

Business Mathematics

Prepared by Dr. Alfaisal Hasan

MTH 112
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Lecture NO 1



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Chapter

Topics

- **Chapter 1: Mathematical preliminaries**
- Chapter 2: The straight line and applications
- Chapter 3: Functions and Graphs
- Chapter 4: Limits and Continuity
- Chapter 5: Financial Mathematics
- Chapter 6: Differentiation
- Chapter 7: Methods of Differentiation
- Chapter 8: Function of several variables
- Chapter 9: Integration and applications
- Chapter 10: Linear algebra and applications

Some Mathematical Preliminaries

Powers: positive whole numbers such as 2^4 which means
 $2 \times 2 \times 2 \times 2 = 16$

$$(\text{anything})^3 = (\text{anything}) \times (\text{anything}) \times (\text{anything})$$

$$(x)^3 = x \times x \times x$$

$$(x + 4)^5 = (x + 4)(x + 4)(x + 4)(x + 4)(x + 4)$$

Some Mathematical Preliminaries

Variables and letters: when we don't know the value of quantity, we give that quantity a symbol, such as x
This statements may be expressed as a mathematical equation:

$$15 \times \text{weekly savings} = 4000$$

$$15 \times x = 4000 \quad \text{or} \quad 15x = 4000$$

Now that the statement has been reduced to a mathematical equation, we may solve the equation for the unknown, x :

$$15x = 4000$$

$$\frac{15x}{15} = \frac{4000}{15} \quad \text{divide both sides of the equation by 15}$$

$$x = 266.67$$

Some Mathematical Preliminaries

Square roots: the square root of a number is the reverse of squaring:

$$(2)^2 = 4 \rightarrow \sqrt{4} = 2$$

$$(2.5)^2 = 6.25 \rightarrow \sqrt{6.25} = 2.5$$

Arithmetic Operations

Adding: If all the signs are the same, simply add all the terms and give the answer with the common overall sign.

Subtracting: When subtract any two numbers or any two similar terms, give the answer with the sign of the largest number or terms.

Arithmetic Operations

Add/subtract with numbers, mostly

$$5 + 8 + 3 = 16 \quad \text{similarity} \rightarrow$$

$$5 + 8 + 3 + y = 16 + y \quad \text{similarity} \rightarrow$$

The y -term is different, so it cannot be added to the others

Add/subtract with variable terms

$$5x + 8x + 3x = 16x$$

$$(i) \quad 5x + 8x + 3x + y = 16x + y$$

$$(ii) \quad 5xy + 8xy + 3xy + y = 16xy + y$$

The y -term is different, so it cannot be added to the others

Arithmetic Operations

$$7 - 10 = -3 \quad \text{similarity} \rightarrow$$

$$7 - 10 - 10x = -3 - 10x \quad \text{similarity} \rightarrow$$

The x -term is different, so it cannot be subtracted from the others

The y -term is different, so it cannot be added to the others

$$(i) \quad 7x - 10x = -3x$$

$$(ii) \quad 7x^2 - 10x^2 = -3x^2$$

$$7x^2 - 10x^2 - 10x = -3x^2 - 10x$$

The x -term is different, so it cannot be subtracted from the others

Fractions

Terminology:

$$\text{fraction} = \frac{\text{numerator}}{\text{denominator}}$$

$$\frac{3}{7} \quad \left| \begin{array}{l} \leftarrow 3 \text{ is called the numerator} \\ \leftarrow 7 \text{ is called the denominator} \end{array} \right.$$

Fractions

Numerical example

$$\frac{1}{7} + \frac{2}{3} - \frac{4}{5}$$

Step 1: The common denominator is $(7)(3)(5)$

$$\text{Step 2: } \frac{1}{7} + \frac{2}{3} - \frac{4}{5}$$

$$= \frac{1(3)(5) + 2(7)(5) - 4(7)(3)}{(7)(3)(5)}$$

$$\text{Step 3: } = \frac{15 + 70 - 84}{105} = \frac{1}{105}$$

Fractions

$$(a) \quad \left(\frac{2}{3}\right)\left(\frac{5}{7}\right) = \frac{(2)(5)}{(3)(7)} = \frac{10}{21}$$

$$(b) \quad \left(-\frac{2}{3}\right)\left(\frac{7}{5}\right) = \frac{(-2)(7)}{(3)(5)} = -\frac{14}{15}$$

$$(c) \quad 3 \times \frac{2}{5} = \left(\frac{3}{1}\right)\left(\frac{2}{5}\right) = \frac{(3)(2)}{(1)(5)} = \frac{6}{5} = 1\frac{1}{5}$$

