

Business Mathematics

Prepared by Dr. Alfaisal Hasan

MTH 112
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Lecture NO 2



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Chapter

Topics

- Chapter 1: Mathematical preliminaries
- **Chapter 2: The straight line and applications**
- Chapter 3: Functions and Graphs
- Chapter 4: Limits and Continuity
- Chapter 5: Financial Mathematics
- Chapter 6: Differentiation
- Chapter 7: Methods of Differentiation
- Chapter 8: Function of several variables
- Chapter 9: Integration and applications
- Chapter 10: Linear algebra and applications

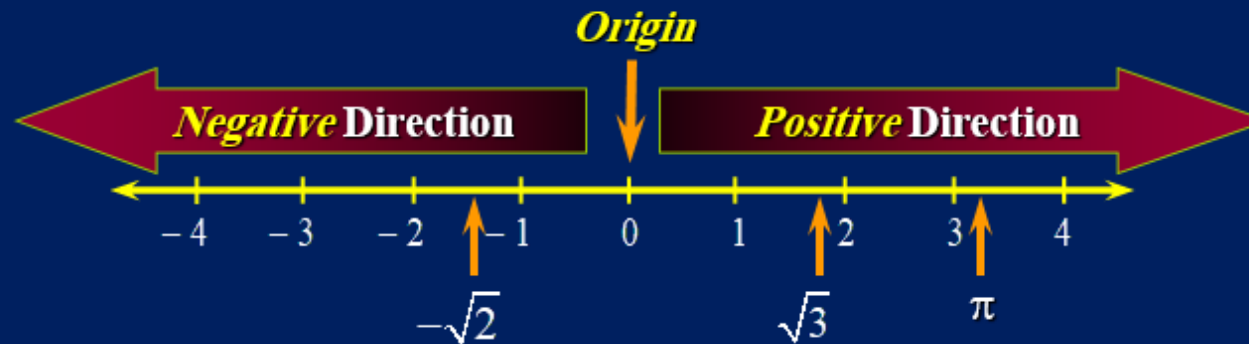
The Straight line and Applications

- ◆ **The Cartesian Coordinate System**
- ◆ **Straight Lines**
- ◆ **Linear Functions and Mathematical Models**
- ◆ **Intersection of Straight Lines**

The Straight line and Applications

The Cartesian Coordinate System

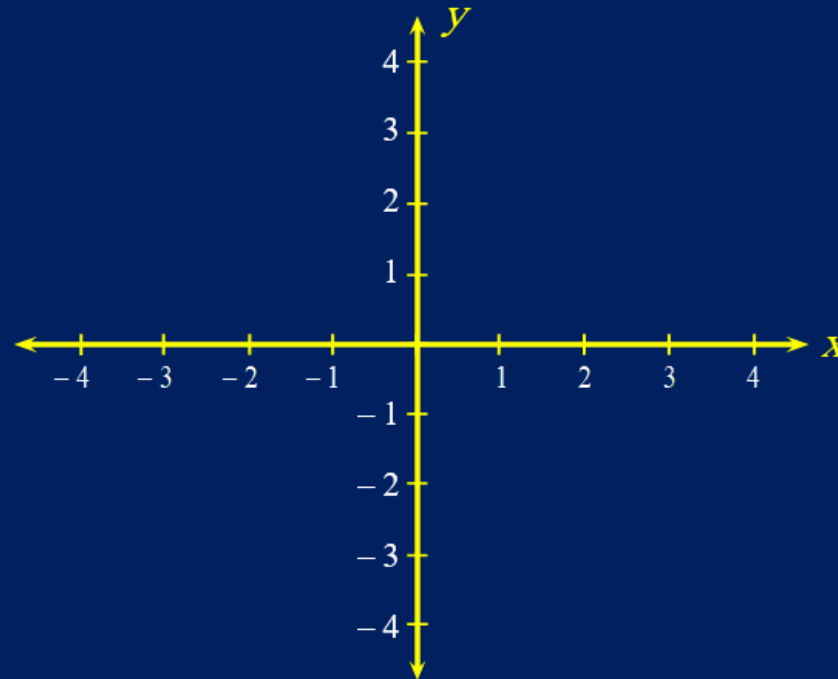
- ◆ We can represent real numbers **geometrically** by points on a **real number**, or **coordinate**, **line**:
- ◆ This line includes **all** real numbers.
- ◆ Exactly **one** point on the line is associated with **each** real number, and vice-versa (one dimensional space).



