

Essential Mathematics

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Lecture 01



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MTH 112
Business Mathematics

At the end of this chapter you should be able to:

- Perform basic arithmetic operations and simplify algebraic expressions
- Perform basic arithmetic operations with fractions
- Solve equations in one unknown, including equations involving fractions
- Understand the meaning of no solution and infinitely many solutions
- Currency conversions
- Solve simple inequalities
- Calculate percentages

In addition, you will be introduced to the calculator and a spreadsheet.

□ Some mathematical preliminaries

Brackets: - in mathematics are used for grouping and clarity
- may also be used to indicate multiplication.
- are used in functions to declare the independent variable (see later).

Powers: positive whole numbers such as 2^3 , which means $2 \times 2 \times 2 = 8$:

$$(\text{anything})^3 = (\text{anything}) \times (\text{anything}) \times (\text{anything})$$

Note:

Brackets: $(A)(B)$ or $A \times B$ or AB all indicate A
multiplied by B . $(x)^3 = x \cdot x \cdot x$

$$(x + 4)^5 = (x + 4)(x + 4)(x + 4)(x + 4)(x + 4)$$

□ Some mathematical preliminaries

Square roots: the square root of a number is the reverse of squaring:

$$(2)^2 = 4 \quad \sqrt{4} = \pm 2$$

$$(2.5)^2 = 6.25 \quad \sqrt{6.25} = \pm 2.5$$

Variables and letters: When we don't know the value of a quantity, we give that quantity a symbol, such as x .

□ Some mathematical preliminaries

Variables and letters:

Ex: 'For the next 15 weeks, if I save £x per week I shall have £4000 to spend on a holiday'.

This statement may be expressed as a mathematical equation:

$$15 \times \text{weekly savings} = 4000$$

$$15 \times x = 4000 \text{ or } 15x = 4000$$

To solve the equation for the unknown, x:

$$15x = 4000 \quad (\text{divide both sides of the equation by } 15)$$

$$x = 266.67$$

□ Arithmetic Operations

Addition and subtraction

Adding: If all the signs are the same, simply add all the terms and give the answer with the common overall sign.

Subtracting: When subtracting any two numbers or two similar terms, give the answer with the sign of the largest number or term.

If terms are identical,, then they may be added or subtracted :

□ Arithmetic Operations

Addition and subtraction

Ex: Add/subtract with numbers,

(i) $5 + 8 + 3 = 16$

(ii) $5 + 8 + 3 + y = 16 + y$

(iii) The y-term is different, so it cannot be added to the others

Ex: Add/subtract with variable terms

(i) $5x + 8x + 3x = 16x$

(ii) $5x + 8x + 3x - 4y = 16x - 4y$

(iii) $5xy + 8xy + 3xy + y = 16xy + y$

□ Arithmetic Operations

Multiplying and dividing

Multiplying or dividing two quantities with like signs gives an answer with a **positive sign**.

Multiplying or dividing two quantities with different signs gives an answer with a **negative sign**.

□ Arithmetic Operations

Multiplying and dividing

Ex:

(i) $(-5)(-7) = 35$

(ii) $(x + 4)(x + 2) = x(x + 2) + 4(x + 2)$
 $= x^2 + 2x + 4x + 8$
 $= x^2 + 6x + 8$

Note:

$$0 \times (\text{any real number}) = 0$$

$$\frac{0}{(\text{any real number})} = 0$$

But you cannot divide by 0

□ Fraction

Add/subtract fraction:

The method for adding or subtracting fractions is:

Step 1: Take a common denominator, that is, a number or term which is divisible by the denominator of each fraction to be added or subtracted. A safe bet is to use the product of all the individual denominators as the common denominator.

Step 2: For each fraction, divide each denominator into the common denominator, then multiply the answer by the numerator.

Step 3: Simplify your answer if possible.

Add/subtract fraction:

Solution: Step 1: The common denominator is $(7)(3)(5) = 105$

$$\text{Step 3: } = \frac{15x + 70x - 84x}{105} = \frac{x}{105}$$

$\frac{3}{7}$ 3 is called the numerator
 7 is called the denominator



□ Fraction

Multiplying fractions

In multiplication, write out the fractions, multiply the numbers across the top lines and multiply the numbers across the bottom lines.

Note: Write whole numbers as fractions by putting them over 1.

