2.5 45° line through the origin

□ Plotting a line

Solution (a):

- Locate the vertical intercept, $y = 0$ on the y -axis. This gives the point $(0,0)$ which is the origin.
- Since slope is 1 and, $\text{slope} = \Delta y / \Delta x$, start at the origin (intercept) and move forward a horizontal distance of 1 unit and move up a vertical height of 1 unit; see Figure 2.5. This gives the point $(1,1)$.

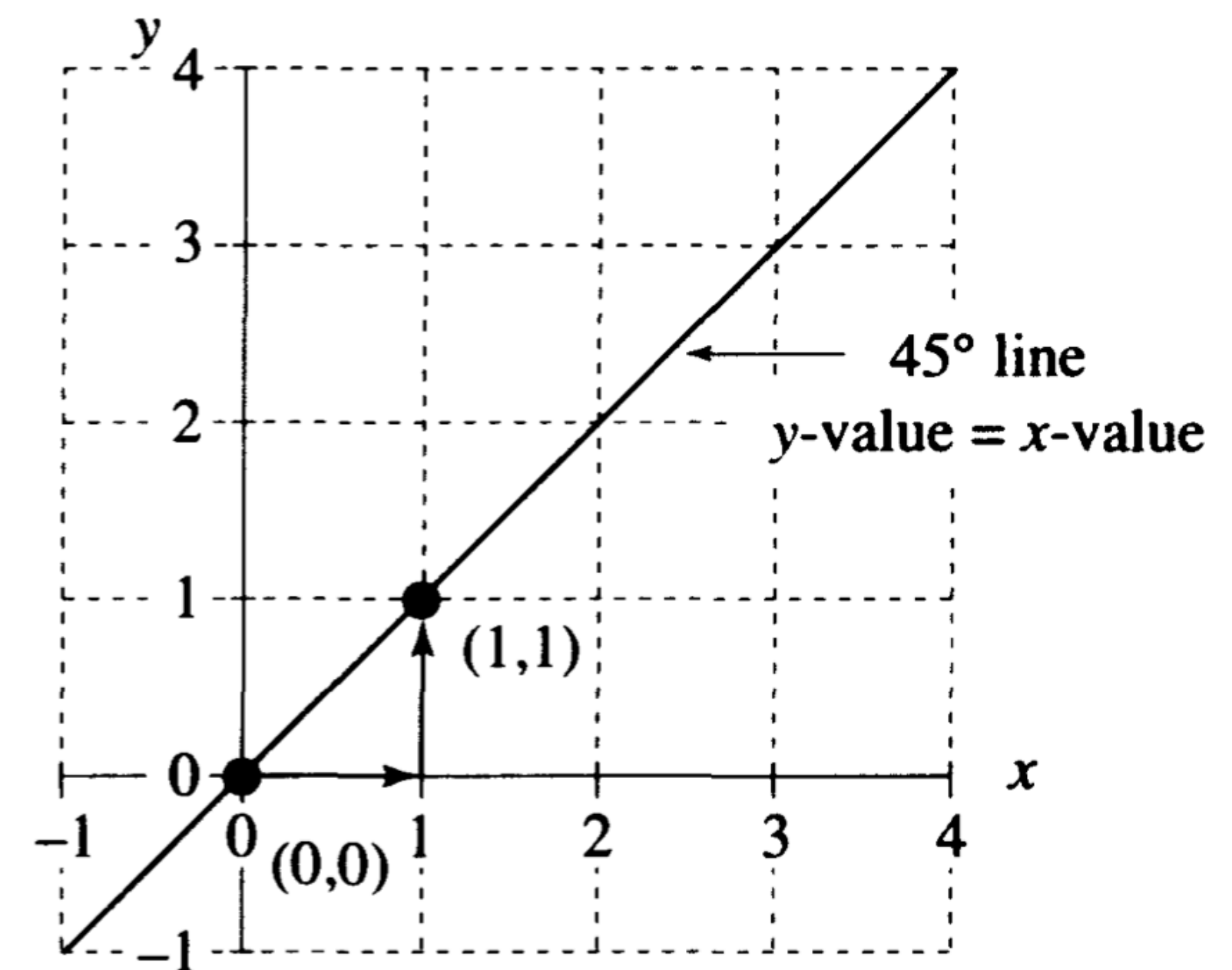


Figure 2.5 45° line through the origin

□ Plotting a line

Solution (a):

- Join the points $(0,0)$ and $(1,1)$; continue the straight line until it is the required length.
- This line is often referred to as the 45° line: a line through the origin which makes an angle of 45° with the horizontal axis (use for simplicity).
- Any change in the horizontal distance is accompanied by an equal change in the vertical height.

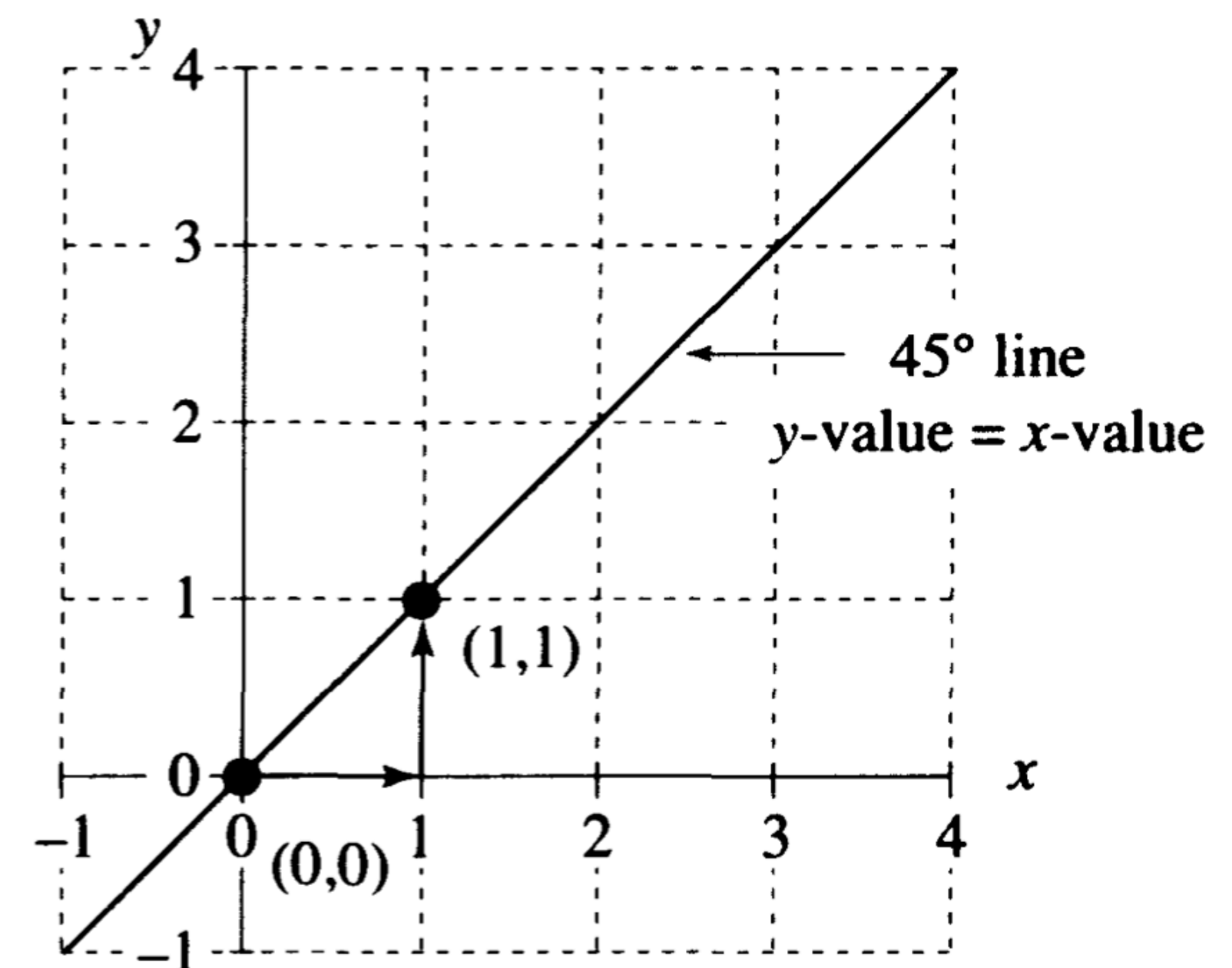


Figure 2.5 45° line through the origin

□ Plotting a line

Ex: (Plotting lines given slope and intercept)

Plot the lines given the following information on slope and intercept:

(a) Slope = 1, intercept = 2 ($m = 1$, $c = 2$)

Solution: Left as a HW.

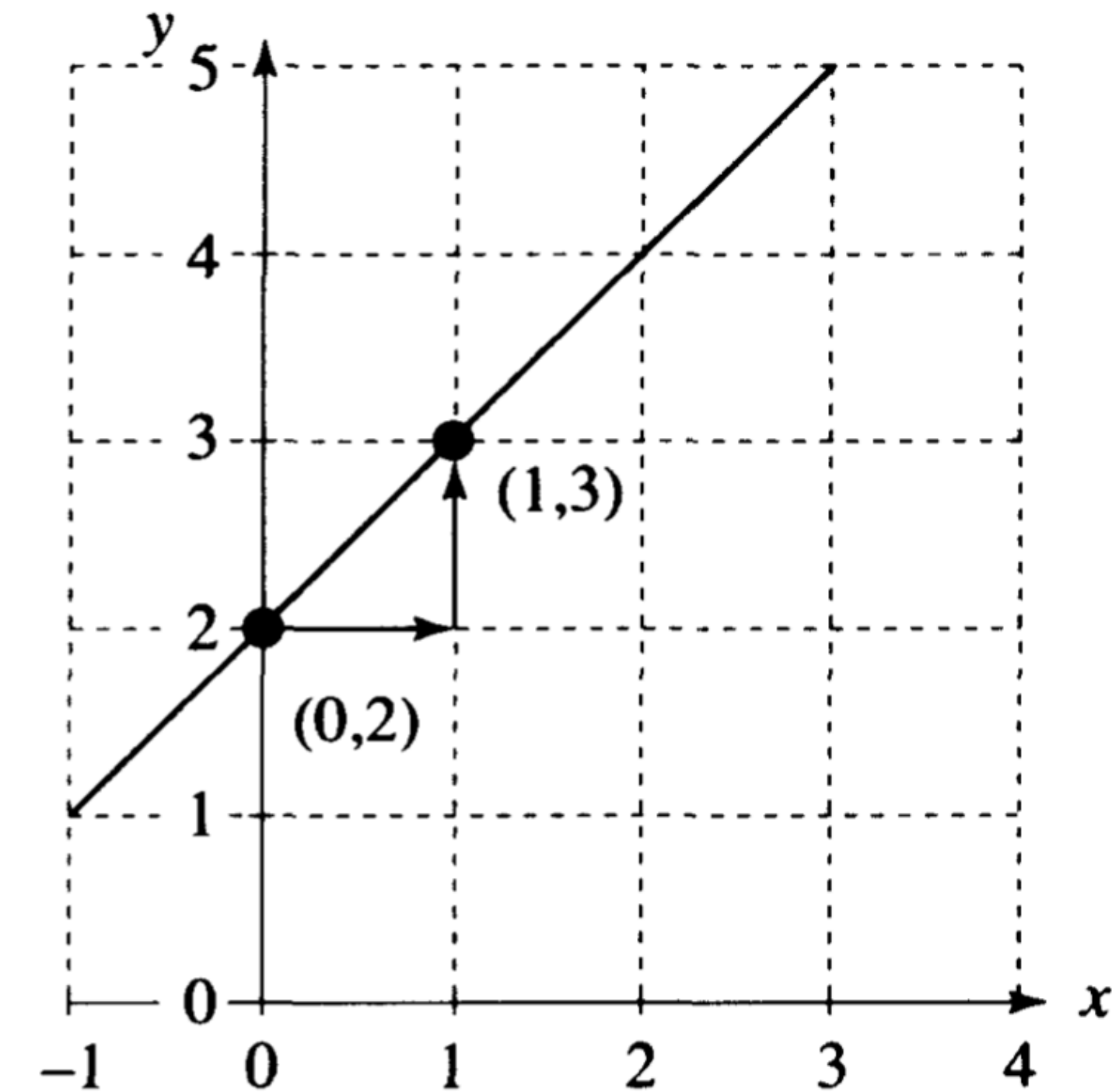


Figure 2.6: Line with slope = 1, intercept = 2

□ The equation of a line:

The equation of a line is simply a formula which shows how the y —coordinate is related to the x —coordinate for every point on the line. The equation of a line is simply a formula which shows how the y —coordinate is related to the x —coordinate for every point on the line.

