

Mathematics

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Lecture 8

جامعة الجميع الذكية

تسعى لتكون الكيان الأول الذي يتبنى مبدأ المجتمع التعليمي
التعاوني ويقدم العلم مقابل العمل لتنمية وبناء المجتمعات.

MAT 122

Advanced Topics in Linear Algebra



جامعة الجميع الذكية
EVERYONE'S SMART UNIVERSITY

- Definition of Inverse
- Inverse of 3×3 matrix using Adjoint matrix
- How to check the inverse matrix is achieved or not
- Inverse of 3×3 matrix using Reduced Echelon Form
- Inverse of 3×3 matrix using Scientific Calculator

Inverse of 3×3 matrix using Adjoint matrix

First Method

Find the inverse of the following:

$$A = \begin{bmatrix} 9 & 2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & -2 \end{bmatrix}$$

Solution of Example 1

$$|A| = \begin{vmatrix} 9 & 2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & -2 \end{vmatrix}$$

$$= 9 ((-1) \times (-2) - 0 \times 6) - 2 (5 \times (-2) - 4 \times 6) + 1 (5 \times 0 - 4 \times (-1))$$

$$= 9 (2 - 0) - 2 (-10 - 24) + 1 (0 + 4)$$

$$= 9 \times 2 - 2 \times (-34) + 1 \times 4$$

$$= 18 + 68 + 4$$

$$= 90$$

Since $|A| \neq 0$,

Inverse of A is possible

Solution of Example 1

Finding cofactors

$$A_{11} = \begin{vmatrix} 9 & 2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & -2 \end{vmatrix} = \begin{vmatrix} -1 & 6 \\ 0 & -2 \end{vmatrix} = (-1) \times (-2) - 0 \times 6 = 2 - 0 = 2$$

$$A_{12} = -1 \times \begin{vmatrix} 9 & 2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & -2 \end{vmatrix} = -1 \times \begin{vmatrix} 5 & 6 \\ 4 & -2 \end{vmatrix} = -(5 \times (-2) - 4 \times 6) = 34$$

$$A_{13} = \begin{vmatrix} 9 & 2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & -2 \end{vmatrix} = \begin{vmatrix} 5 & -1 \\ 4 & 0 \end{vmatrix} = 5 \times 0 - 4 \times (-1) = 0 + 4 = 4$$

Solution of Example 1

$$A_{21} = -1 \times \begin{vmatrix} 9 & 2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & -2 \end{vmatrix} = -1 \times \begin{vmatrix} 2 & 1 \\ 0 & -2 \end{vmatrix} = -(2 \times (-2) - 0 \times 1) = 4$$

$$A_{22} = \begin{vmatrix} 9 & 2 & 1 \\ 5 & 1 & 6 \\ 4 & 0 & -2 \end{vmatrix} = \begin{vmatrix} 9 & 1 \\ 4 & -2 \end{vmatrix} = (9 \times (-2) - 4 \times 1) = -22$$

Solution of Example 1

$$A_{23} = -1 \times \begin{vmatrix} 9 & 2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & -2 \end{vmatrix} = -1 \times \begin{vmatrix} 9 & 2 \\ 4 & 0 \end{vmatrix} = -(9 \times 0 - 4 \times 2) = 8$$

$$A_{31} = \begin{vmatrix} 9 & 2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ -1 & 6 \end{vmatrix} = 2 \times 6 - (-1) \times 1 = 12 + 1 = 13$$

Solution of Example 1

$$A_{32} = -1 \times \begin{vmatrix} 9 & 2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & 2 \end{vmatrix} = -1 \times \begin{vmatrix} 9 & 1 \\ 5 & 6 \end{vmatrix} = -(9 \times 6 - 5 \times 1) = -49$$

$$A_{33} = \begin{vmatrix} 9 & 2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & -2 \end{vmatrix} = \begin{vmatrix} 9 & 2 \\ 5 & -1 \end{vmatrix} = 9 \times (-1) - 5 \times 2 = -9 - 10 = -19$$

Solution of Example 1

Thus,

$$\text{adj } A = \begin{bmatrix} 2 & 4 & 13 \\ 34 & -22 & -49 \\ 4 & 8 & -19 \end{bmatrix}$$

And,

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$= \frac{1}{90} \begin{bmatrix} 2 & 4 & 13 \\ 34 & -22 & -49 \\ 4 & 8 & -19 \end{bmatrix}$$