The Straight line and Applications

The Cartesian Coordinate System

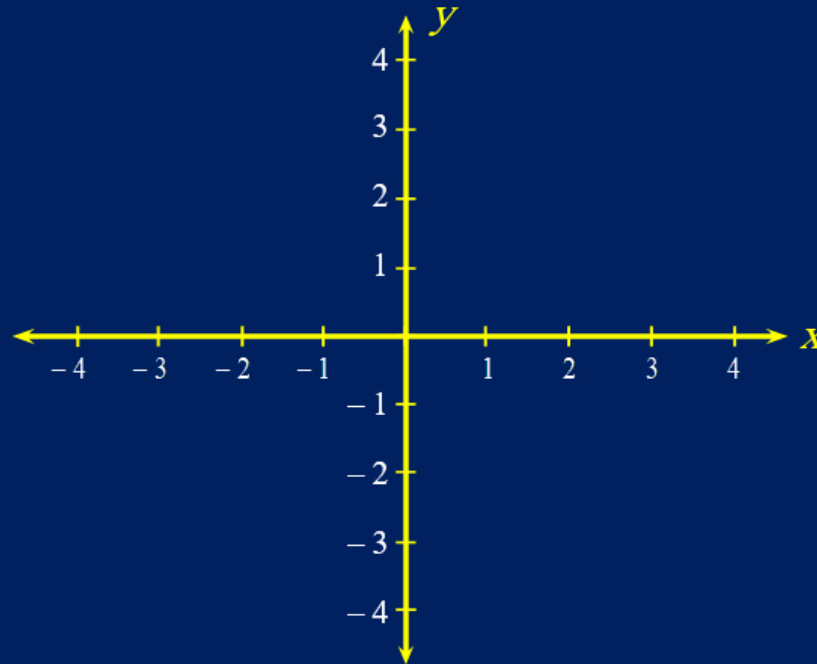
- ◆ The Cartesian coordinate system extends this concept to a plane (two dimensional space) by adding a vertical axis.



The Straight line and Applications

The Cartesian Coordinate System

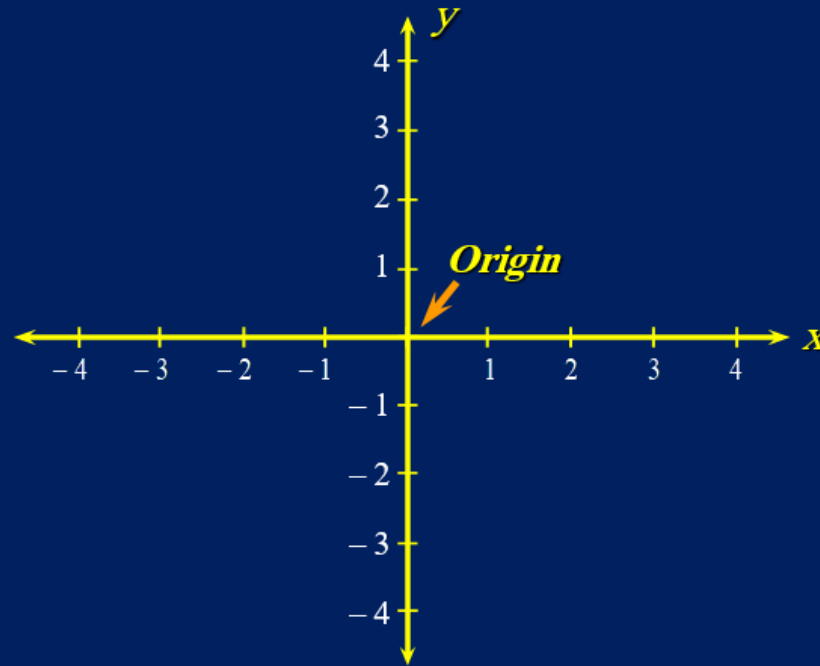
- ◆ The **horizontal** line is called the x -axis, and the **vertical** line is called the y -axis.