□ The equation of a line:

The general form of the equation of a line

$$y = mx + c$$

where

- m is the symbol representing slope.
- c is the symbol representing the intercept on the y -axis.

□ The equation of a line:

Ex: (Determine the equation of a line given slope and intercept)

(a) Demonstrate that the equation of the line with slope $m = 3$ and intercept $c = 1$ is:

$$y = 1 + 3x.$$

Hence deduce that the equation of any line is:

$$y = mx + c.$$

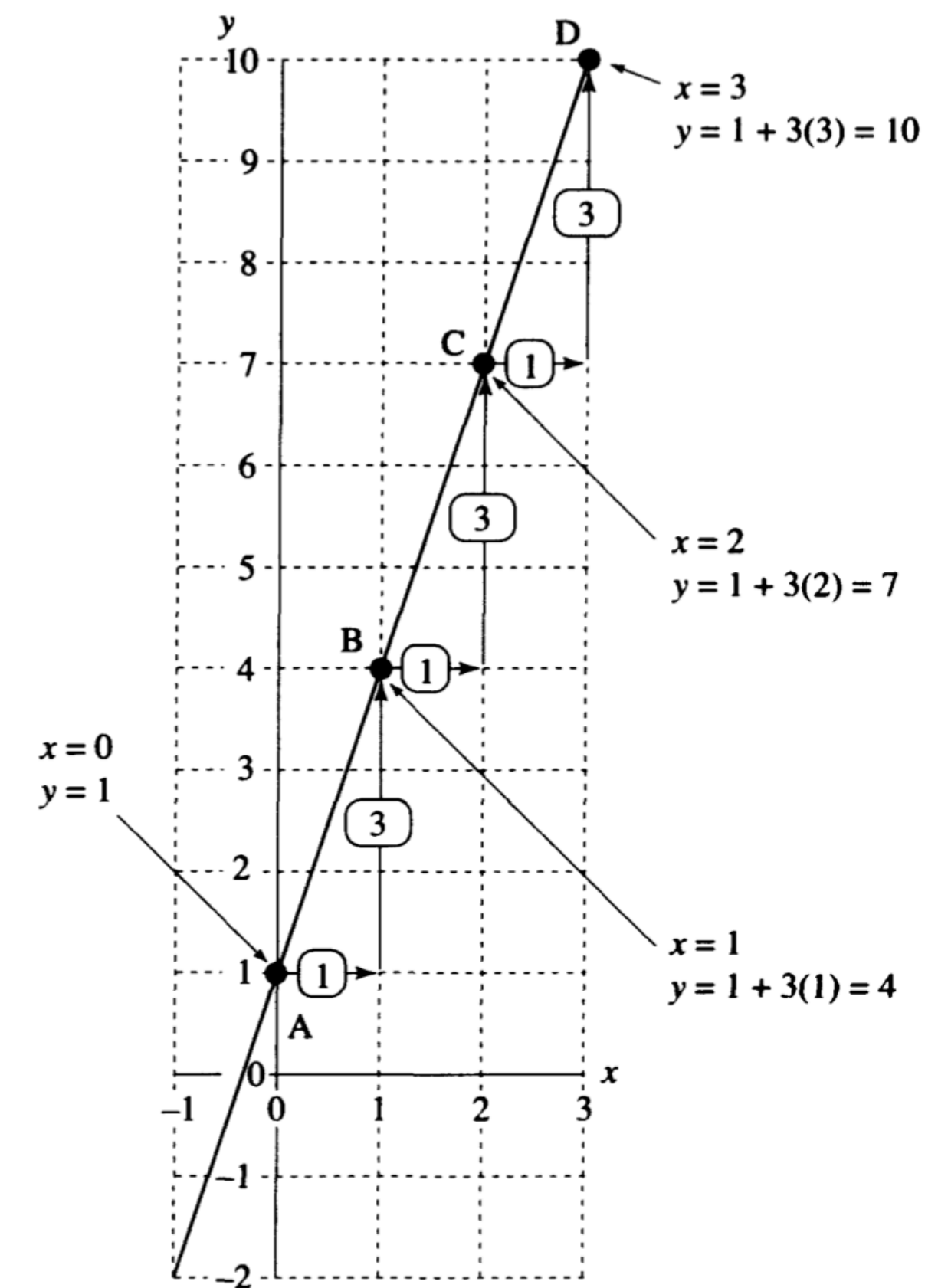


Figure 2.7: Deducing the equation of the line $y = 1 + 3x$.

□ The equation of a line:

Solution:

(a) See **Figure 2.7**. The line is drawn starting at the intercept $c = 1$ on the y -axis. that is at the point $x = 0, y = 1$.

Then, for each subsequent unit increase in x , y increases by 3 units since slope $m = 3$.

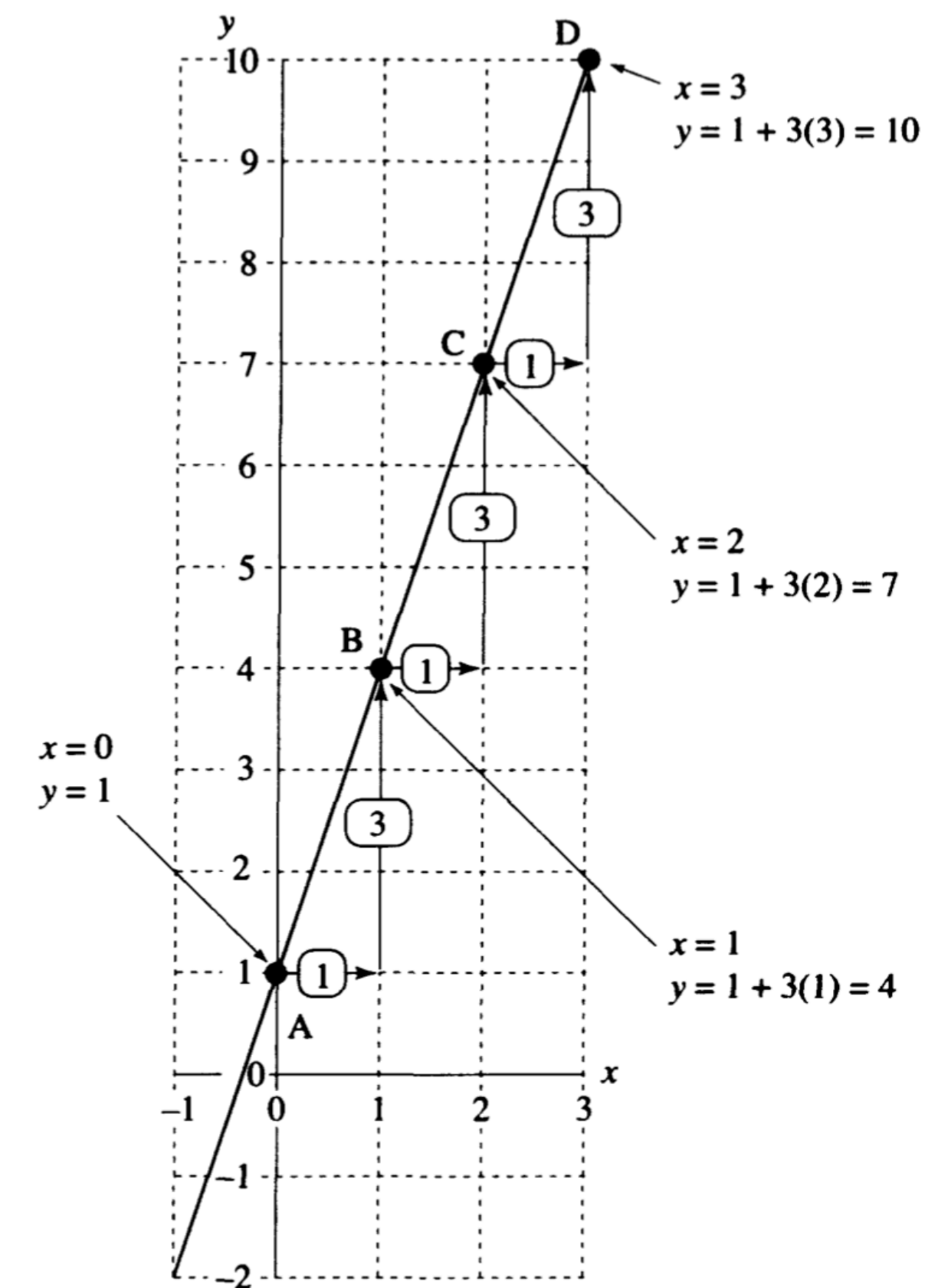


Figure 2.7: Deducing the equation of the line $y = 1 + 3x$.

□ The equation of a line:

Solution:

(a) Now observe the pattern relating the x and y values for selected points on the line.

x-value	y-value	Calculation of y-value
$x = 0$	$y = 1$	this is the intercept
$x = 1$	$y = 4$	$y = 1 + 3x$ (x -value)
$x = 2$	$y = 7$	$y = 1 + 3x$ (x -value)
$x = 3$	$y = 10$	$y = 1 + 3x$ (x -value)
any -value	$y = 1 + 3x$	$y = 1 + 3x$ (x -value)

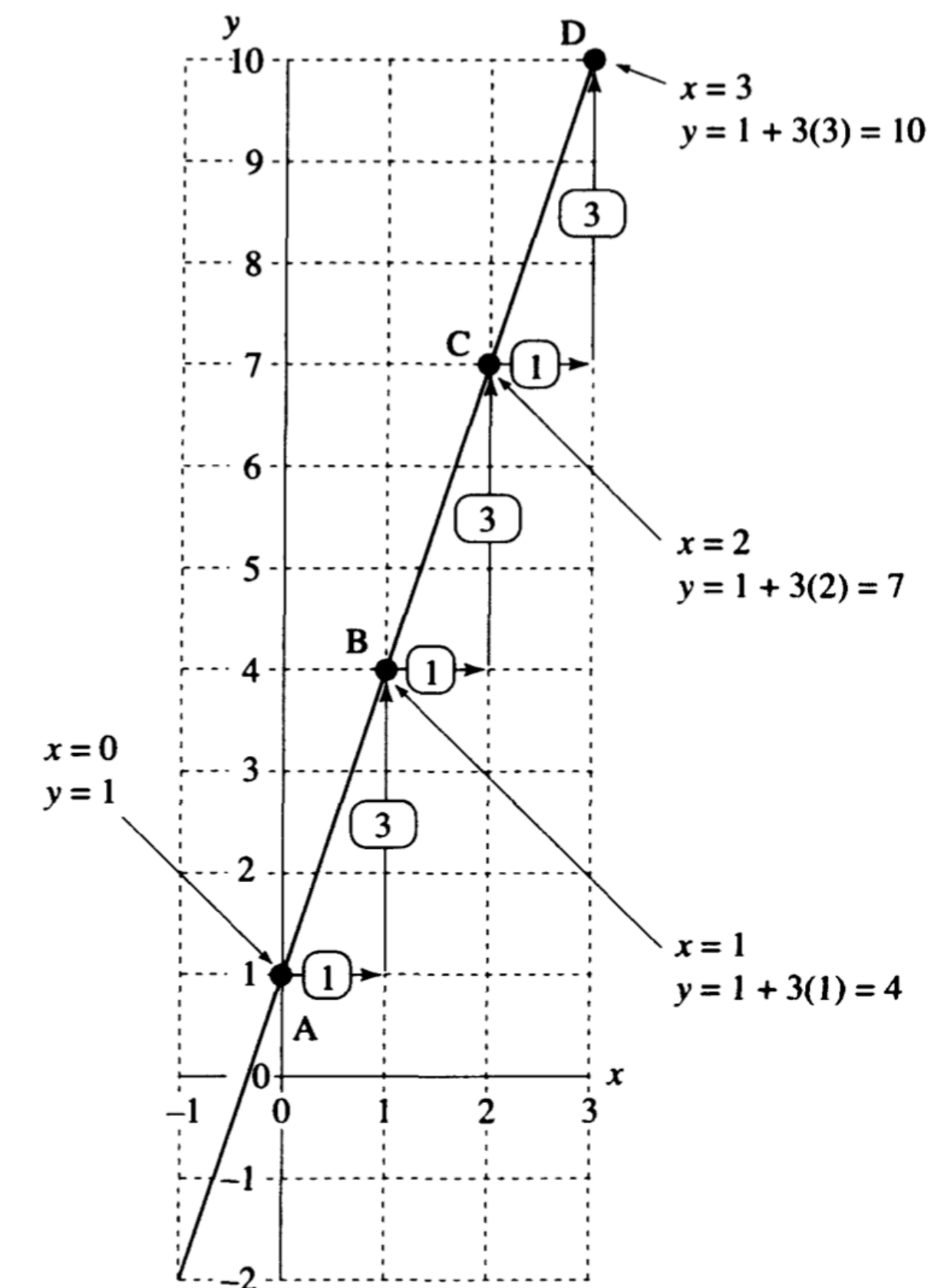


Figure 2.7: Deducing the equation of the line $y = 1 + 3x$.