□ Fraction

Multiplying fractions

Ex: (i) $3 \times \frac{2}{5} = \left(\frac{3}{1}\right)\left(\frac{2}{5}\right) = \frac{3 \times 2}{1 \times 5} = \frac{6}{5} = 1\frac{1}{5}.$

(ii) $\left(\frac{3}{x}\right)\frac{(x+3)}{(x-5)} = \frac{3(x+3)}{x(x-5)} = \frac{3x+9}{x^2-5x}.$

Terminology: RHS means right-hand side and LHS means left-hand side.

□ Fraction

Dividing fractions

Dividing by a fraction is the same as multiplying by the fraction inverted.

Ex: (i) $\frac{\left(\frac{2}{3}\right)}{\left(\frac{5}{11}\right)} = \left(\frac{2}{3}\right)\left(\frac{11}{5}\right) = \frac{22}{15}.$

(ii) $\frac{\frac{2x}{x+1}}{\frac{3x}{2(x-y)}} = \frac{2x}{(x+1)} \frac{2(x-y)}{3x} = \frac{4x(x-y)}{3x(x+1)} = \frac{4(x-y)}{3(x+1)}.$

□ Solving Equations

The solution of an equation is simply the value or values of the unknown(s) for which the left-hand side (LHS) of the equation is equal to the right-hand side (RHS).

For example, the equation, $x + 4 = 10$, has the solution $x = 6$. We say $x = 6$ 'satisfies' the equation. We say this equation has a **unique solution**.

Not all equations have solutions. In fact, equations may have no solutions at all or may have infinitely many solutions. Each of these situations is demonstrated in the following examples.

□ Solving Equations

Case 1: Unique solutions An example of this is given above: $x + 4 = 10$ etc.

Case 2: Infinitely many solutions The equation, $x + y = 10$ has solutions $(x = 5, y = 5)$; $(x = 4, y = 6)$; $(x = 3, y = 7)$, etc. In fact, this equation has infinitely many solutions or pairs of values (x, y) which satisfy the formula, $x + y = 10$.

Case 3: No solution The equation, $0(x) = 5$ has no solution. There is simply no value of x which can be multiplied by 0 to give 5.

□ Solving Equations

Ex: Given the equation $x + 3 = 2x - 6 + 5x$, solve for x .

Solution: $x + 3 = 2x - 6 + 5x$

$x + 3 = 7x - 6$ adding the x terms on the RHS

$$9 = 6x$$

$$\frac{9}{6} = x \quad \text{dividing both sides by 6}$$

$$1.5 = x$$

□ Currency Conversions

You may have browsed the internet in order to purchase books, software, music, etc. Frequently prices will be quoted in some currency other than your own. With some simple maths and knowledge of the current rate of exchange, you should have no difficulty converting the price to your own currency. The following worked examples use the Euro exchange rates in Table 1.1. This table equates each of the currencies listed to 1 Euro on a given day in August 2001.

□ Currency Conversions

Table 1.1

Currency	Rate	Currency	Rate
British pound	0.6187	Canadian dollar	1.3460
US dollar	0.8770	Australian dollar	1.6988
Japanese yen	108.5400	Polish zloty	3.7143
Danish krone	7.4424	Hungarian forint	247.5000
Swedish krona	9.1550	Hong Kong dollar	6.8399
Swiss franc	1.5060	Singapore dollar	1.5607
Norwegian krone	7.9566		

□ Currency Conversions

Ex:

- (a) A book is priced at US\$20. Calculate the price of the book in
(i) Euro, (ii) British pounds and (iii) Australian dollars.
- (b) How many British pounds are equivalent to (i) \$500 Australian and (ii) \$10000 Singapore?

□ Currency Conversions

Solution: The calculations (correct to 4 decimal places) may be carried out as follows:

Step 1 State the appropriate rates from Table 1.1.

Step 2 Set up the equation: 1 unit of *given* currency = y units of *required* currency.

Step 3 Solve the equation: x units of *given* currency = $x \times (y$ units of *required* currency).

□ Currency Conversions

Solution: (a) (i) The price is given in US dollars, the required price (currency) is in euros, hence

Step 1 \$0.8770 US = 1 Euro (from Table 1.1)

Step 2 \$1 US = $\frac{1}{0.8770}$ Euro dividing both sides by 0.8770 to get rate for \$1.

Step 3 \$20 US = $\frac{20}{0.8770}$ Euro multiplying both sides by 20
= 22.8050 Euro.