The same rules apply for fractions involving variables, x , y , etc.

$$(d) \quad \left(\frac{3}{x}\right)\frac{(x+3)}{(x-5)} = \frac{3(x+3)}{x(x-5)} = \frac{3x+9}{x^2-5x}$$

Find the value of

$$\left(\frac{4}{9}\right)\left(\frac{1}{5}\right)$$

$$\left(\frac{4}{9}\right)\left(\frac{1}{5}\right) = \left(\frac{4 \times 1}{9 \times 5}\right) = \frac{4}{45}$$

Find the value of

$$\left(\frac{3}{10}\right)\left(\frac{6}{5}\right)$$

$$\left(\frac{3}{10}\right)\left(\frac{6}{5}\right) = \left(\frac{3 \times 6}{10 \times 5}\right) = \frac{18}{50}$$

Chapter

Find the value of

$${}_{(6)}\left(\frac{3}{7}\right)$$

$$\left(\frac{6}{1}\right)\left(\frac{3}{7}\right) = \left(\frac{6 \times 3}{1 \times 7}\right) = \frac{18}{7}$$

Solving Equations

- What do you notice about each of the following?

$$6 + 3 = 9$$

$$5 - 3 = 2$$

$$5 + 3 = 1 + 7$$

$$x = 4.$$

$$2x = x + x$$

$$3x = 2x + x = x + x + x$$

- What does this picture represent?
- What is this apparatus called?

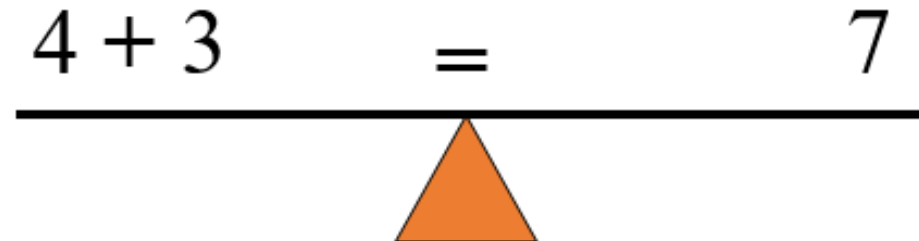


Solving Equations

What is an equation?

Equation - a balanced statement of equality between two quantities.

$$4 + 3 = 7$$


$$\begin{array}{ccc} 4 + 3 & = & 7 \\ \hline \end{array}$$

Solving Equations

Solve.

$$\begin{array}{r} \text{a) } m + 2 = 10 \\ -2 \quad -2 \\ \hline m = 8 \end{array}$$

$$\begin{array}{r} \text{b) } x - 4 = 6 \\ +4 \quad +4 \\ \hline x = 10 \end{array}$$

$$\begin{array}{r} \text{c) } m - 4 = 9 \\ +4 \quad +4 \\ \hline m = 13 \end{array}$$

$$\begin{array}{r} \text{d) } x + 7 = 3 \\ -7 \quad -7 \\ \hline x = -4 \end{array}$$

Solving Equations

Solve.

$$\begin{array}{r} \text{a) } -4 + x = 15 \\ +4 \quad \quad +4 \\ \hline x = 19 \end{array}$$

$$\begin{array}{r} \text{b) } 7 = x + 13 \\ -13 \quad \quad -13 \\ \hline -6 = x \end{array}$$

$$\begin{array}{r} \text{c) } -4 = m + 9 \\ -9 \quad \quad -9 \\ \hline -13 = m \end{array}$$

$$\begin{array}{r} \text{d) } 9 + 7 = x \\ \hline 16 = x \end{array}$$

Solving Equations

Solve.