Solution of Example 1

$$= \frac{1}{90} \begin{bmatrix} 2 & 4 & 13 \\ 34 & -22 & -49 \\ 4 & 8 & -19 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{90} & \frac{4}{90} & \frac{13}{90} \\ \frac{34}{90} & \frac{-22}{90} & \frac{-49}{90} \\ \frac{4}{90} & \frac{8}{90} & \frac{-19}{90} \end{bmatrix} = \begin{bmatrix} \frac{1}{45} & \frac{2}{45} & \frac{13}{90} \\ \frac{17}{45} & \frac{-11}{45} & \frac{-49}{90} \\ \frac{2}{45} & \frac{4}{45} & \frac{-19}{90} \end{bmatrix}$$

$$AA^{-1}$$

$$= \begin{bmatrix} 9 & 2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & -2 \end{bmatrix} \begin{bmatrix} \frac{1}{45} & \frac{2}{45} & \frac{13}{90} \\ \frac{17}{45} & \frac{-11}{45} & \frac{-49}{90} \\ \frac{2}{45} & \frac{4}{45} & \frac{-19}{90} \end{bmatrix}$$

$$= \begin{bmatrix} 9\left(\frac{1}{45}\right) + 2\left(\frac{17}{45}\right) + 1\left(\frac{2}{45}\right) & 9\left(\frac{2}{45}\right) + 2\left(\frac{-11}{45}\right) + 1\left(\frac{4}{45}\right) & 9\left(\frac{13}{90}\right) + 2\left(\frac{-49}{90}\right) + 1\left(\frac{-19}{90}\right) \\ 5\left(\frac{1}{45}\right) + (-1)\left(\frac{17}{45}\right) + 6\left(\frac{2}{45}\right) & 5\left(\frac{2}{45}\right) + (-1)\left(\frac{-11}{45}\right) + 6\left(\frac{4}{45}\right) & 5\left(\frac{13}{90}\right) + (-1)\left(\frac{-49}{90}\right) + 6\left(\frac{-19}{90}\right) \\ 4\left(\frac{1}{45}\right) + 0\left(\frac{17}{45}\right) + (-2)\left(\frac{2}{45}\right) & 4\left(\frac{2}{45}\right) + 0\left(\frac{-11}{45}\right) + (-2)\left(\frac{4}{45}\right) & 9\left(\frac{13}{90}\right) + 0\left(\frac{-49}{90}\right) + (-2)\left(\frac{-19}{90}\right) \end{bmatrix}$$

How to check the inverse matrix is achieved

$$= \begin{bmatrix} \frac{9}{45} + \frac{34}{45} + \frac{2}{45} & \frac{18}{45} - \frac{22}{45} + \frac{4}{45} & \frac{117}{90} - \frac{98}{90} - \frac{19}{90} \\ \frac{5}{45} - \frac{17}{45} + \frac{12}{45} & \frac{10}{45} + \frac{11}{45} + \frac{24}{45} & \frac{65}{90} + \frac{49}{90} - \frac{114}{90} \\ \frac{4}{45} - \frac{4}{45} & \frac{8}{45} - \frac{8}{45} & \frac{52}{90} + \frac{38}{90} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{45}{45} & 0 & 0 \\ 0 & \frac{45}{45} & 0 \\ 0 & 0 & \frac{90}{90} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathbf{I}$$

Thus,
 $AA^{-1} = \mathbf{I}$

Inverse of 3×3 matrix using Reduced Echelon Form
Second Method

$$M = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}; \quad M^{-1} = ?$$
$$\left[\begin{array}{ccc|ccc} 3 & 0 & 2 & 1 & 0 & 0 \\ 2 & 0 & -2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array} \right]$$

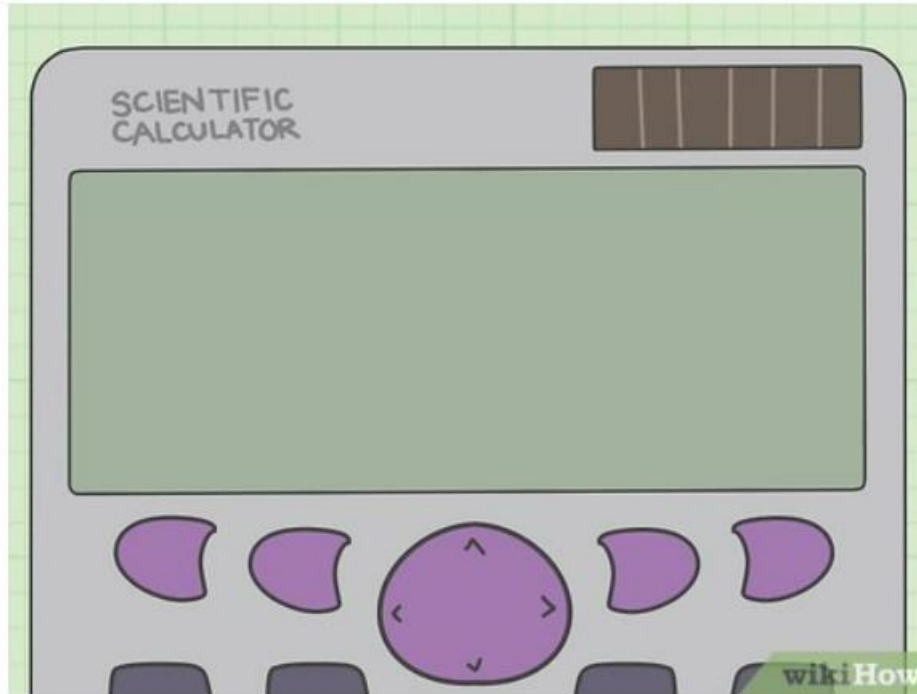
$$\begin{bmatrix} 3 & 0 & 2 & | & 1 & 0 & 0 \\ 2 & 0 & -2 & | & 0 & 1 & 0 \\ 0 & 1 & 1 & | & 0 & 0 & 1 \end{bmatrix} \quad R1' = R1 + R2$$
$$\begin{bmatrix} 5 & 0 & 0 & | & 1 & 1 & 0 \\ 2 & 0 & -2 & | & 0 & 1 & 0 \\ 0 & 1 & 1 & | & 0 & 0 & 1 \end{bmatrix} \quad R1' = R1 \div 5$$
$$\begin{bmatrix} 1 & 0 & 0 & | & 0.2 & 0.2 & 0 \\ 2 & 0 & -2 & | & 0 & 1 & 0 \\ 0 & 1 & 1 & | & 0 & 0 & 1 \end{bmatrix} \quad \dots$$