The Straight line and Applications

The Cartesian Coordinate System

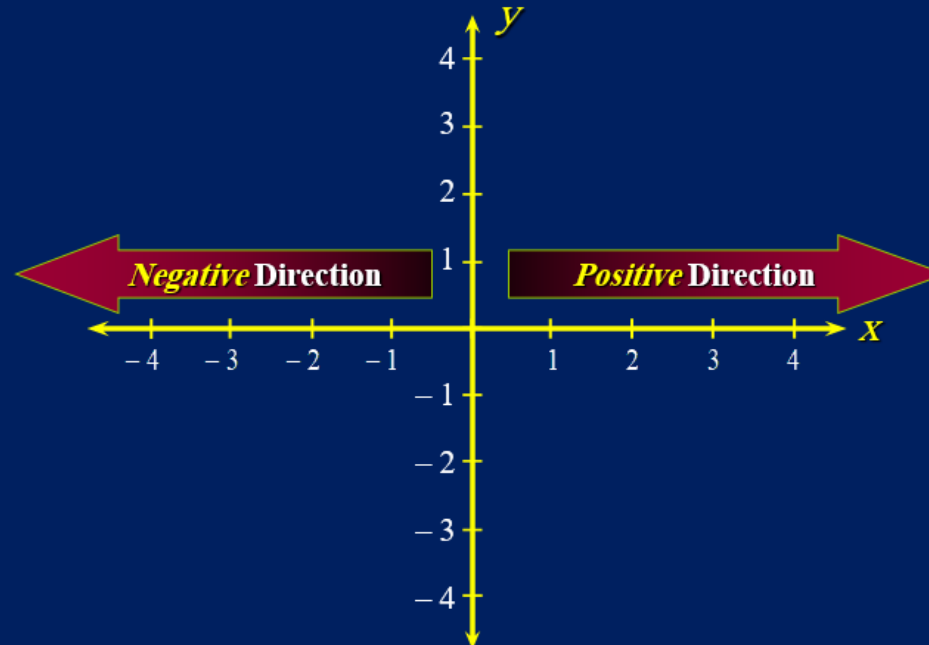
- ◆ The point where these two lines **intersect** is called the *origin*.



The Straight line and Applications

The Cartesian Coordinate System

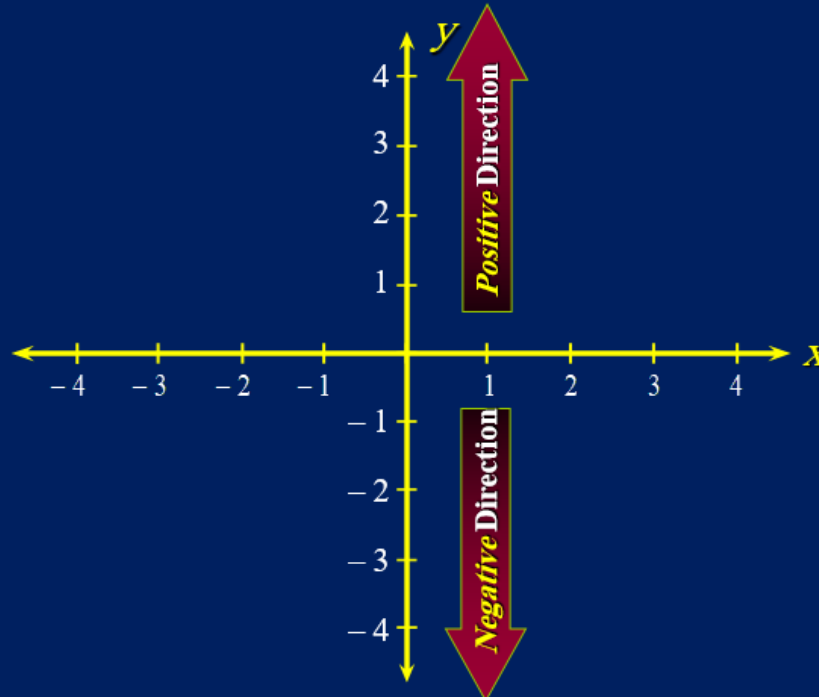
- ◆ In the x -axis, **positive numbers** are to the **right** and **negative numbers** are to the **left** of the origin.