$$\text{a) } \frac{5x}{5} = \frac{35}{5}$$
$$x = 7$$

$$\text{b) } \frac{9}{-3} = \frac{-3m}{-3}$$
$$-3 = m$$

Solving Equations

$$\frac{-4k}{-4} = \frac{14}{-4}$$

$$k = \left(-\frac{7}{2}\right) = \left(-3\frac{1}{2}\right) = \left(-3.5\right)$$

All answers must be fully reduced

Solving Equations involving fractions

Solve. $\frac{3}{5}x = 9$

~~$\left(\frac{5}{3}\right)\frac{3}{5}x = 9\left(\frac{5}{3}\right)$~~

$$x = \frac{9}{1} \bullet \frac{5}{3} = \frac{45}{3}$$

$$x = 15$$

Easier to multiply by the reciprocal!

Solving Equations involving fractions

Solve.

$$\begin{aligned} \text{a) } \left(\frac{7}{2} \right) \frac{2}{7} y &= 14 \left(\frac{7}{2} \right) \\ y &= \frac{14}{1} \bullet \frac{7}{2} \\ x &= 49 \end{aligned}$$

$$\begin{aligned} \text{b) } \left(\frac{7}{2} \right) 16 &= \frac{2}{7} x \left(\frac{7}{2} \right) \\ \frac{7}{2} \bullet \frac{16}{1} &= x \\ 56 &= x \end{aligned}$$

Solving Equations involving fractions

Solve.

$$\text{a) } \left(\frac{\cancel{-3}}{\cancel{2}} \right) \frac{\cancel{-2}}{\cancel{3}} m = 18 \left(\frac{-3}{2} \right)$$

$$m = \frac{18}{1} \bullet \frac{-3}{2}$$

$$m = -27$$

$$\text{b) } \left(\frac{\cancel{-5}}{\cancel{4}} \right) \frac{\cancel{-4}}{\cancel{5}} m = 9 \left(\frac{-5}{4} \right)$$

$$m = \frac{9}{1} \bullet \frac{-5}{4}$$

$$m = \frac{-45}{4} = -11\frac{1}{4}$$

Chapter

Solve the equation

$$4x - 3 = x + 9$$

Solve the equation

$$4x - 3 = x + 9$$

$$4x - 3 = x + 9$$



$$4x - x - 3 = 9$$

$$3x = 9 + 3$$

$$3x = 12$$

$$x = \frac{12}{3}$$

$$x = 4$$

Chapter

Solve the equation

$$5x - 2 = 2x + 7$$

Solve the equation

$$5x - 2 = 2x + 7$$

$$5x - 2 = 2x + 7$$



$$5x - 2x - 2 = 7$$

$$3x = 7 + 2$$

$$3x = 9$$

$$x = \frac{9}{3}$$

$$x = 3$$

Simple Inequalities

An **inequality** is like an equation, but instead of an equal sign (=) it has one of these signs:

$<$: less than

$\leq$: less than or equal to

$>$: greater than

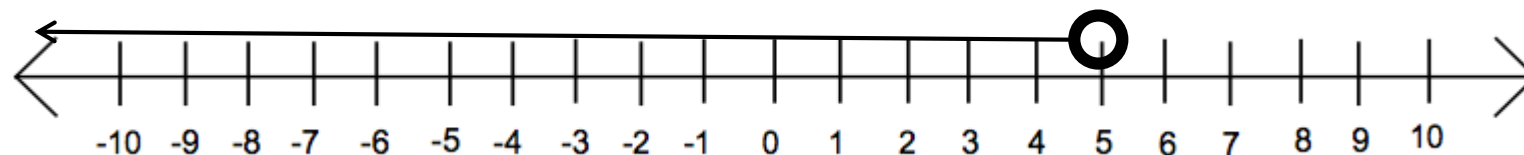
$\geq$: greater than or equal to

Simple Inequalities

$$x < 5$$

means that whatever value x has,
it must be less than 5.

What could x be?