$$\left[\begin{array}{ccc|ccc} 3 & 0 & 2 & 1 & 0 & 0 \\ 2 & 0 & -2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array} \right] \rightarrow \dots \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.2 & 0.2 & 0 \\ 0 & 1 & 0 & -0.2 & 0.3 & 1 \\ 0 & 0 & 1 & 0.2 & -0.3 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.2 & 0.2 & 0 \\ 0 & 1 & 0 & -0.2 & 0.3 & 1 \\ 0 & 0 & 1 & 0.2 & -0.3 & 0 \end{array} \right]$$
$$M^{-1} = \begin{bmatrix} 0.2 & 0.2 & 0 \\ -0.2 & 0.3 & 1 \\ 0.2 & -0.3 & 0 \end{bmatrix}$$

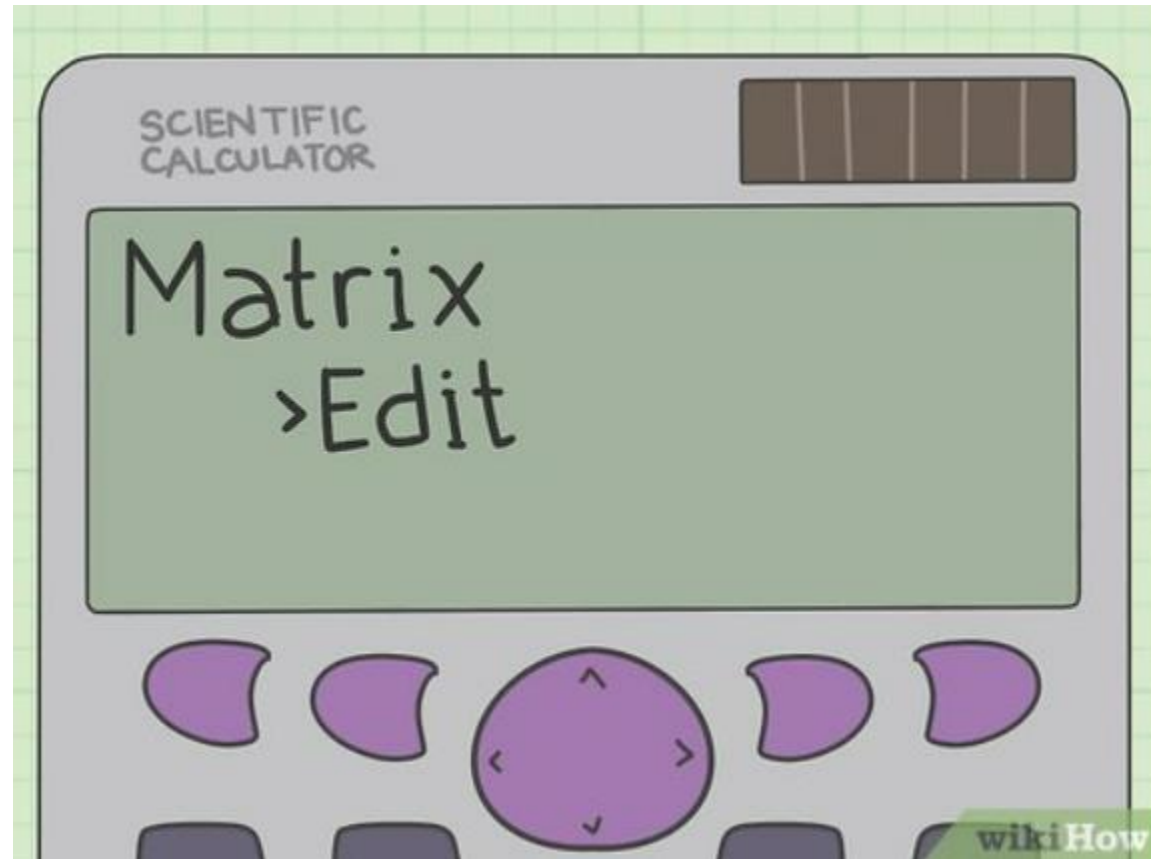
Inverse of 3×3 matrix using Calculator Third Method