The Straight line and Applications

The Cartesian Coordinate System

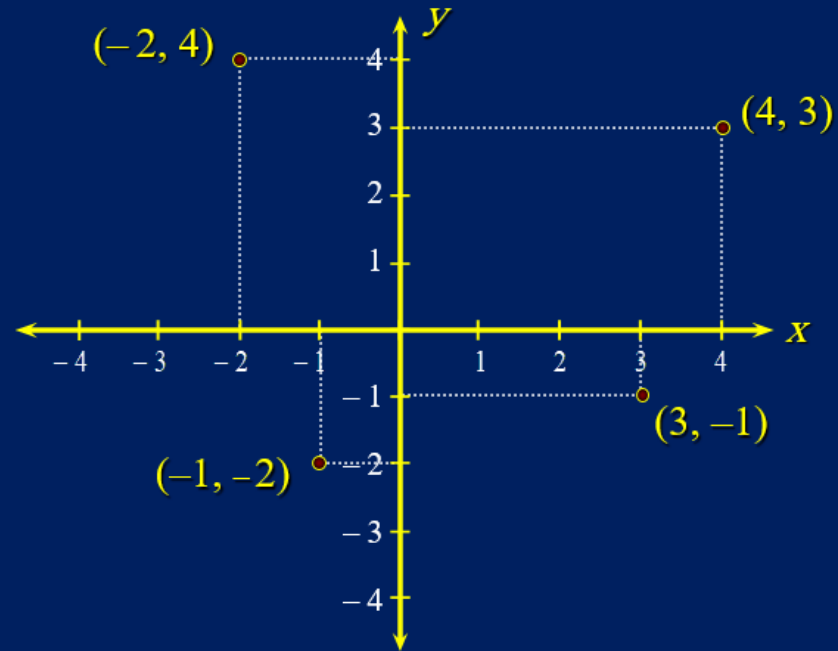
- ◆ In the y -axis, **positive numbers** are **above** and **negative numbers** are **below** the origin.



The Straight line and Applications

The Cartesian Coordinate System

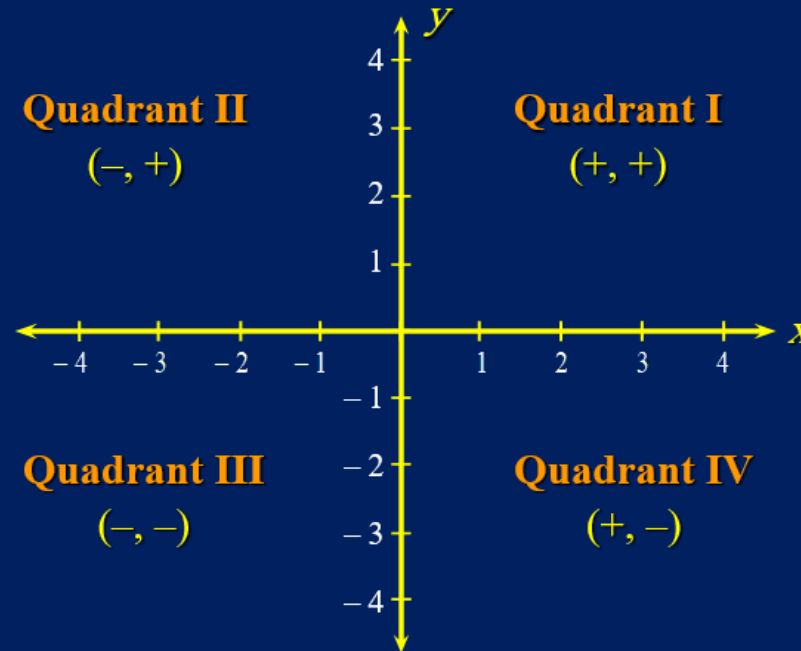
- ◆ A **point** in the **plane** can now be represented uniquely in this coordinate system by an **ordered pair of numbers** (x, y) .