□ The equation of a line:

Solution: (a)

$y = 1 + 3x$ is the equation of the line

Note: slope is the change in y per unit increases in x .

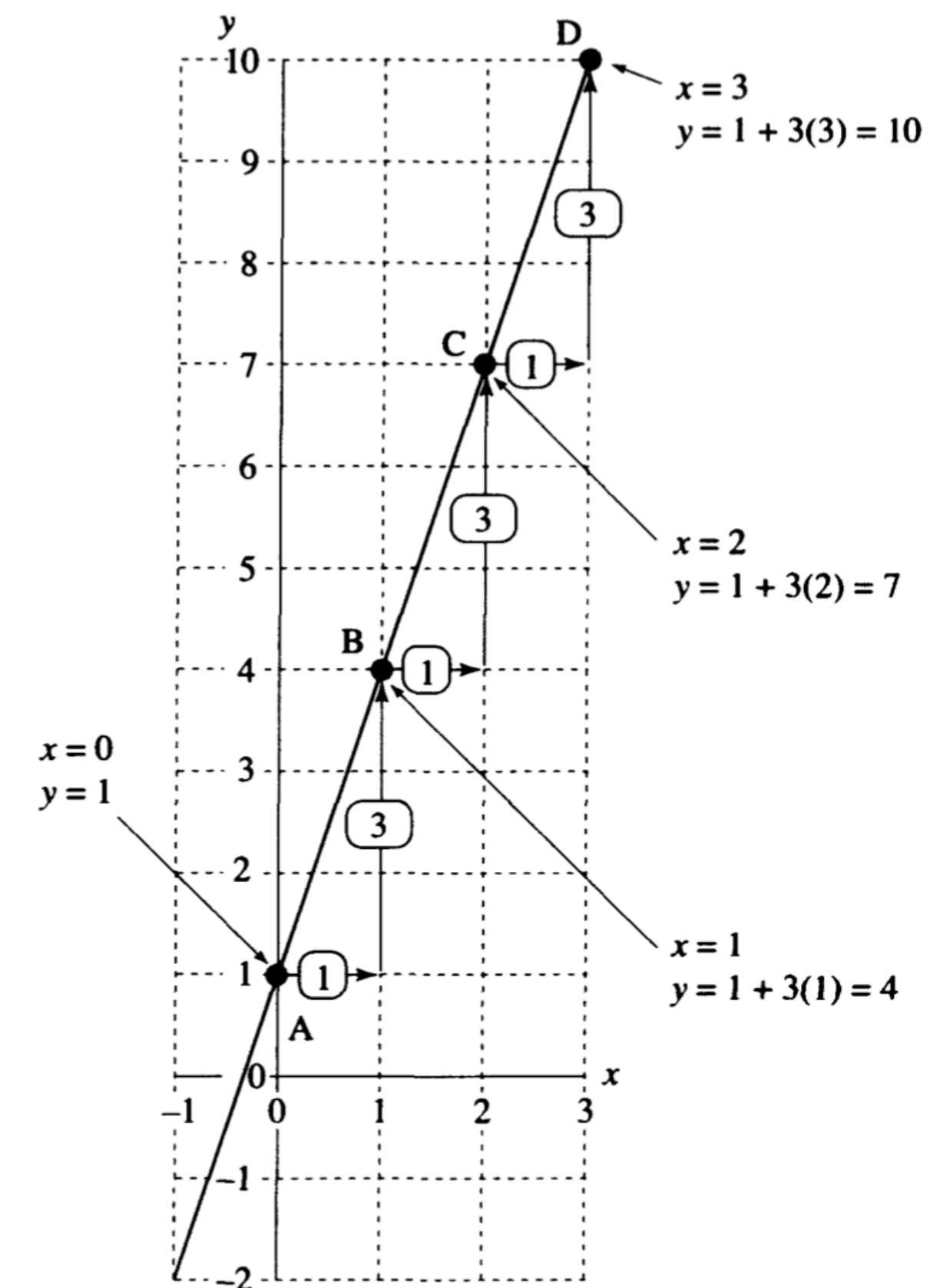


Figure 2.7: Deducing the equation of the line $y = 1 + 3x$.

- The equation of a line:

Solution: (a)

Figure 2.8 is simply **Figure 2.7** redrawn with the intercept referred to as c , slope as m .

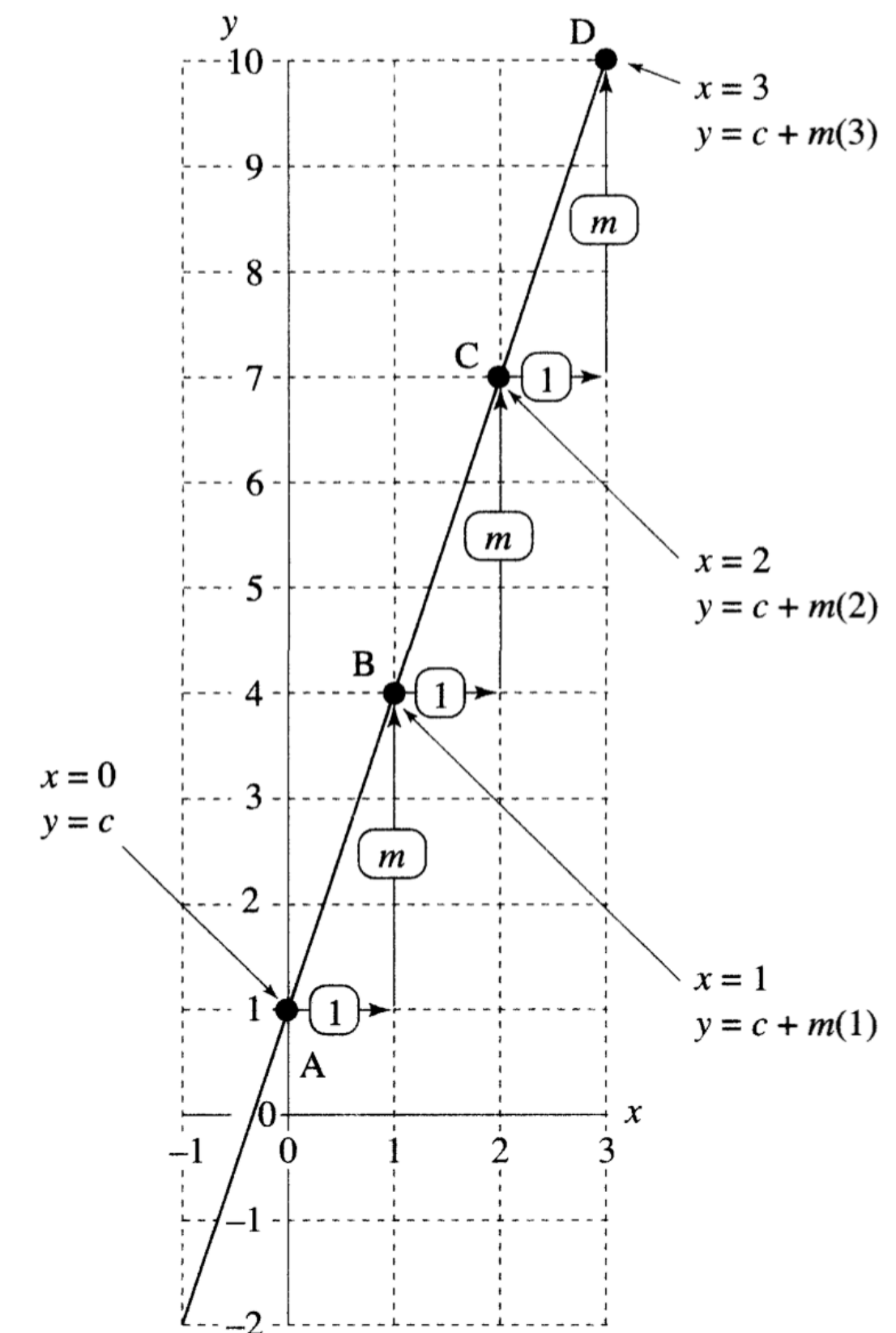


Figure 2.8 Deducing the equation of the line $y = mx + c$ with $m = 3$, $c = 1$

□ The equation of a line:

Solution: (a) Now observe the pattern relating the x and y values for selected points on the line.

x-value	y-value	Calculation of y-value
$x = 0$	$y = c$	this is the intercept
$x = 1$	$y = c + m(1)$	$y = c + m(x)$ (x-value)
$x = 2$	$y = c + m(2)$	$y = c + m(x)$ (x-value)
$x = 3$	$y = c + m(3)$	$y = c + m(x)$ (x-value)
any -value	$y = c + m(x)$	$y = c + m(x)$ (x-value)