□ Currency Conversions

Solution: (a) (ii) The price is given in US dollars, the required price (currency) is in British pounds, hence

Step 1 From Table 1.1 write down the exchange rates for 1 Euro with both British pounds and US dollars:

$$\$0.8770 = 1 \text{ Euro}$$

$$£0.6187 = 1 \text{ Euro}$$

Thus, $\$0.8770 = £0.6187$ (since they are each equivalent to 1 Euro).

Step 2 $\$1 \text{ US} = £1 \times \frac{0.6178}{0.8770} \text{ Euro}$ (dividing both sides by 0.8770 to get rate for \$1.)

Step 3 $\$20 \text{ US} = £20 \times \frac{0.6187}{0.8770} \text{ Euro}$ (multiplying both sides by 20)
 $= £14.1095 .$

□ Currency Conversions

Ex Continued: (b) How many British pounds are equivalent to (i) \$500 Australian and (ii) \$10000 Singapore?

Solution: (b) (i)

Step 1 From Table 1.1 write down the exchange rates for 1 Euro with Australian dollars and British pounds:

$$£0.6178 = 1 \text{ Euro}$$

$$\$1.6988 \text{ Aus} = 1 \text{ Euro}$$

Here we are *given* \$500 Aus and we require its equivalent in British pounds
 $\$1.6988 \text{ Aus} = £0.6187$ since they are each equivalent to 1 Euro

□ Currency Conversions

Ex Continued: (b) How many British pounds are equivalent to (i) \$500 Australian and (ii) \$10000 Singapore?

Solution: (b) (i)

Step 2 \$1 Aus = £1 $\times \frac{0.6178}{1.6988}$ Euro dividing both sides by 1.6988 to get rate for \$1 Aus.

Step 3 \$500 Aus = £500 $\times \frac{0.6187}{0.8770}$ Euro multiplying both sides by 500
= £182.0991.

□ Simple Inequalities

An equation is an equality. It states that the expression on the LHS of the '=' sign is equal to the expression on the RHS. An inequality is a statement in which the expression on the LHS is either greater than (denoted by the symbol $>$) or less than (denoted by the symbol $<$) the expression on the RHS.

□ Simple Inequalities

For example,

$5 = 5$ or $5x = 5x$ are equations, while

$5 > 3$ $5x > 3x$ are inequalities which read, '5 is greater than 3'; ' $5x$ is greater than $3x$ ' (for any positive value of x).

Note: Inequalities may be read from left to right, as above, or the inequality may be read from right to left, in which case the above inequalities are

$5 > 3$ (5 is greater than 3) is the same as $3 < 5$ ('3 is less than 5')

$5x > 3x$ (' $5x$ is greater than $3x$ ') is the same as $3x < 5x$ (' $3x$ is less than $5x$ ')

□ Simple Inequalities

Inequality symbols

$>$ greater than

$\geq$ greater than or equal to

$<$ less than

$\leq$ less than or equal to

□ Simple Inequalities

The number line

The number line is a horizontal line on which every point represents a real number. The central point is zero, the numbers on the left are negative, numbers on the right are positive, as illustrated for selected numbers in Figure 1.1.