The Straight line and Applications

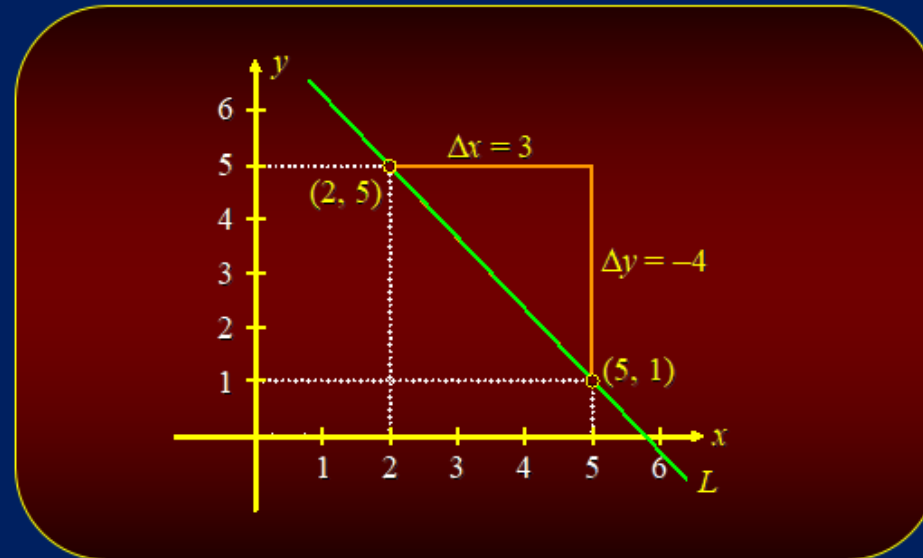
The Cartesian Coordinate System

- ◆ The axes divide the plane into **four quadrants** as shown below.



The Straight line and Applications

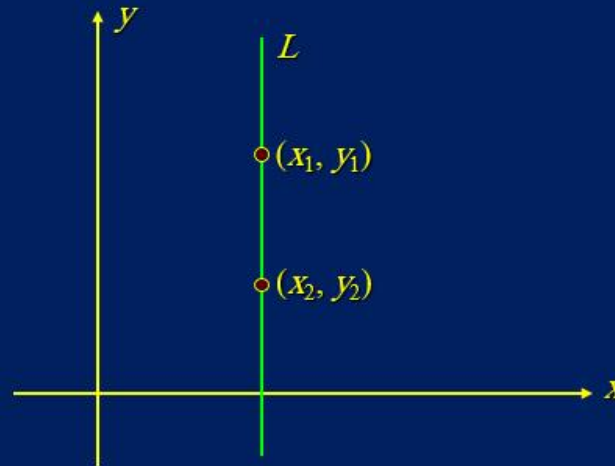
Straight Lines



The Straight line and Applications

Slope of a Vertical Line

- ◆ Let L denote the unique straight line that passes through the two distinct points (x_1, y_1) and (x_2, y_2) .
- ◆ If $x_1 = x_2$, then L is a **vertical line**, and the **slope is undefined**.

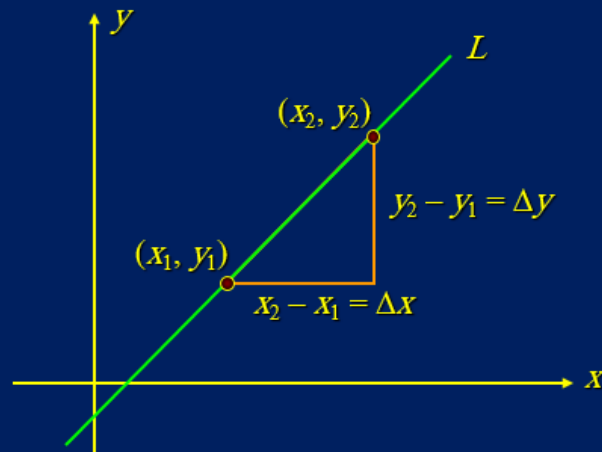


The Straight line and Applications

Slope of a Nonvertical Line

- ◆ If (x_1, y_1) and (x_2, y_2) are two distinct points on a **nonvertical line** L , then the **slope** m of L is given by

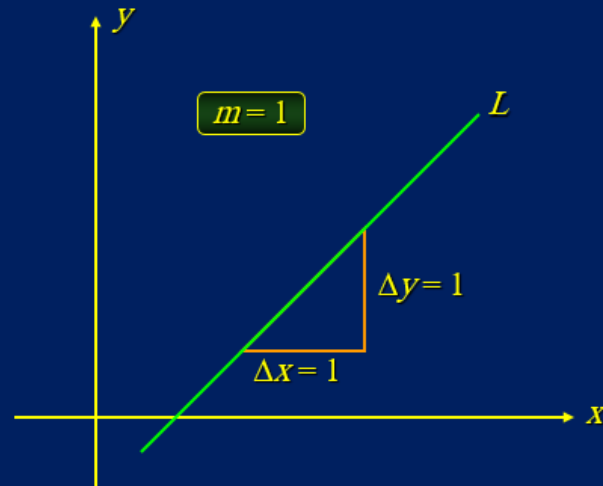
$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