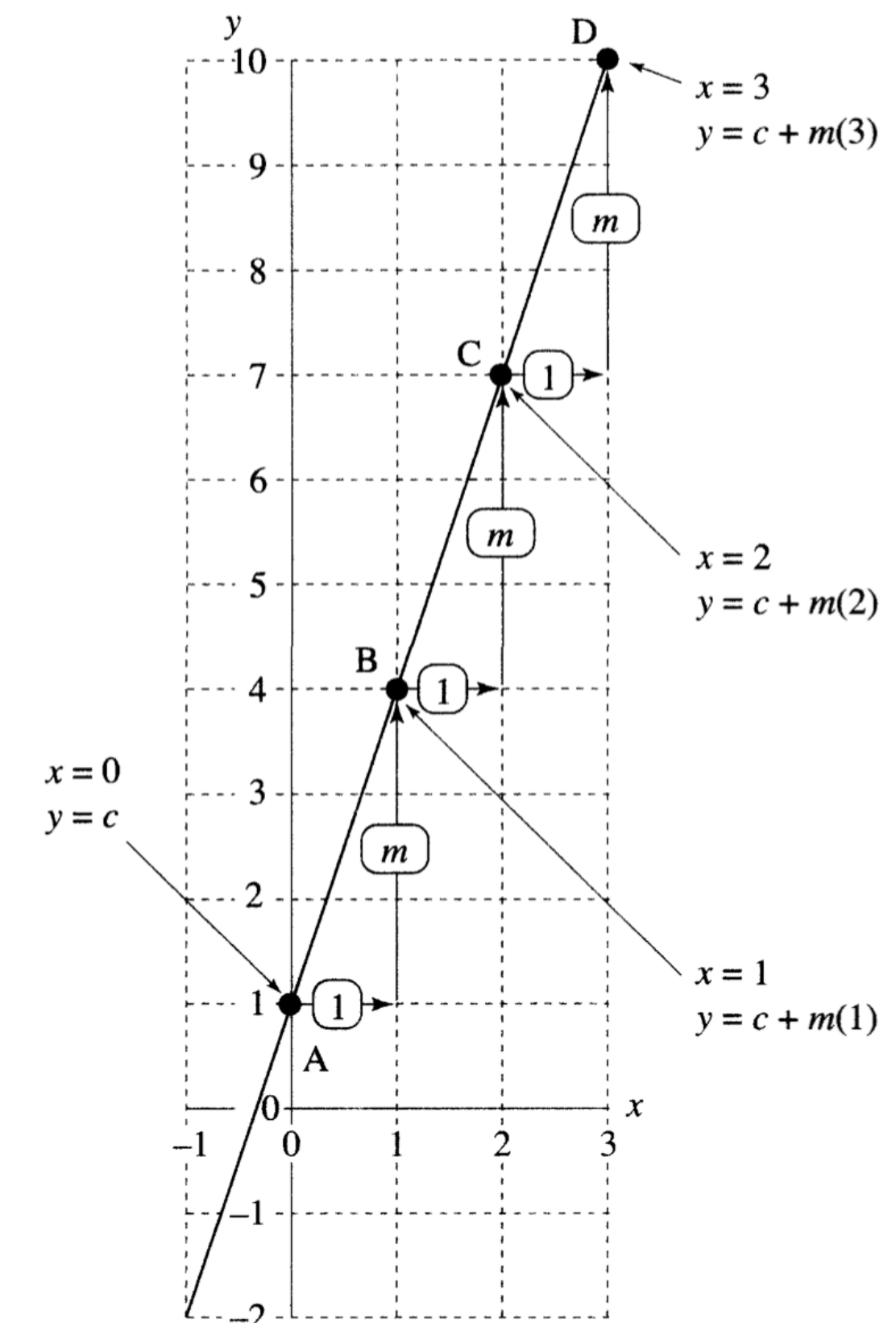


Figure 2.8 Deducing the equation of the line $y = mx + c$ with $m = 3$, $c = 1$

- The equation of a line:

Solution: (a)

For each value of x , the y -coordinate is calculated by the formula:

$$y = c + mx \text{ or } y = mx + c$$

is the equation of the line

Hence, if the slope and intercept of a line are given, the equation of that line may be obtained by substituting the given values of m and c into the general formula $y = mx + c$.

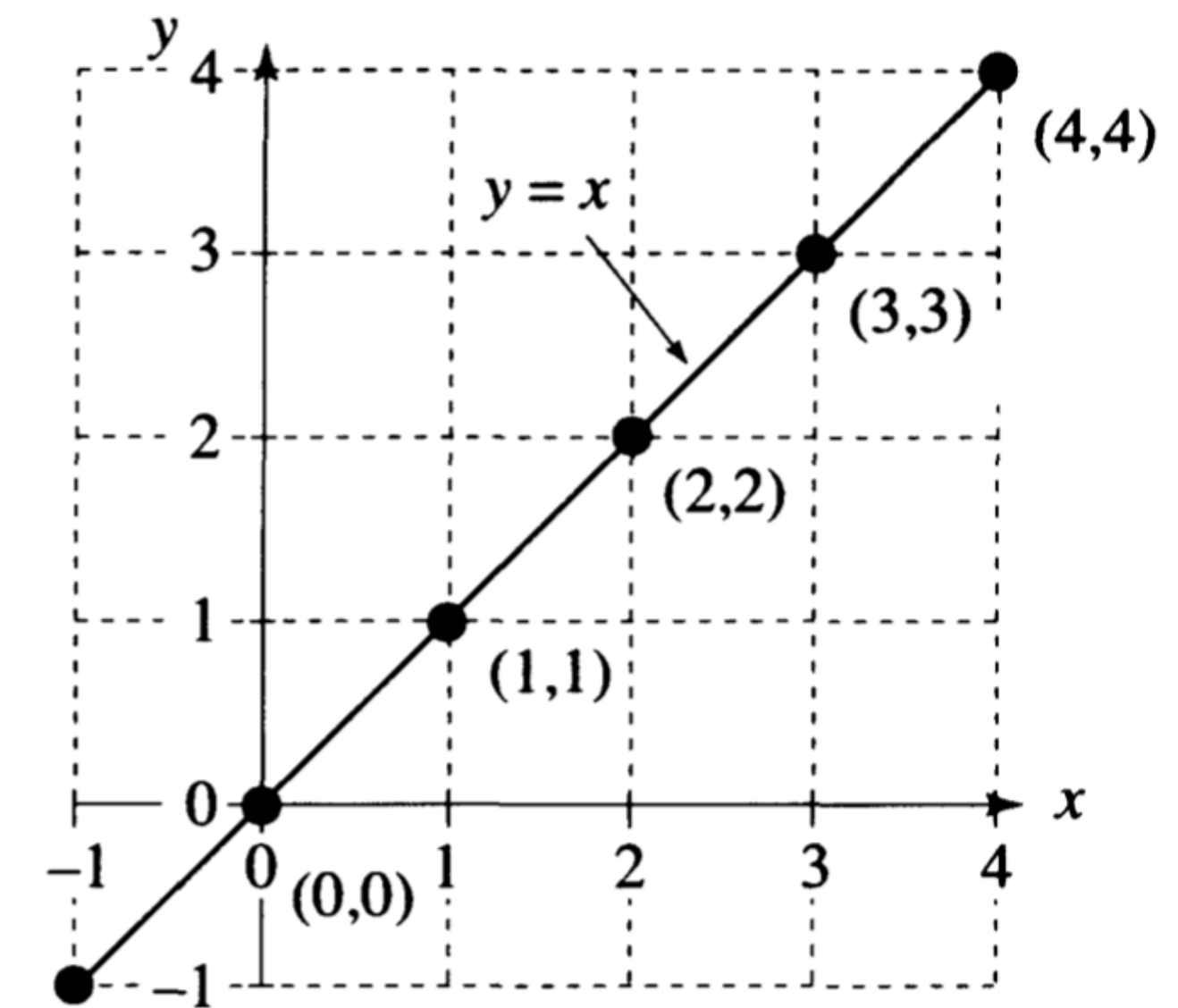


Figure 2.9 Graph of the line $y = x$

□ Horizontal intercepts

The point where the line crosses the horizontal axis. This calculation uses the fact that every point on the horizontal axis has a y -coordinate $y = 0$. Since y is known, substitute $y = 0$ into the equation of the line, $y = mx + c$, and solve for x : this value of $(x = -c/m)$ is the horizontal intercept.

Ex: Calculate the horizontal and vertical intercepts for each of the following lines:

(i) $y = mx + c$

□ Horizontal intercepts

Solution: (i)

Equation of line: $y = mx + c$

Horizontal or x -intercept Line cuts x -axis at $y = 0 : 0 = mx + c$

Solve for x : $-c = mx$

$$x = -c/m$$

This is the horizontal intercept: $x = -\frac{c}{m}, y = 0$

□ The straight line

Equation of horizontal and vertical lines:

Figure 2.11 illustrates that the equation of a horizontal line simply takes the value of the coordinate where the line crosses the y -axis, and the equation of a vertical line simply takes the value of the coordinate where the line crosses the x -axis.

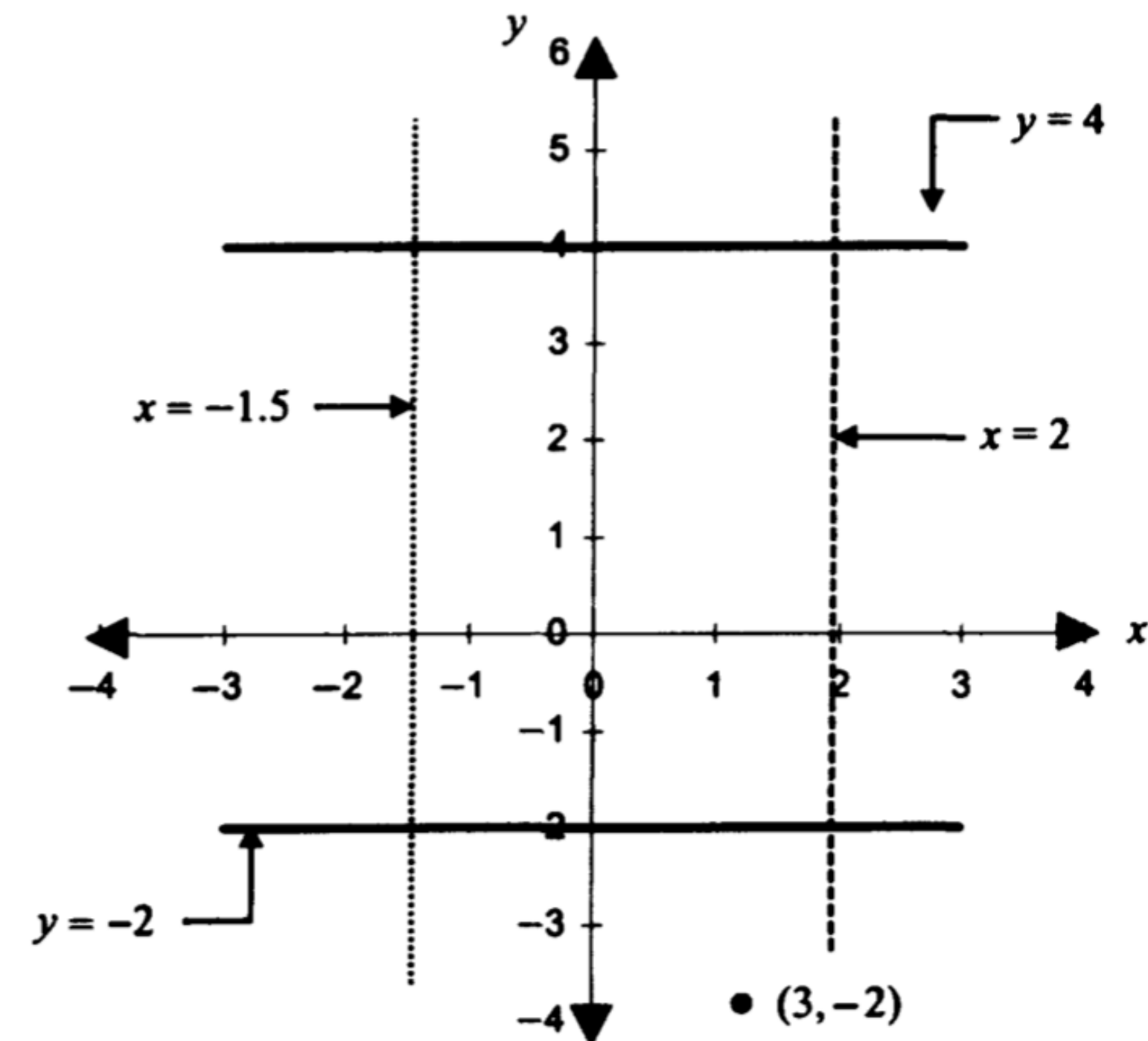


Figure 2.11 Equation of horizontal and vertical lines

□ The straight line

To graph a straight line from its equation $y = mx + c$

- Earlier in this Lecture lines were plotted, starting at the intercept (first point), then plotting further points using the fact that the slope is the change in y per unit increase in x .
- In this section a straight line is graphed from its equation by calculating at least two points then drawing a line through them. To calculate the required points, use either of the following methods.

□ The straight line

To graph a straight line from its equation $y = mx + c$

Method A (Calculate a table of points (at least two points).)

Step 1: Rearrange the equation into the form $y = mx + c$, if it is not already in this form.

Step 2: Choose any x -values (choose convenient values of x , such as $(x = 0, x = 10, x = -2)$); calculate the corresponding y -values, using the rearranged formula.

Step 3: Plot the points and draw the line through these points.

□ The straight line

To graph a straight line from its equation $y = mx + c$

In economics, graphs are frequently required in the region where x -values and y -values are positive. To plot a line in this region, calculate the horizontal and vertical intercepts.

Method B

Step 1: Rearrange the equation into the form $y = mx + c$ if necessary.