Inverse of 3×3 matrix using Calculator

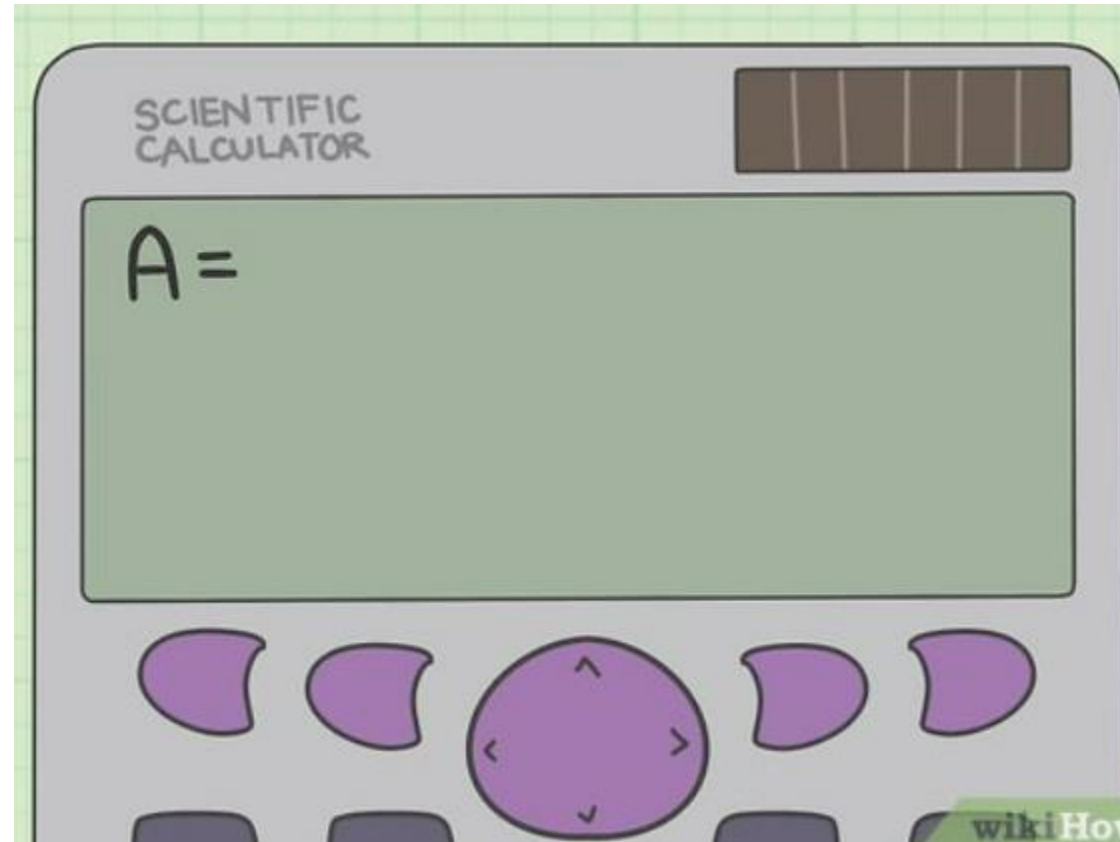


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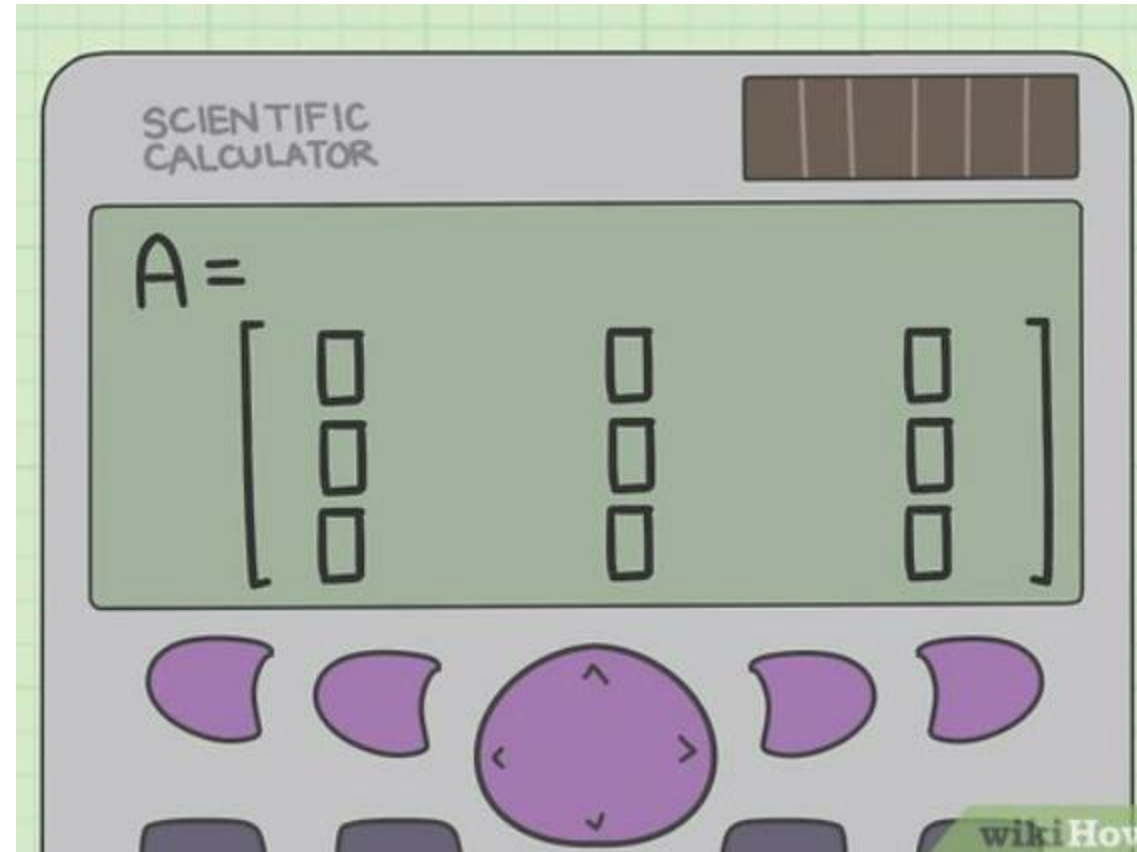
Inverse of 3×3 matrix using Calculator