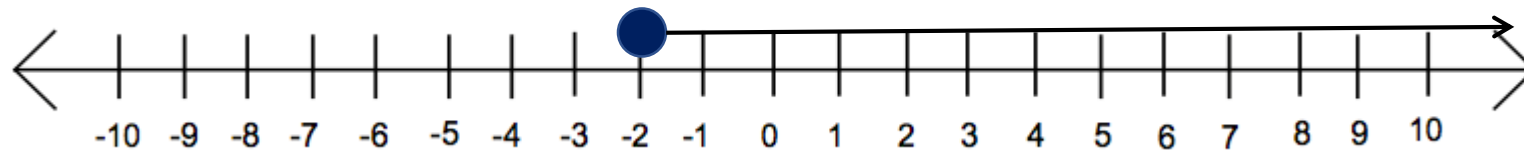


Simple Inequalities

$$x \geq -2$$

means that whatever value x has,
it must be greater than or equal to -2 .

What could x be?



Simple Inequalities

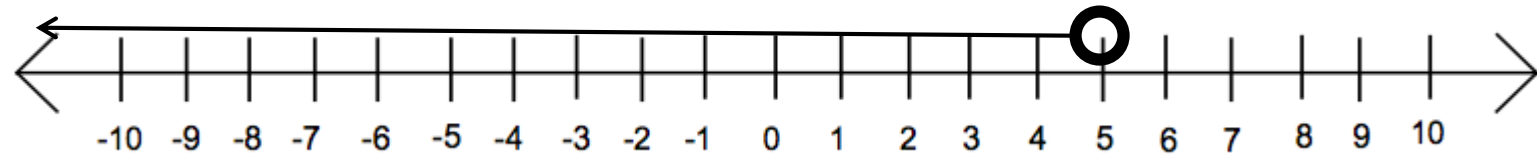
Graphing Rules

Symbol	Circle	Direction of Arrow
$<$	Open	Left
$>$	Open	Right
$\leq$	Closed	Left
$\geq$	Closed	Right

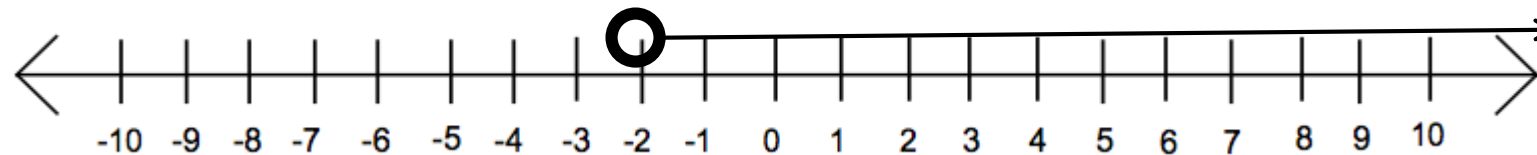
Simple Inequalities

Examples:

$$x < 5$$



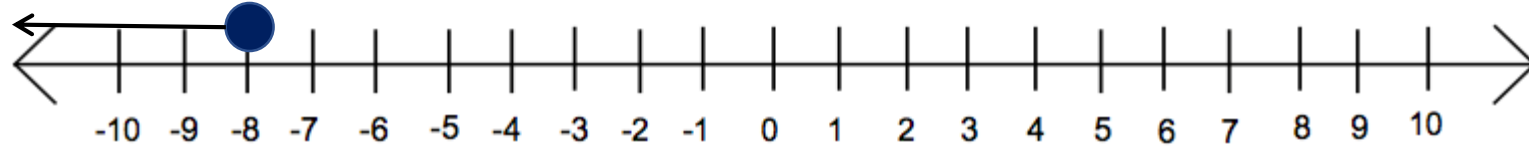
$$x > -2$$



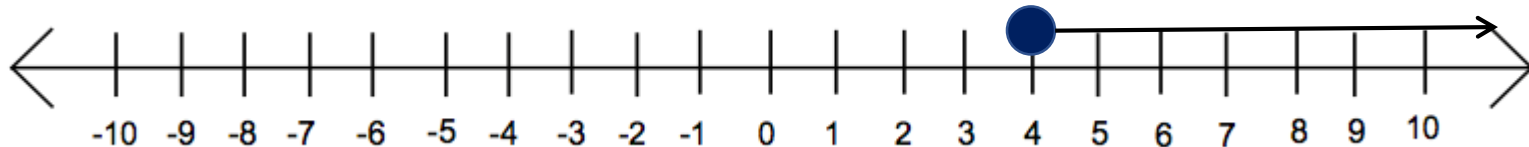
Simple Inequalities

Examples:

$$x \leq -8$$



$$x \geq 4$$



Chapter

Solving one-step Inequalities

Divide both
sides by the
coefficient of x

$$\frac{5x}{5} > \frac{20}{5}$$

$$5x > 20$$

$$x > 4$$

Solving one-step Inequalities

Multiply both sides by the reciprocal of the coefficient

$$5 \times \frac{x}{5} = 10 \times 5$$

$$\frac{x}{5} = 10$$

$$x = 50$$

Solving Inequalities

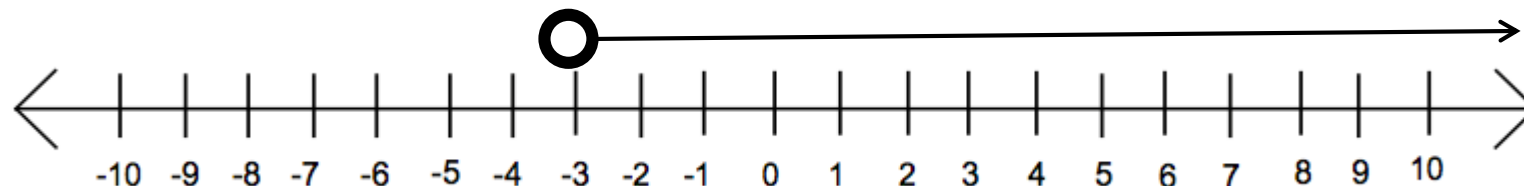
- Solving inequalities is the same as solving equations.
- There are only 2 things you need to know...
 - 1.) If you multiply or divide by a negative number you must switch the sign.

$$\frac{-7x}{-7} < \frac{21}{-7}$$

$$x > -3$$

Dividing by a negative means switch the sign!!

2.) You will graph your solutions.



Solving Inequalities

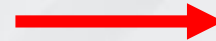
- When solving inequalities, if you multiply or divide by a negative you must switch the signs.
- **Switching the signs:**
 - Less than becomes Greater than $<$ **switches to** $>$
 - Greater than becomes Less than $>$ **switches to** $<$
 - Less than or equal to becomes Greater than or equal to
 $\leq$ **switches to** $\geq$
 - Greater than or equal to becomes Less than or equal to
 $\geq$ **switches to** $\leq$

Solving Inequalities

$$-5x < 20$$



$$\frac{-5x}{-5} < \frac{20}{-5}$$



$$x > -4$$

Caution! Dividing by
a negative number

Same if multiplying?

Notice: Sign
CHANGED

Solving Inequalities

YES!

$$\frac{-x}{5} > 2$$

$$\frac{(-5) \cdot \frac{-x}{5}}{1} > 2(-5)$$

$$x < -10$$

Caution! When you multiply by a negative number...

...the sign
CHANGES

Solving Inequalities

- Follow same steps used to solve equations:

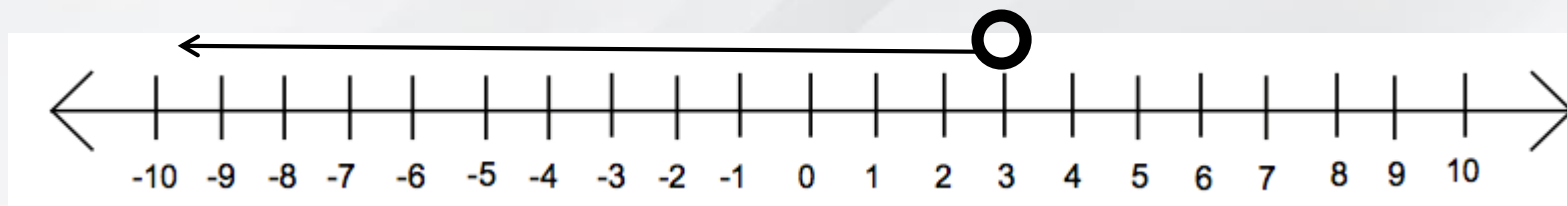
$$3x + 4 < 13$$

$$\begin{array}{r} -4 \\ -4 \end{array}$$

$$3x < 9$$

$$\begin{array}{r} 3 \\ 3 \end{array}$$

$$x < 3$$



Chapter

Solve the inequality

$$4x - 3 \leq x + 9$$

Solve the inequality

$$4x - 3 \leq x + 9$$

$$4x - 3 \leq x + 9$$



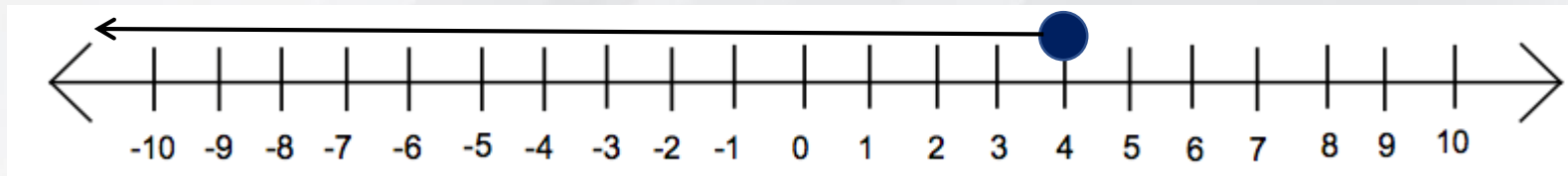
$$4x - x - 3 \leq 9$$

$$3x \leq 9 + 3$$

$$3x \leq 12$$

$$x \leq \frac{12}{3}$$

$$x \leq 4$$



Solve the inequality

$$15x - 2 > 5x + 28$$

Solve the inequality

$$15x - 2 > 5x + 28$$

$$15x - 2 > 5x + 28$$



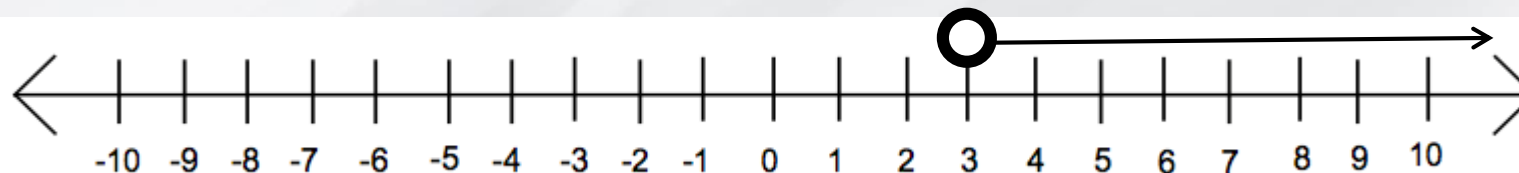
$$15x - 5x - 2 > 28$$

$$10x > 28 + 2$$

$$10x > 30$$

$$x > \frac{30}{10}$$

$$x > 3$$



Chapter

Solve the inequality

$$-6x - 3 \leq 27$$

Solve the inequality

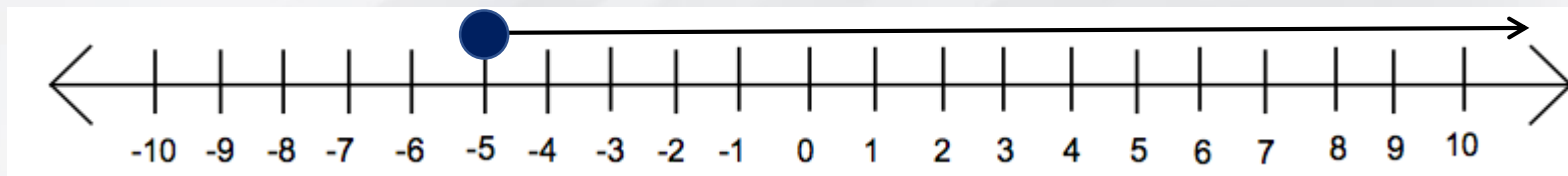
$$-6x - 3 \leq 27$$


$$\begin{aligned} -6x - 3 &\leq 27 \\ -6x &\leq 27 + 3 \\ -6x &\leq 30 \end{aligned}$$

Caution! When you multiply by a negative number...

$$\begin{aligned} \frac{-6x}{-6} &\geq \frac{30}{-6} \\ x &\geq -5 \end{aligned}$$

...the sign
CHANGES



Solving Equations

- What do you notice about each of the following?

$$6 + 3 = 9$$

$$5 - 3 = 2$$

$$5 + 3 = 1 + 7$$

$$x = 4.$$

$$2x = x + x$$

$$3x = 2x + x = x + x + x$$

- What does this picture represent?
- What is this apparatus called?