Step 2: The x -intercept is at $(x = -c/m, y = 0)$; the y -intercept is at $(x = 0, y = c)$.

Step 3: Draw the line through the points.

□ The straight line

Ex: A line has the equation (or formula) $y = 2x - 1$. Plot the line over the interval $x = -2$ to $x = 3$.

Note: While a minimum of two points are required to plot a line, it is a good idea to calculate at least three points, just in case of mistakes in calculations. All points should lie perfectly on the line.

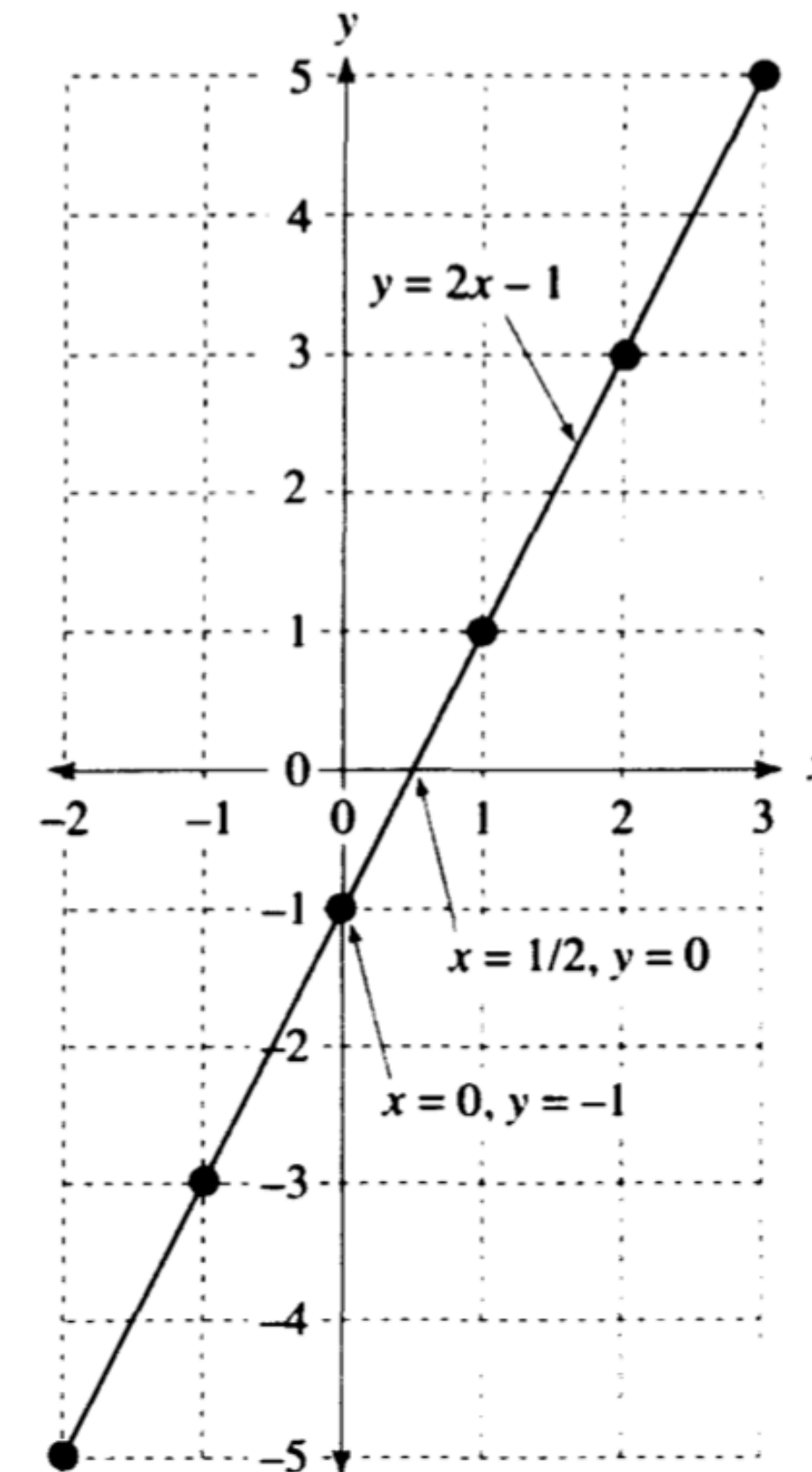


Figure 2.12: Plot the line $y = 2x - 1$

Solution: (Method A)

Step 1: $y = 2x - 1$. No rearranging necessary.

Step 2: Since the graph is required for values of x from $x = -2$ to $x = 3$, calculate y for several x -values within this interval. You may choose any x -values you wish, for example $x = -2, -1, 0, 1, 2, 3$. Calculate the corresponding y -coordinates by simply substituting the chosen x -value into the equation (Table 2.2).

Solution: (Method A)

Step 2: Table 2.2 Calculating y -coordinate given the x -coordinate

x	$y = 2x - 1$	Points (x, y)
-2	$y = 2(-2) - 1 = (-4) - 1 = -5$	$(-2, -5)$
-1	$y = 2(-1) - 1 = (-2) - 1 = -3$	$(-1, -3)$
0	$y = 2(0) - 1 = (0) - 1 = -1$	$(0, -1)$
1	$y = 2(1) - 1 = (2) - 1 = 1$	$(1, 1)$
2	$y = 2(2) - 1 = (4) - 1 = 3$	$(2, 3)$
3	$y = 2(3) - 1 = (6) - 1 = 5$	$(3, 5)$

Step 3: Plot the points, draw the line; see Figure 2.12.

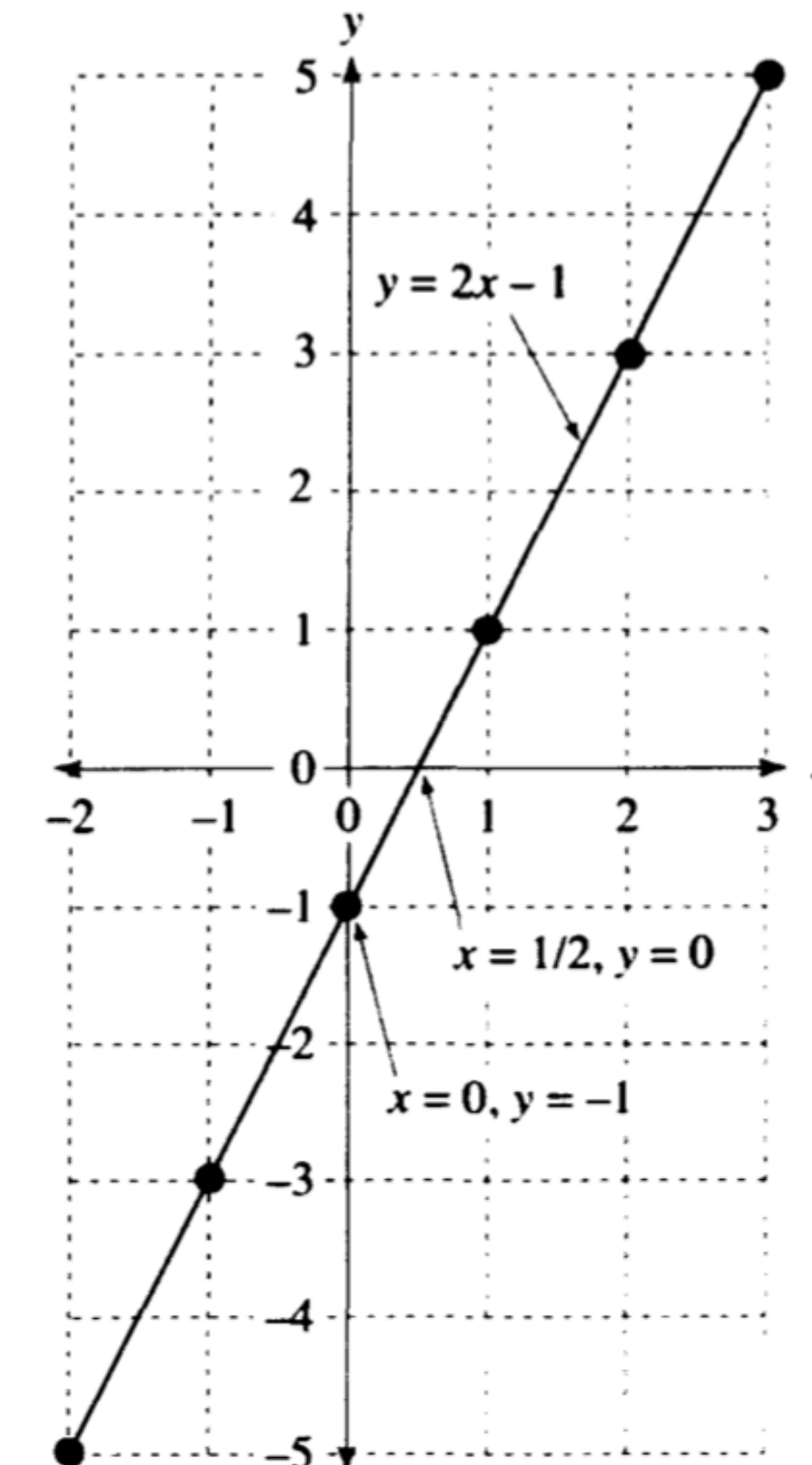


Figure 2.12: Plot the line $y = 2x - 1$

Solution: (Method B)

Step 1: $y = 2x - 1$. No rearranging necessary.

Step 2: *Vertical intercept:* the line cuts the vertical axis at $x = 0$, $y = -1$, hence vertical intercept is $(0, -1)$.

Horizontal intercept: the line cuts the horizontal axis at $x = -c/m$, $x = 0.5$, hence horizontal intercept is $(0.5, 0)$.

Step 3: Plot the vertical and horizontal intercept on the axis, then draw the line through these points. See **Figure 2.12**.