The Straight line and Applications

Slope of a Nonvertical Line

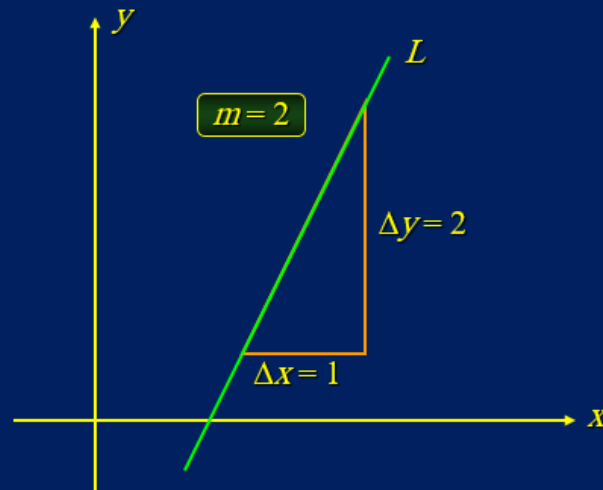
◆ If $m > 0$, the line **slants upward** from left to right.



The Straight line and Applications

Slope of a Nonvertical Line

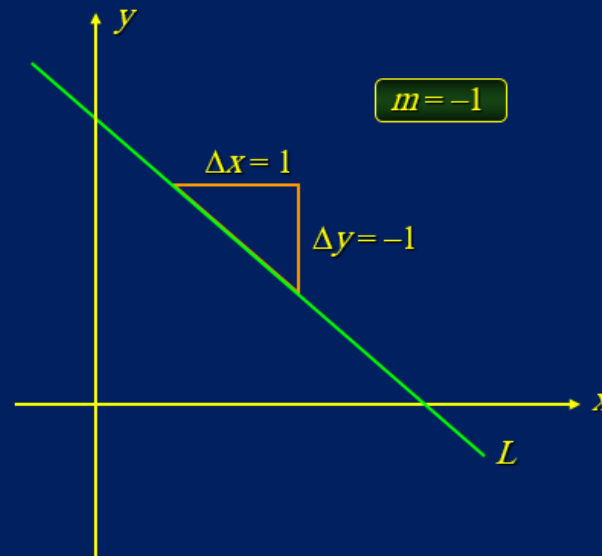
◆ If $m > 0$, the line **slants upward** from left to right.



The Straight line and Applications

Slope of a Nonvertical Line

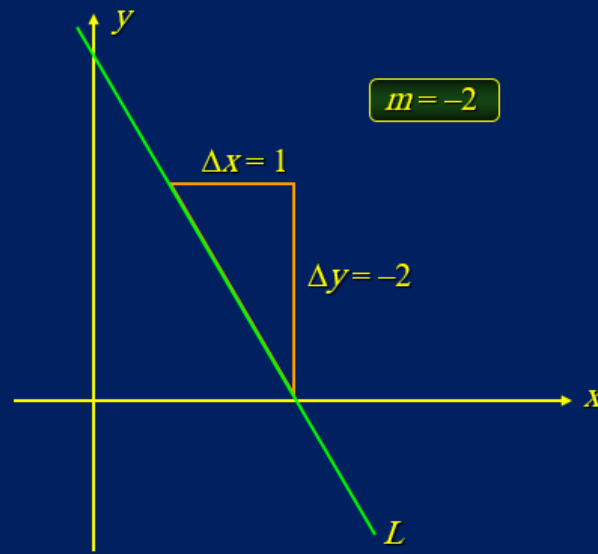
◆ If $m < 0$, the line **slants downward** from left to right.



The Straight line and Applications

Slope of a Nonvertical Line

◆ If $m < 0$, the line **slants downward** from left to right.