Simple Inequalities

An **inequality** is like an equation, but instead of an equal sign (=) it has one of these signs:

$<$: less than

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Simple Inequalities

Graphing Rules

Symbol	Circle	Direction of Arrow
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$\geq$	Closed	Right

Chapter

Un Solved Examples

Find the value of

$$\left(\frac{2}{9}\right)\left(\frac{1}{7}\right)$$

Chapter

Un Solved Examples**Find the value of**

$$\left(\frac{2}{9}\right)\left(\frac{1}{7}\right)$$

$$\left(\frac{2}{9}\right)\left(\frac{1}{7}\right) = \frac{2 \times 1}{9 \times 7} = \frac{2}{63}$$

Chapter

Un Solved Examples

Find the value of

$$\begin{pmatrix} \frac{3}{11} \end{pmatrix} \begin{pmatrix} \frac{2}{5} \end{pmatrix}$$

Chapter

Un Solved Examples**Find the value of**

$$\left(\frac{3}{11}\right)\left(\frac{2}{5}\right)$$

$$\left(\frac{3}{11}\right)\left(\frac{2}{5}\right) = \frac{3 \times 2}{11 \times 5} = \frac{6}{55}$$

Chapter

Un Solved Examples

Find the value of

$${}_{(4)}\left(\frac{2}{5}\right)$$

Chapter

Un Solved Examples**Find the value of**

$$({}_4) \left(\frac{2}{5} \right)$$

$$({}_4) \left(\frac{2}{5} \right) = \left(\frac{4}{1} \right) \left(\frac{2}{5} \right) = \frac{4 \times 2}{1 \times 5} = \frac{8}{5}$$