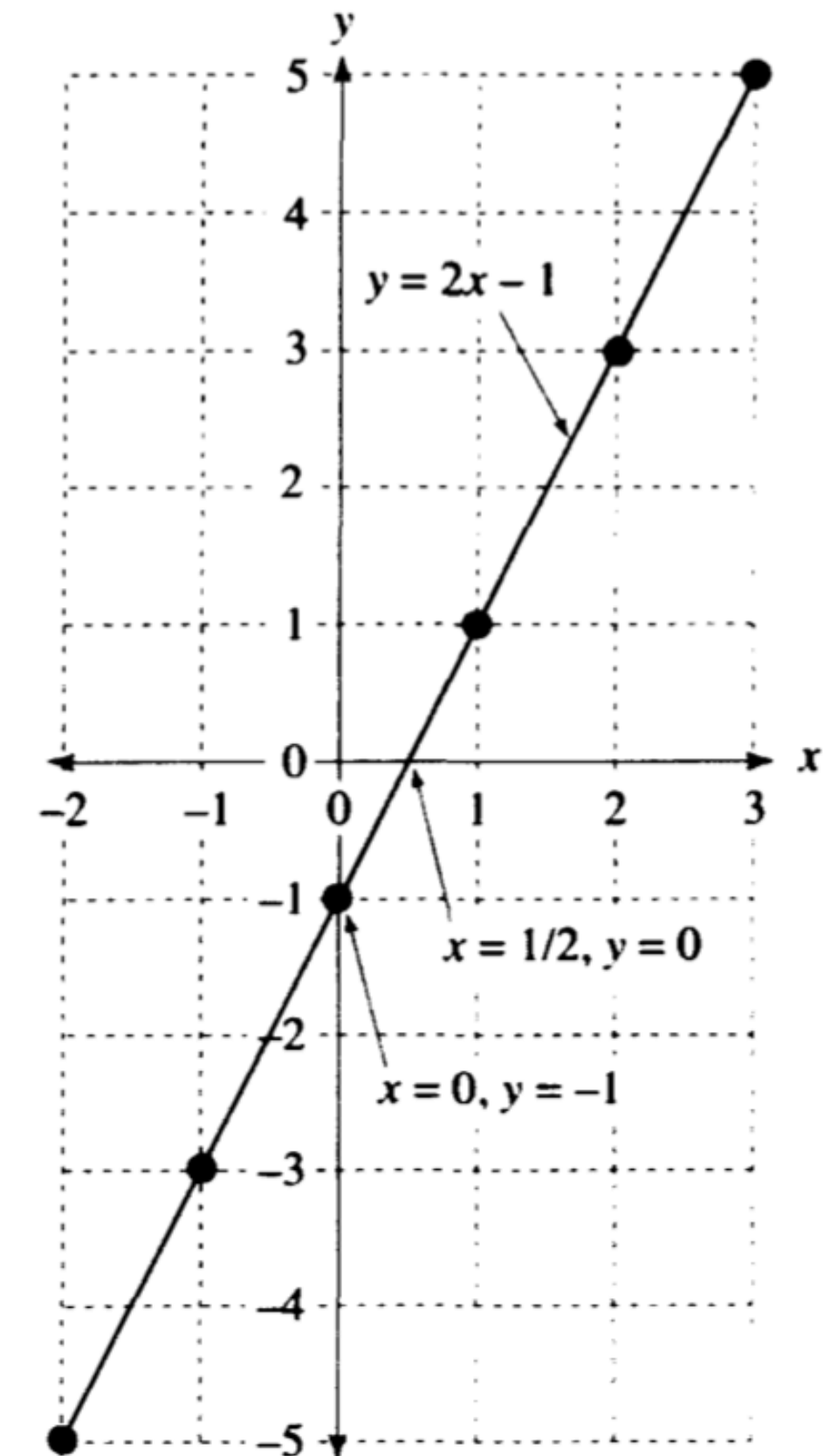


Figure 2.12: Plot the line $y = 2x - 1$

Chapter 02

Introducing Excel

A spreadsheet is another aid to mathematical and statistical problem solving. An electronic spreadsheet is a software package that accepts data in the rows and columns of its worksheet as shown in **Figure 1.6**.

One single location on the spreadsheet is called a 'cell'. Cells A4, C2, C7, C9, E2 and H8 are highlighted in **Figure 1.6**. A cell is referenced by the column letter followed by the row number, for example, cell C7, is located in column C, row 7.

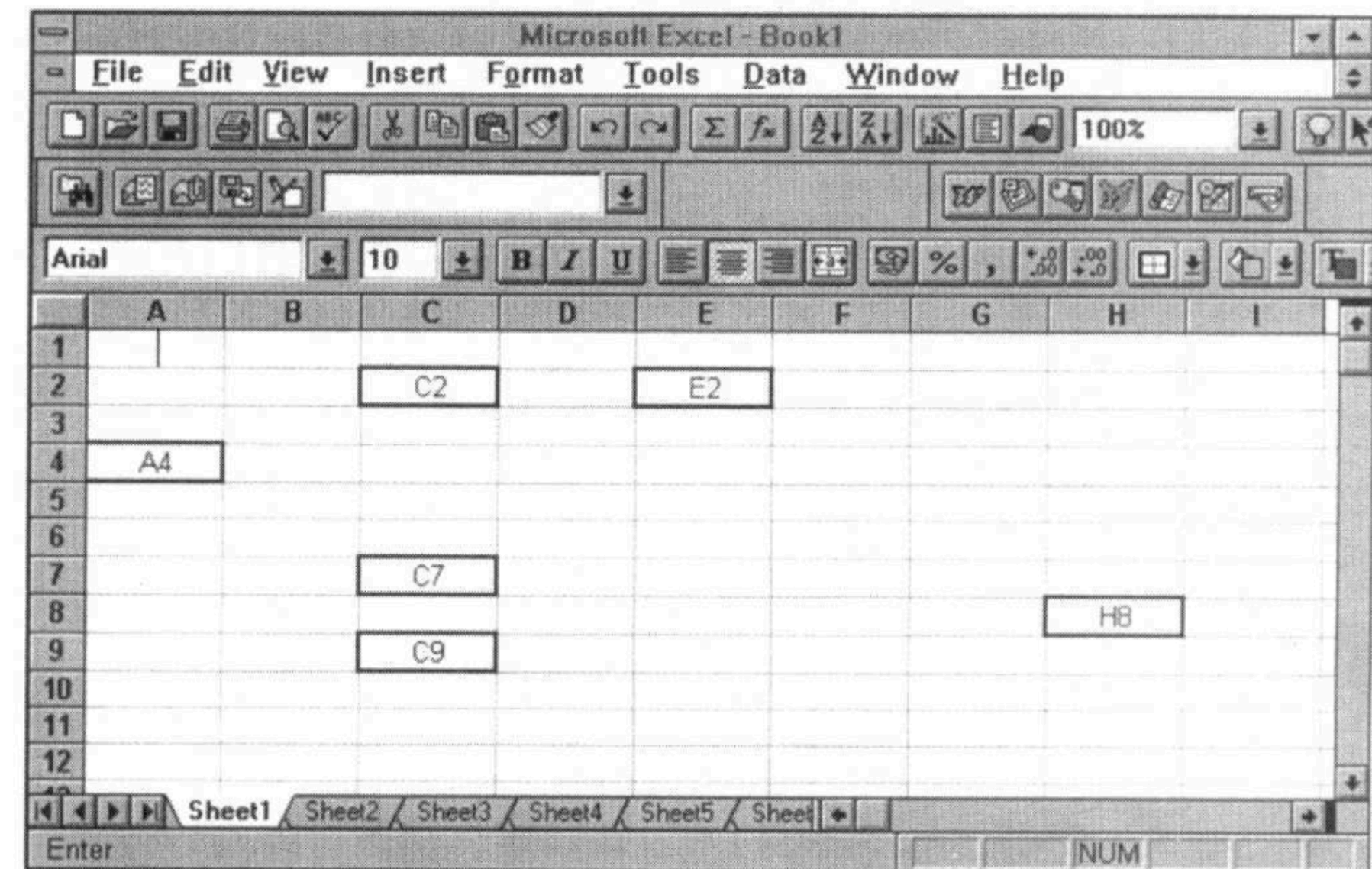


Figure 1.6 Cell reference on a spreadsheet

When the data is entered, the user may then perform calculations, such as:

- Sum the data in specified rows and/or columns of the spreadsheet.
- Enter formula which calculates results from data in the spreadsheet.
- Plot graphs from data in the spreadsheet.

There is a variety of different spreadsheet packages available. We shall use Excel, which is available in Microsoft Office.

Ex: (Using Excel to perform calculations and plot graphs)

Part-time staff are paid on an hourly basis. The number of hours worked with the hourly rate of pay for seven staff are as follows:

Name	J.M	P.M	D.H	K.C	J.McM	A.B	C.McK
Hours	6	12.6	34	23	45.8	18	12.6
Rate	£27.5	£27.5	£16.6	£19.2	£50.5	£27.5	£27.5

- (a) Enter the data onto a spreadsheet.
- (b) Enter a formula to calculate the total pay for each member of staff.
- (c) Plot a barchart, showing the total pay received by each member of staff

Solution:

- (a) Enter the data onto the spreadsheet as shown in **Figure 1.8**. You may start entering the data anywhere in the spreadsheet.

The screenshot shows the Microsoft Excel interface with the following data in the spreadsheet:

	A	B	C	D	E	F	G	H	I
1									
2		J M	P M	D H	K C	J McM	A B	C Mck	
3	Hours	6	12.6	34	23	45.8	18	12.6	
4	Rate	27.5	27.5	16.6	19.2	50.5	27.5	27.5	
5	Pay	165	346.5	564.4	441.6	2312.9	495	346.5	
6									
7									
8		=B3*B4							
9									
10									
11									
12									
13									

The formula bar shows the formula `=B3*B4` for cell A8. The status bar at the bottom indicates the active sheet is 'Sheet3'.

Figure 1.8 Type the formula `=3*B4` into cell B5 to calculate pa= 165 for JM. Copy the formula across row 5 to calculate pay for the remaining employees.

Solution:

(b) Next, enter a formula to calculate total pay for each individual in the table.

Total pay = number of hours x hourly rate

To indicate to the computer that a formula is being entered into a given cell, the first character typed should be, = , followed by the formula as follows:

- Position the cursor in cell B5 (so that hours worked, the hourly rate and total pay for J.M are all in the same column).
- Now, from the keyboard, enter the formula to calculate total pay by typing = B3*B4
- Repeat this process for each cell in row 5: '= C3*C4': '= D3*D4' etc.

The results of the calculations are given in row 5, **Figure 1.8**.

Note: the advantage of entering the cell names instead of the actual values is that the result is automatically recalculated if the numbers in these cells are changed. Try it!

It is possible to copy the first formula in cell B5 across row 5, instead of keying the formula into each individual cell. This may be done by clicking on cell B5. It is now the active cell with a black box around it. Point to the bottom right-hand corner of the black box until a solid black cross appears, hold the left click on the mouse down and drag across cells C5 to H5. (Use the HELP button or consult some of the many reference books on Excel.)

Solution:

(c) To simplify the graph sketching, copy or move the rows containing the data required for plotting together. In this example, it is required to plot total pay for each person, so the row of initials is moved from row 2 to row 6, and the row of total pay from row 5 to row 7 as shown in **Figure 1.9**.