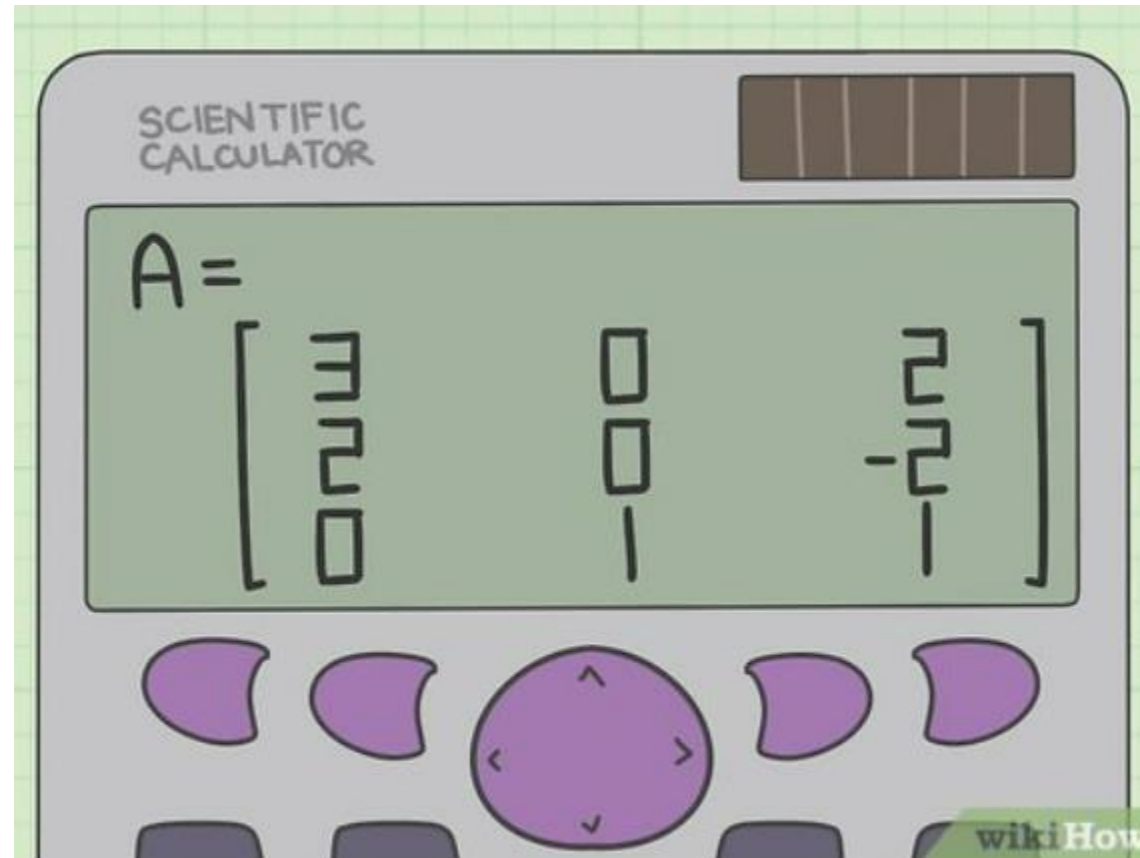
Inverse of 3×3 matrix using Calculator



Inverse of 3×3 matrix using Calculator

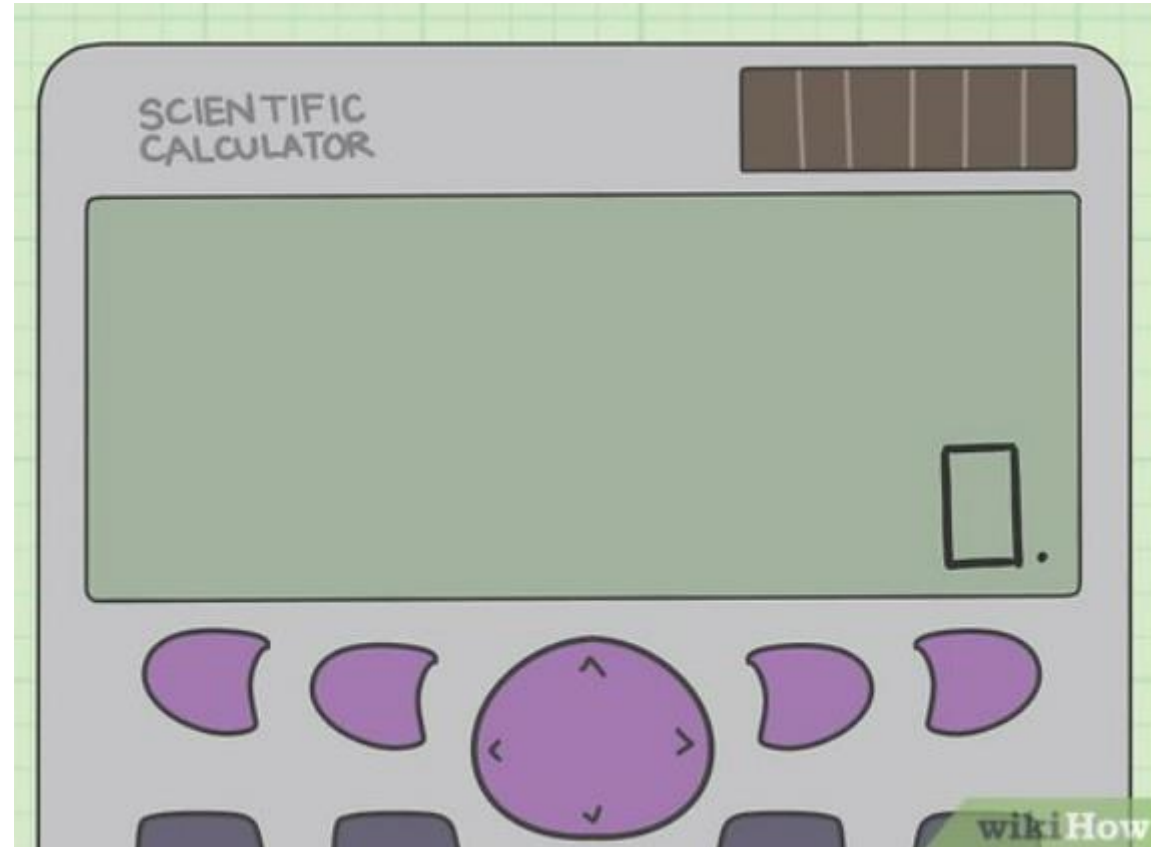


Inverse of 3×3 matrix using Calculator

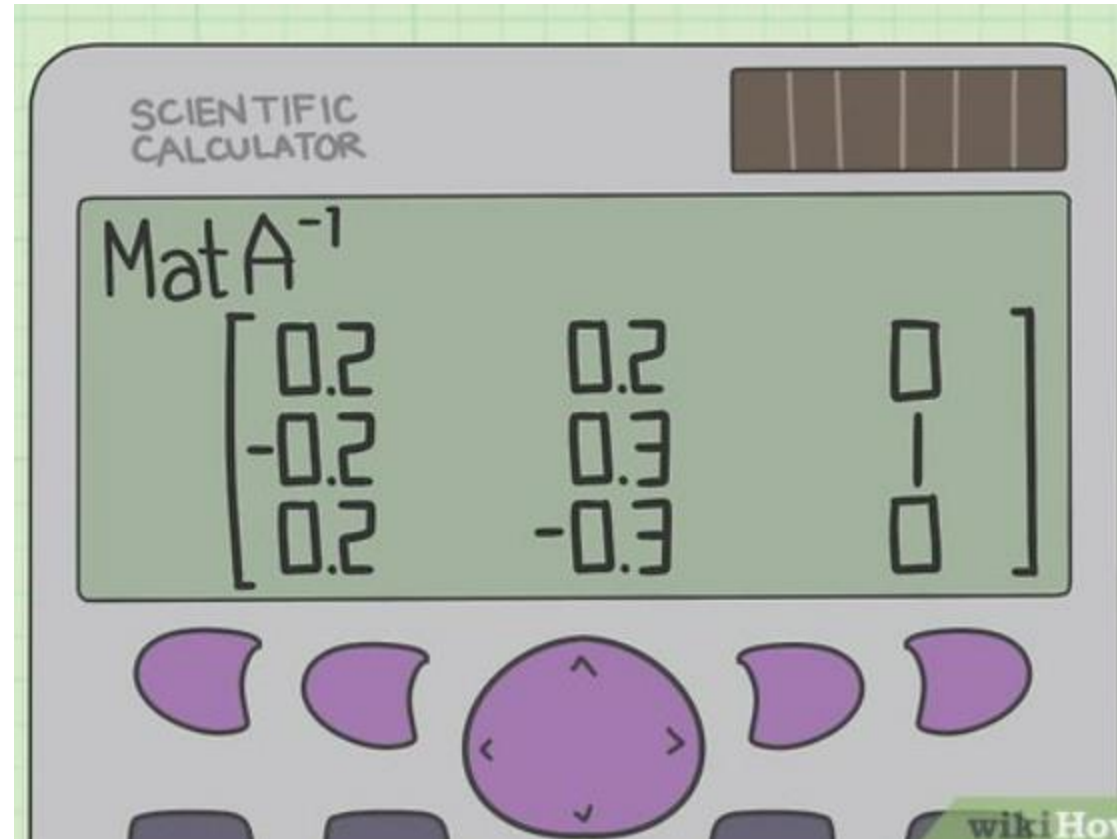