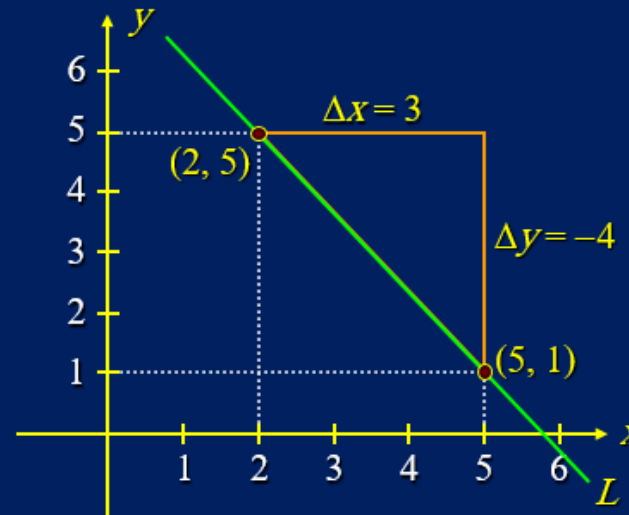
The Straight line and Applications

Examples

- ◆ Sketch the straight line that passes through the point $(2, 5)$ and has slope $-4/3$.

Solution

1. Plot the **point** $(2, 5)$.
2. A **slope** of $-4/3$ means that if x **increases** by 3, y **decreases** by 4.
3. Plot the **resulting point** $(5, 1)$.
4. Draw a **line** through the two points.



The Straight line and Applications

Examples

- ◆ Find the **slope** m of the line that goes through the **points** $(-1, 1)$ and $(5, 3)$.

Solution

- ◆ Choose (x_1, y_1) to be $(-1, 1)$ and (x_2, y_2) to be $(5, 3)$.
- ◆ With $x_1 = -1, y_1 = 1, x_2 = 5, y_2 = 3$, we find

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{5 - (-1)} = \frac{2}{6} = \frac{1}{3}$$

The Straight line and Applications

Examples

- ◆ Find the **slope** m of the line that goes through the **points** $(-2, 5)$ and $(3, 5)$.

Solution

- ◆ Choose (x_1, y_1) to be $(-2, 5)$ and (x_2, y_2) to be $(3, 5)$.
- ◆ With $x_1 = -2, y_1 = 5, x_2 = 3, y_2 = 5$, we find

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 5}{3 - (-2)} = \frac{0}{5} = 0$$

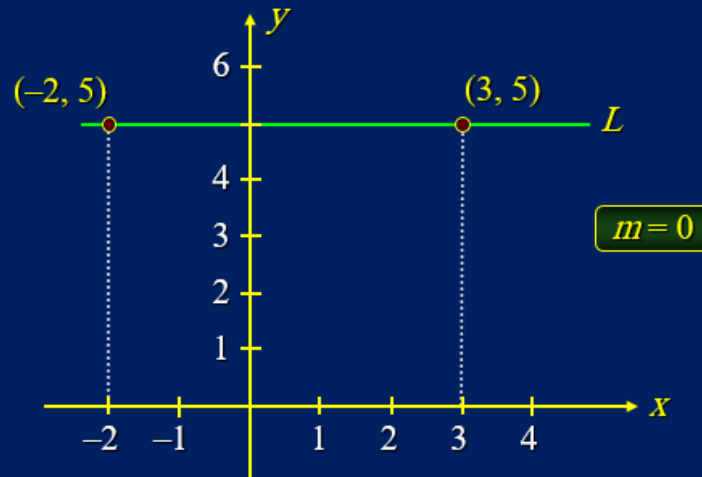
The Straight line and Applications

Examples

- ◆ Find the **slope** m of the line that goes through the **points** $(-2, 5)$ and $(3, 5)$.

Solution

- ◆ The slope of a **horizontal line** is **zero**:



The Straight line and Applications

Parallel Lines

- ◆ Two distinct lines are **parallel** if and only if their **slopes are equal** or their **slopes are undefined**.

The Straight line and Applications

Example

- ◆ Let L_1 be a line that passes through the points $(-2, 9)$ and $(1, 3)$, and let L_2 be the line that passes through the points $(-4, 10)$ and $(3, -4)$.
- ◆ Determine whether L_1 and L_2 are parallel.

Solution

- ◆ The **slope** m_1 of L_1 is given by

$$m_1 = \frac{3-9}{1-(-2)} = -2$$

- ◆ The **slope** m_2 of L_2 is given by

$$m_2 = \frac{-4-10}{3-(-4)} = -2$$

- ◆ Since $m_1 = m_2$, the lines L_1 and L_2 are in fact **parallel**.