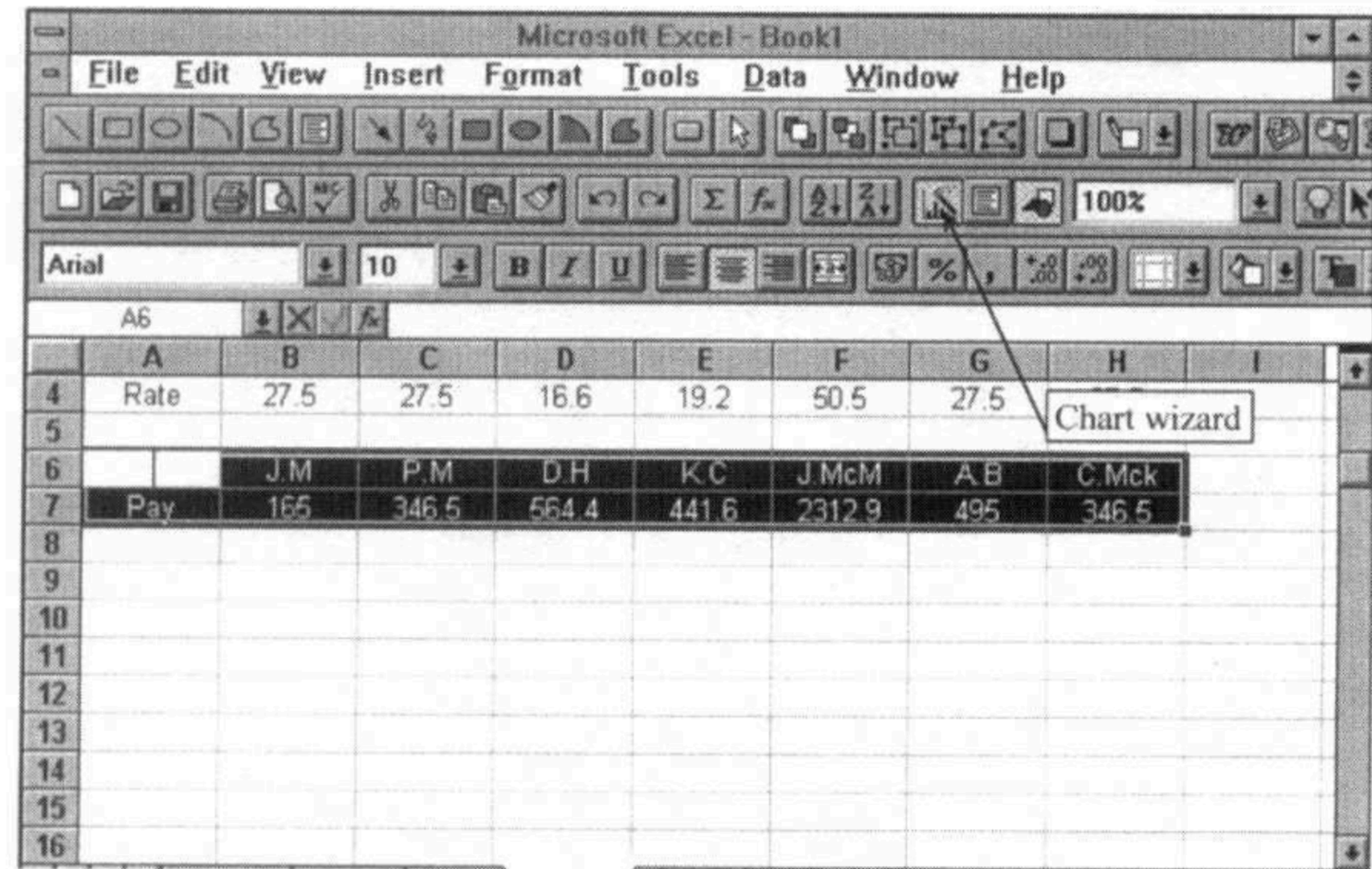
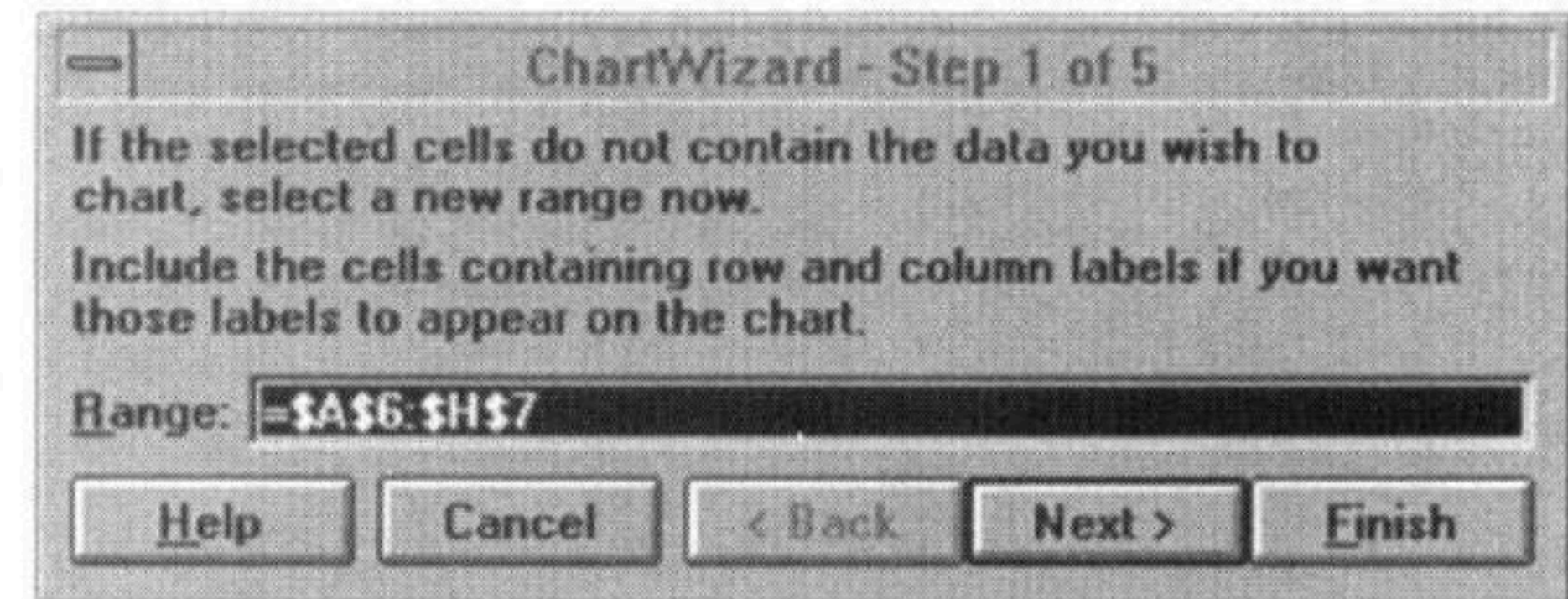


Figure 1.9 Highlight the data required for graph plotting then click on the chart wizard

To plot the graph, select the data required for the graph plotting. Then click on 'chart wizard' from the standard toolbar, as shown in **Figure 1.9**. A cross should now appear with the chart wizard symbol attached. Select any location on the spreadsheet for the final graph. This location and its size is not crucial since both may be changed when the graph is finished.

Excel will now guide you through the graph plotting by presenting a series of five menus as shown in **Figure 1.10(a)** to (e).

(a)



Excel will now guide you through the graph plotting by presenting a series of five menus as shown in Figure 1.10(a) to (e).

Step 1 of 5: Select the rows or columns of data on the spreadsheet for which the graph is required (if different from those selected already). In this example, rows 6 and 7 should already be selected, ranging from cell A6 to cell H7, as shown in Figure 1.10(a).

(a)

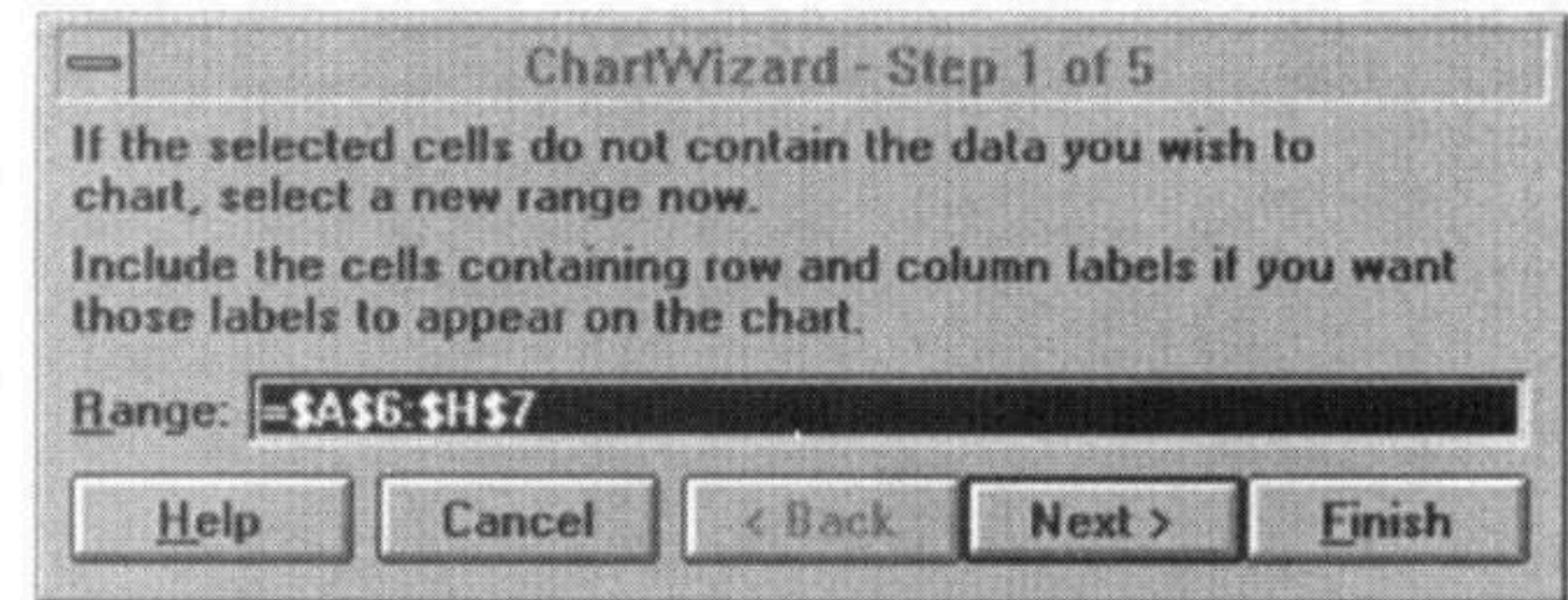


Figure 1.10 Excel menus for graph plotting

Step 2 of 5: Select the chart or graph type. A column bar chart is selected. Figure 1.10(b).

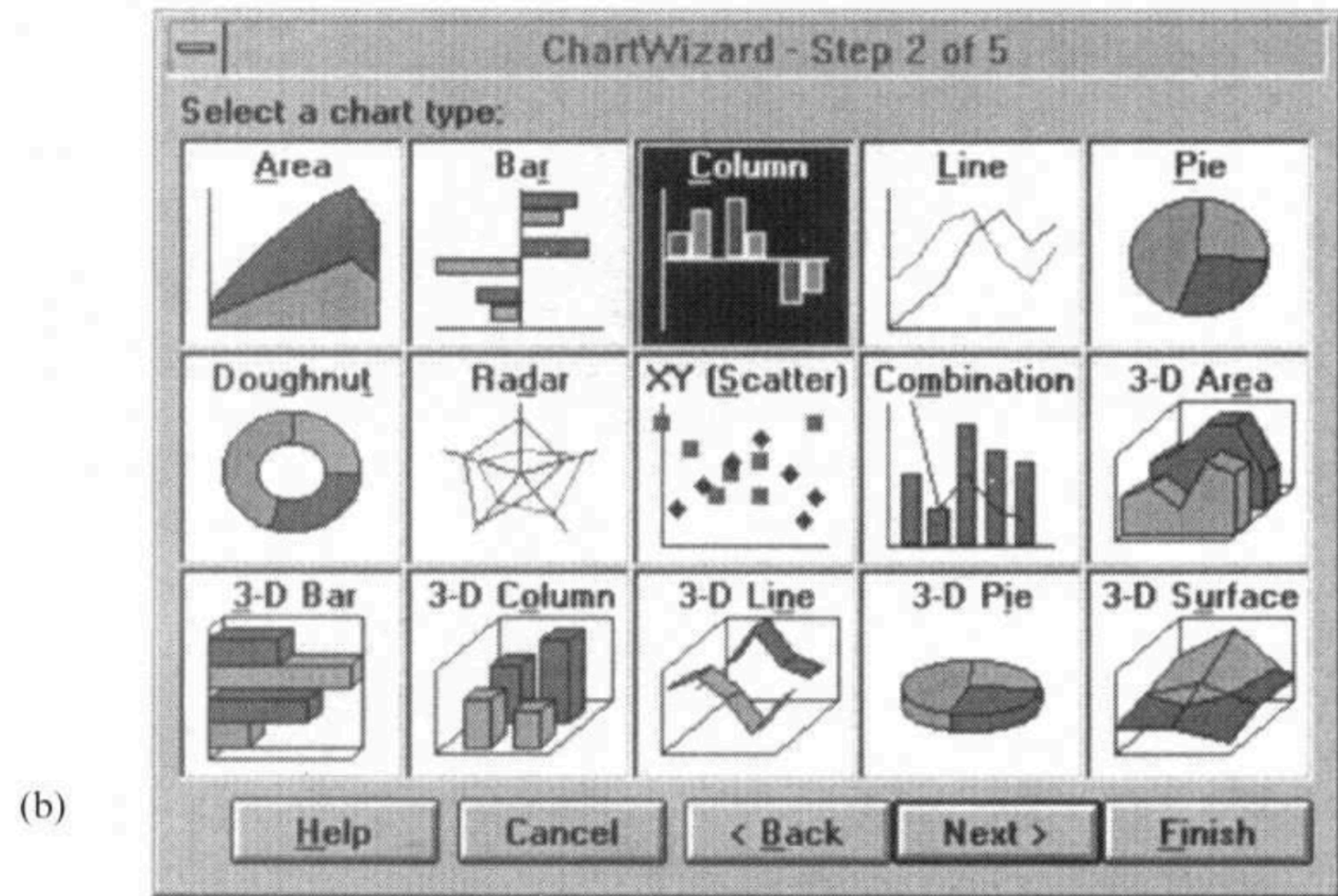


Figure 1.10 Excel menus for graph plotting

Step 3 of 5: Select the type of column chart. Type 6 was selected, though type 1 is also suitable, see Figure 1.10(c).