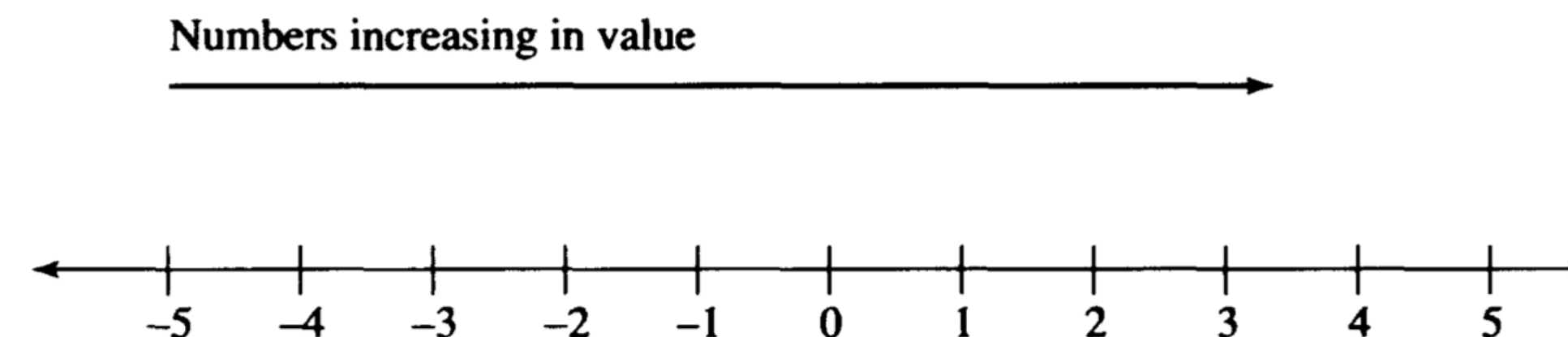


Figure 1.1 Number line, numbers increasing from left to right

□ Simple Inequalities

In Economics It is meaningful to talk about positive prices and quantities only, so in solving inequalities, we shall assume that the variable in the inequality is always positive.

Manipulating inequalities

The inequality remains true when constants are added or subtracted to both sides of the inequality sign, or when both sides of the inequality are multiplied or divided by positive numbers or variables.

However, when both sides of the inequality are multiplied or divided by negative numbers or variables, then the statement remains true only when the direction of the inequality changes: $>$ becomes $<$ and vice versa.

Ex: $5(-2) > 3(-2)$ or $-10 > -6$ is not true

$5(-2) < 3(-2)$ or $-10 < -6$ is true



□ Simple Inequalities

Solving inequalities

The solution of an equation is a distinct value(s) for which the equation statement is true.

For example, $x + 4 = 10$ is true when $x = 6$ only. On the other hand, the solution of an inequality is a range of values for which the inequality statement is true; for example, $x + 4 > 10$ is true when $x > 6$.

□ Simple Inequalities

Intervals defined by inequality statements

When an application uses values within a certain range only, then inequality signs are often used to define this interval precisely. For example, suppose a tax is imposed on all incomes, (£ x), between £10000 and £15 000 inclusive, we say the tax is imposed on all salaries within the interval, $10000 < x < 15000$.

□ Calculating Percentages

When we speak of 5% of a number we mean $\frac{5}{100} \times \text{number}$

When we say a number increases by $x\%$, then the increase in the number
 $= \text{number} \times \frac{x}{100}$

□ Calculating Percentages

Ex: A salary of £55,240 is to be increased by 12%. Calculate (i) the increase, (ii) the new salary.

Solution:

$$(i) \ 12\% \text{ of } 55,240 = \frac{12}{100} \frac{55240}{1} = \frac{12(5524)}{10} = \frac{66288}{10} = 6628.8 = \text{increase. So the}$$

increase in salary is £6,628.8

$$(ii) \text{ The new salary is } £55,240 + £6,628.8 = £61,868.8.$$

□ Using the Calculator

You will require a scientific calculator for the remainder of the text. The keys which shall be required most frequently are:

Mathematical functions

add: [+]
subtract: [-]
multiply: [x]
divide: [/]
power: [^]
change of sign [+/-]
squares: [x^2]
square roots: [$\sqrt{}$]
powers: [x^y]
roots: [$x^{\frac{1}{y}}$]
logs to base 10: [log]
antilog to base 10 [10^x]
logs to base e: [ln]
antilog to base e: [e^x]

□ Using the Calculator

Accuracy: rounding numbers correct to x decimal places

When you use the calculator you will frequently end up with a string of numbers after the decimal point. For example, $(15)/7 = 2.1428571$. For most purposes you do not require all these numbers. However, if some of the numbers are dropped, subsequent calculations are less accurate. To minimise this loss of accuracy, there are rules for 'rounding' numbers correct to a specified number of decimal places, as illustrated by the following example.

□ Using the Calculator

Accuracy: rounding numbers correct to x decimal places

Ex: Consider (a) $(15)/7 = 2.1428571$, (b) $6/7 = 0.8571428$. Assume that three numbers after the decimal point are required. To 'round' correct to three decimal places, denoted as 3D, inspect the number in the fourth decimal place:

□ Using the Calculator

Accuracy: rounding numbers correct to x decimal places

Solution:

- If the number in the fourth decimal place is less than 5, simply retain the first three numbers after the decimal place:
(b) $6/7 = 0.8571428$: use 0.857, when rounded correct to 3D.

□ Using the Calculator

Accuracy: rounding numbers correct to x decimal places

Solution:

- If the number in the fourth decimal place is 5 or greater, then increase the number in the third decimal place by 1, before dropping the remaining numbers:
(a) $(15)/7 = 2.1428571$: use 2.143, when rounded correct to 3D.