Lecture 08

Inverse of 3×3 matrix using Calculator

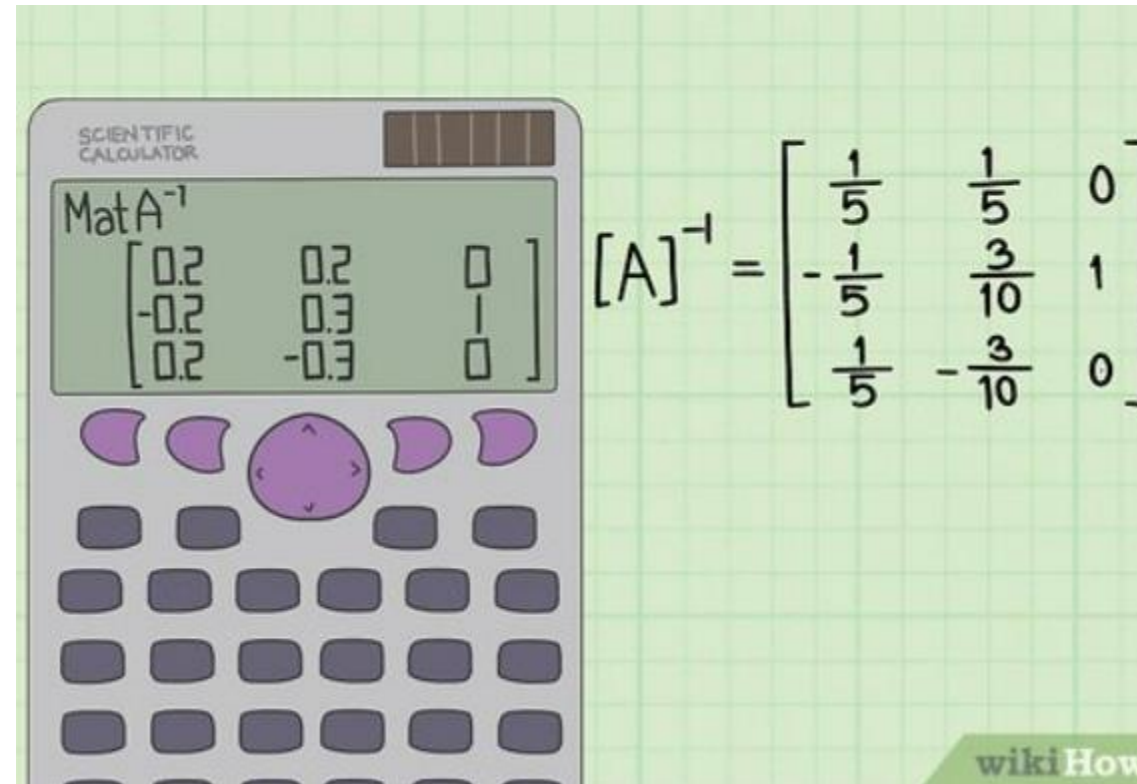


Inverse of 3×3 matrix using Calculator



Lecture 08

Inverse of 3×3 matrix using Calculator



Let's Practice

Find the inverse of the following:

$$A = \begin{bmatrix} 1 & 5 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$

Solution of Example 1

$$A = \begin{bmatrix} 1 & 5 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$

Thus we have $\det(A) = -1 \neq 0$, and hence A is invertible.