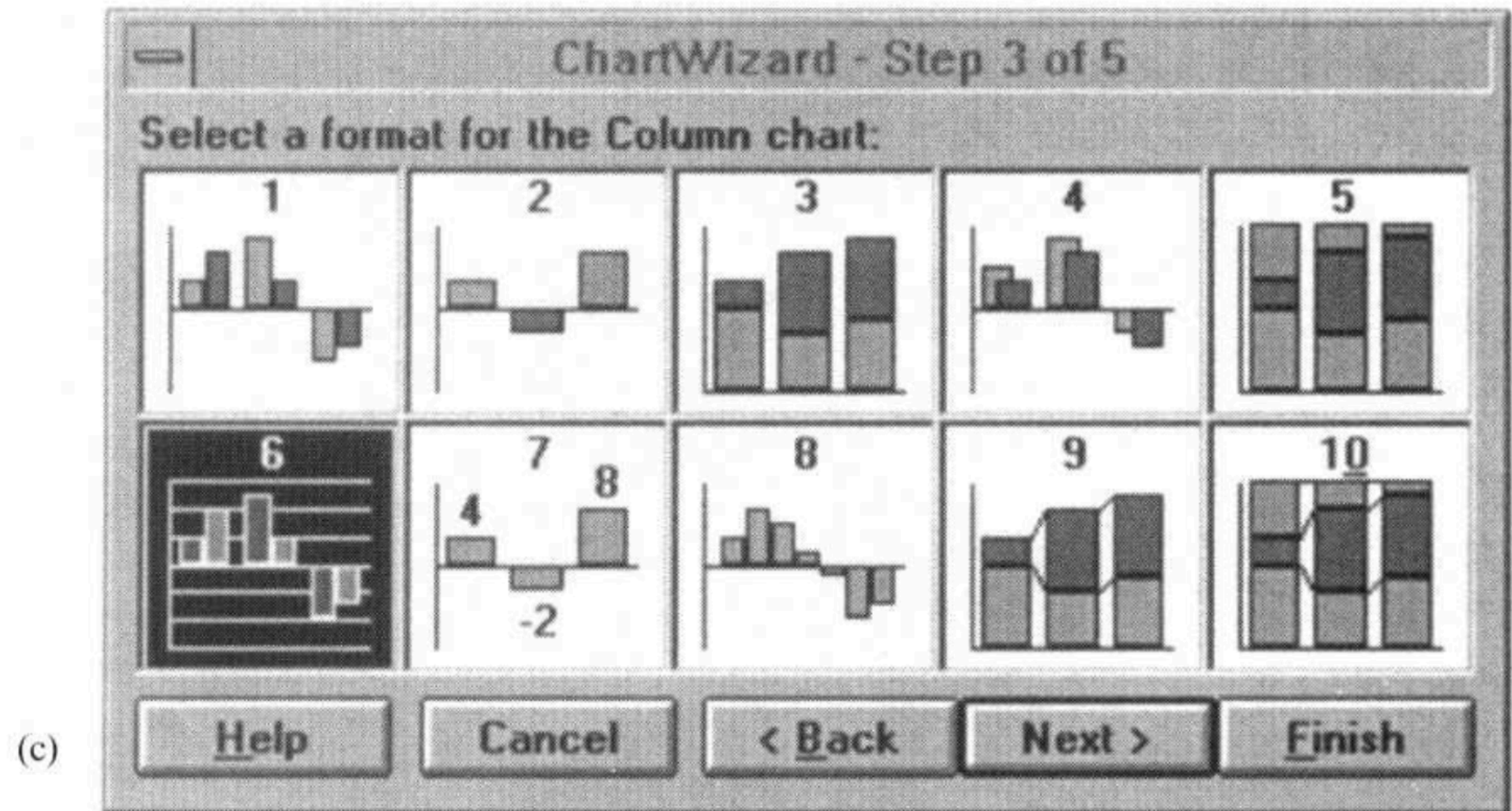


Figure 1.10 Excel menus for graph plotting

Step 4 of 5: Sample chart. This menu asks if you wish to use any row of data as names for the x-values. In this example, the first of the data rows (the initials) is to be used as the names for the x-values, while the first column represents the value of the pay for each employee.

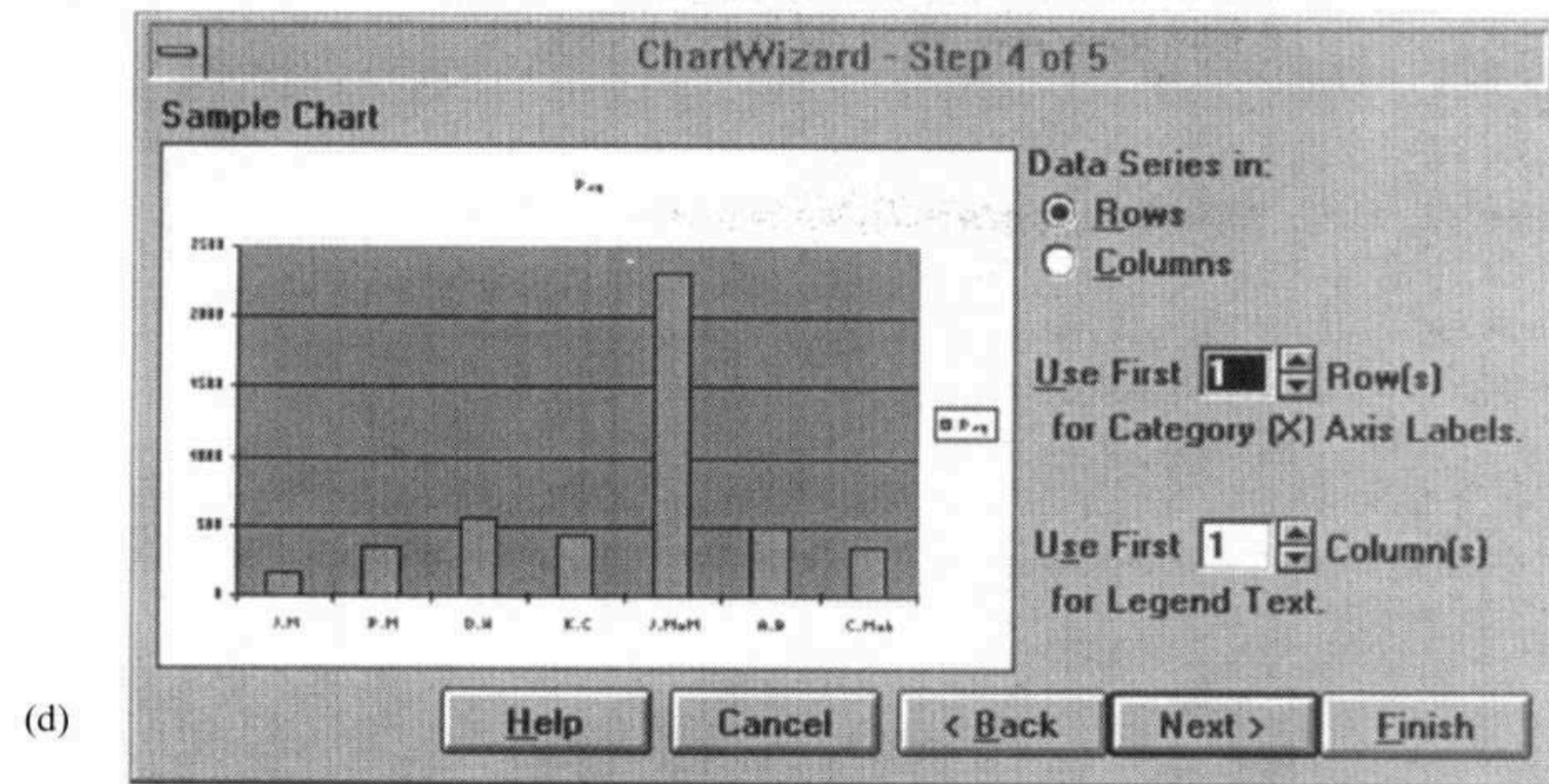


Figure 1.10 Excel menus for graph plotting

Step 5 of 5: This menu allows you to enter a chart title and labels for axes. In Figure 1.10(e), 'Employees' was entered for the .x-category, pay in £ for the rvalues. Finish.

The finished graph now appears at the preselected location on the spreadsheet. However, the location may be changed and the chart edited. Consult HELP and reference manuals for further details.

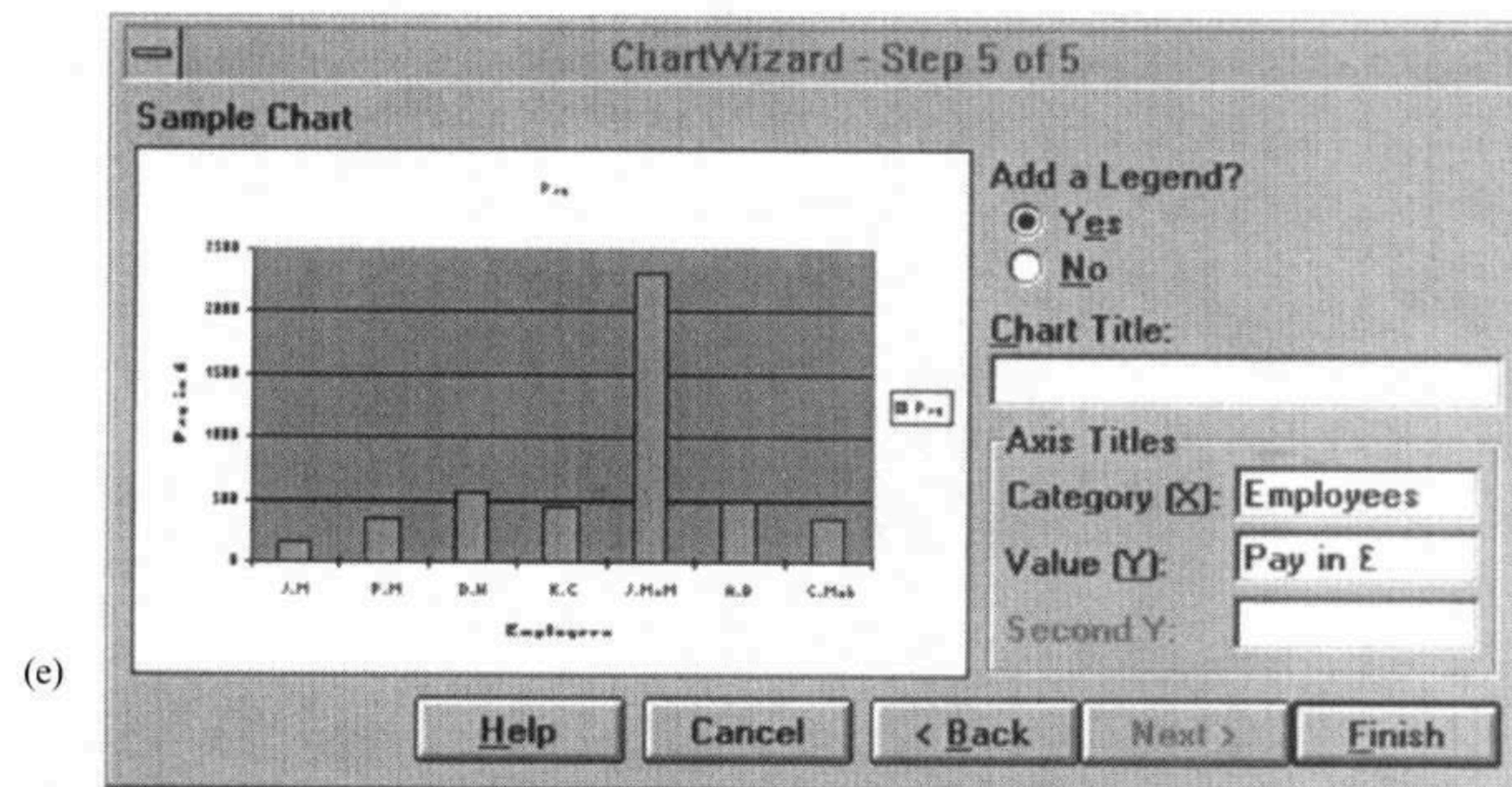


Figure 1.10 Excel menus for graph plotting