Un Solved Examples

Solve the equation

$$3x - 4 = x + 5$$

Solve the equation

$$3x - 4 = x + 5$$

$$3x - 4 = x + 5$$



$$3x - x - 4 = 5$$



$$2x = 5 + 4$$

$$2x = 9$$

$$x = \frac{9}{2}$$

Un Solved Examples

Solve the inequality

$$3x - 4 < x + 5$$

Solve the inequality

$$3x - 4 < x + 5$$

$$3x - 4 < x + 5$$

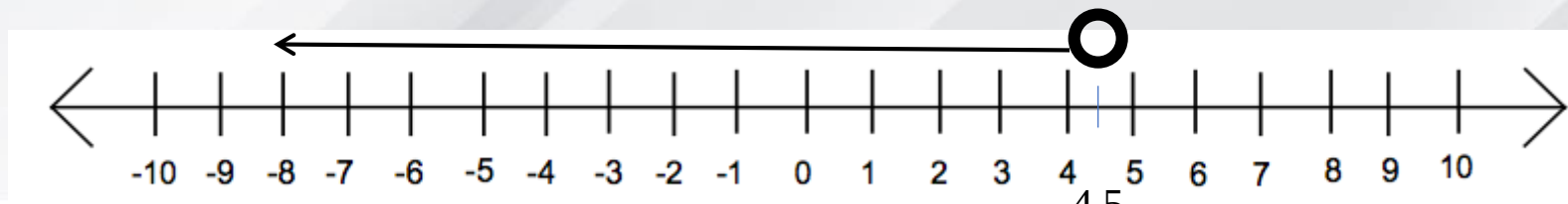


$$3x - x - 4 < 5$$

$$2x < 5 + 4$$

$$2x < 9$$

$$x < \frac{9}{2}$$



Un Solved Examples

Solve the inequality

$$-3x \leq 30$$

Solve the inequality

$$-3x \leq 30$$

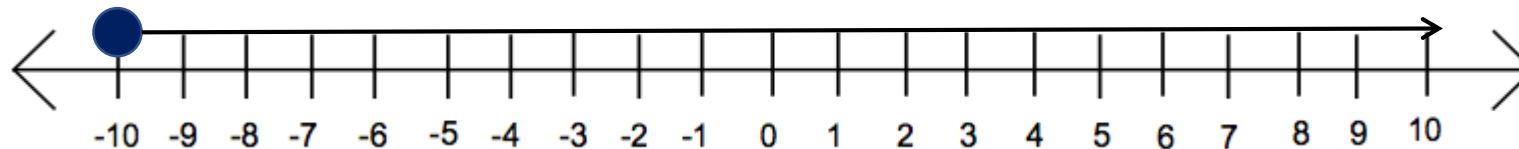
$$-3x \leq 30$$

divided by -3

$$x \geq -10$$

Caution! When you multiply by a negative number...

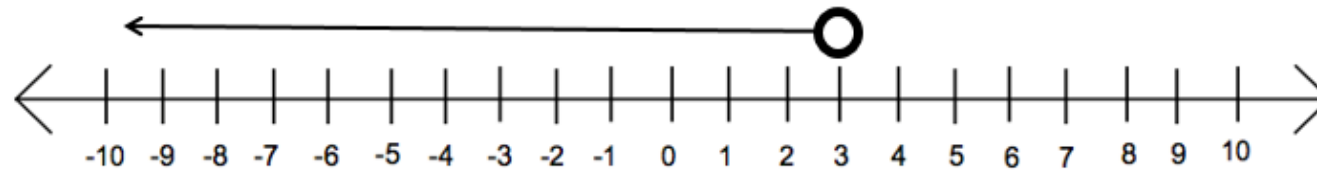
...the sign
CHANGES



Chapter

Un Solved Examples

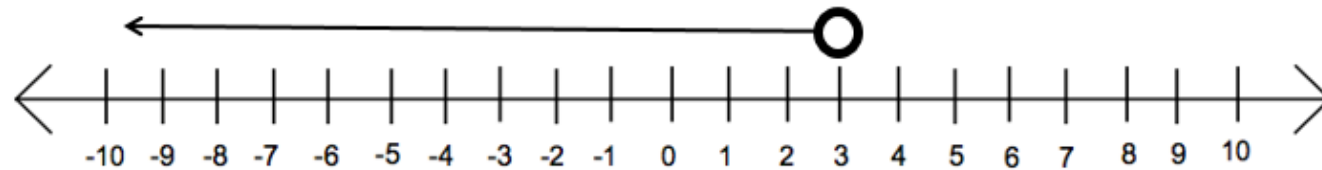
Write the solution



Chapter

Un Solved Examples

Write the solution

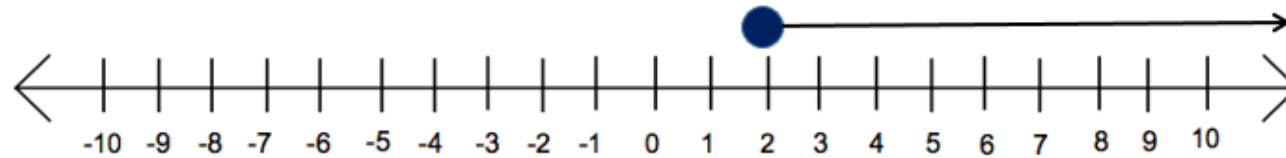


$$x < 3$$

Chapter

Un Solved Examples

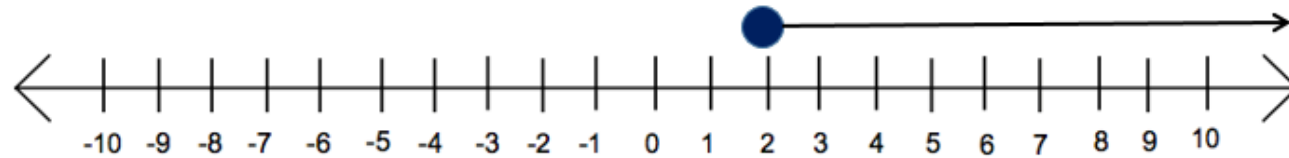
Write the solution



Chapter

Un Solved Examples

Write the solution



$$x \geq 2$$



Thank You