To find the inverse using the formula, we first determine the cofactors C_{ij} of A .

We have

$$C_{11} = \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -1, \quad C_{12} = -\begin{vmatrix} 0 & 2 \\ 0 & 1 \end{vmatrix} = 0, \quad C_{13} = \begin{vmatrix} 0 & -1 \\ 0 & 0 \end{vmatrix} = 0$$

$$C_{21} = -\begin{vmatrix} 5 & 2 \\ 0 & 1 \end{vmatrix} = -5, \quad C_{22} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1, \quad C_{23} = -\begin{vmatrix} 1 & 5 \\ 0 & 0 \end{vmatrix} = 0$$

$$C_{31} = \begin{vmatrix} 5 & 2 \\ -1 & 2 \end{vmatrix} = 12, \quad C_{32} = -\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = -2, \quad C_{33} = \begin{vmatrix} 1 & 5 \\ 0 & -1 \end{vmatrix} = -1.$$

The the adjoint matrix of A is

$$\text{Adj}(A) = C^T = \begin{bmatrix} -1 & -5 & 12 \\ 0 & 1 & -2 \\ 0 & 0 & -1 \end{bmatrix}.$$

Solution of Example 1

Using the formula, we obtain the inverse matrix

$$A^{-1} = \frac{1}{\det(A)} \text{Adj}(A) = \begin{bmatrix} 1 & 5 & -12 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$

Example 2

Find the inverse of the following:

$$B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \\ 3 & 0 & 1 \end{bmatrix}.$$

First, we have to obtain the value of the determinant

$$\det(B) = -5 \neq 0.$$

So the matrix B is invertible.

Solution of Example 2

$$B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \\ 3 & 0 & 1 \end{bmatrix}.$$

Now the cofactors C_{ij} of B are

$$C_{11} = \begin{vmatrix} 1 & 4 \\ 0 & 1 \end{vmatrix} = 1, \quad C_{12} = -\begin{vmatrix} 0 & 4 \\ 3 & 1 \end{vmatrix} = 12, \quad C_{13} = \begin{vmatrix} 0 & 1 \\ 3 & 0 \end{vmatrix} = -3$$

$$C_{21} = -\begin{vmatrix} 0 & 2 \\ 0 & 1 \end{vmatrix} = 0, \quad C_{22} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -5, \quad C_{23} = -\begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} = 0$$

$$C_{31} = \begin{vmatrix} 0 & 2 \\ 1 & 4 \end{vmatrix} = -2, \quad C_{32} = -\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} = -4, \quad C_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1.$$



Solution of Example 2

$$B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \\ 3 & 0 & 1 \end{bmatrix}.$$

Now the cofactors C_{ij} of B are

$$C_{11} = \begin{vmatrix} 1 & 4 \\ 0 & 1 \end{vmatrix} = 1, \quad C_{12} = -\begin{vmatrix} 0 & 4 \\ 3 & 1 \end{vmatrix} = 12, \quad C_{13} = \begin{vmatrix} 0 & 1 \\ 3 & 0 \end{vmatrix} = -3$$

$$C_{21} = -\begin{vmatrix} 0 & 2 \\ 0 & 1 \end{vmatrix} = 0, \quad C_{22} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -5, \quad C_{23} = -\begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} = 0$$

$$C_{31} = \begin{vmatrix} 0 & 2 \\ 1 & 4 \end{vmatrix} = -2, \quad C_{32} = -\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} = -4, \quad C_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1.$$



Hence the adjoint matrix of B is

$$\text{Adj}(B) = C^T = \begin{bmatrix} 1 & 0 & -2 \\ 12 & -5 & -4 \\ -3 & 0 & 1 \end{bmatrix}.$$

It follows from the formula that the inverse matrix of B is

$$B^{-1} = \frac{1}{\det(B)} \text{Adj}(B) = \frac{1}{5} \begin{bmatrix} -1 & 0 & 2 \\ -12 & 5 & 4 \\ 3 & 0 & -1 \end{bmatrix}.$$

Find the inverse of the following:

$$\begin{bmatrix} 1 & 0 & -2 \\ 12 & 0 & -4 \\ -3 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 12 & 0 & -4 \\ -3 & 0 & 1 \end{bmatrix}$$

First, we will obtain the value of the determinant for this matrix:

Det of the matrix = Zero

So, this matrix is noninvertible.

- Definition of Inverse
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- How to check the inverse matrix is achieved or not
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