What the Slope of line that parallel to line
with slope = 3

$$\text{slope} = 3$$

What the Slope of line that parallel to line
with slope = -5

$$\text{slope} = -5$$

What the Slope of line that parallel to line
with slope = 0

$$\text{slope} = 0$$

Notice that the two lines are Horizontal
lines and Parallel.

What the Slope of line that parallel to line with undefined slope

Slope is undefined

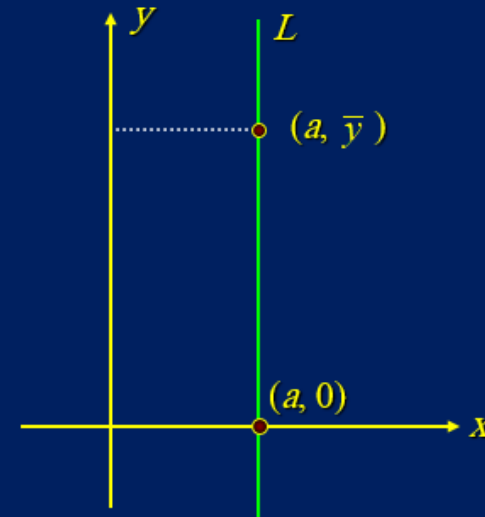
Notice that the two lines are Vertical lines and Parallel.

The Straight line and Applications

Equations of Lines

- ◆ Let L be a **straight line parallel** to the y -axis.
- ◆ Then L **crosses** the x -axis at some **point** $(a, 0)$, with the **x -coordinate** given by $x = a$, where a is a real number.
- ◆ Any other point on L has the form $(a, \bar{y})$, where $\bar{y}$ is an appropriate number.
- ◆ The **vertical line** L can therefore be described as

$$x = a$$



The Straight line and Applications

Equations of Lines

- ◆ Let L be a **nonvertical line** with a slope m .
- ◆ Let (x_1, y_1) be a **fixed point** lying on L , and let (x, y) be a variable point on L distinct from (x_1, y_1) .
- ◆ Using the slope formula by letting $(x, y) = (x_2, y_2)$, we get

$$m = \frac{y - y_1}{x - x_1}$$

- ◆ Multiplying both sides by $x - x_1$ we get

$$y - y_1 = m(x - x_1)$$

The Straight line and Applications

Point-Slope Form

- ◆ An equation of the line that has slope m and passes through point (x_1, y_1) is given by

$$y - y_1 = m(x - x_1)$$

The Straight line and Applications

Examples

- ◆ Find an equation of the line that passes through the point $(1, 3)$ and has slope 2.

Solution

- ◆ Use the **point-slope form**

$$y - y_1 = m(x - x_1)$$

- ◆ Substituting for **point** $(1, 3)$ and **slope** $m = 2$, we obtain

$$y - 3 = 2(x - 1)$$

- ◆ **Simplifying** we get

$$2x - y + 1 = 0$$

The Straight line and Applications

Examples

- ◆ Find an equation of the line that passes through the points $(-3, 2)$ and $(4, -1)$.

Solution

- ◆ The slope is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 2}{4 - (-3)} = -\frac{3}{7}$$

- ◆ Substituting in the **point-slope form** for point $(4, -1)$ and slope $m = -3/7$, we obtain

$$y + 1 = -\frac{3}{7}(x - 4)$$

$$7y + 7 = -3x + 12$$

$$3x + 7y - 5 = 0$$

Find the slope of line with equation

$$(y - 3) = 7(x - 2)$$

Slope = 7

The line with equation $(y - 3) = 7(x - 2)$
is passing through point

$$(y - y_1) = m(x - x_1)$$

Point is (2,3)

The Straight line and Applications

Perpendicular Lines

- ◆ If L_1 and L_2 are two distinct nonvertical lines that have slopes m_1 and m_2 , respectively, then L_1 is **perpendicular** to L_2 (written $L_1 \perp L_2$) if and only if

$$m_1 = -\frac{1}{m_2}$$

What the Slope of line that perpendicular
to line with slope = 3

$$\text{slope} = -\frac{1}{3}$$

What the Slope of line that perpendicular
to line with slope = -5

$$\text{slope} = \frac{1}{5}$$

What the Slope of line that perpendicular

to line with slope $= \frac{3}{2}$

$$\text{slope} = -\frac{2}{3}$$

The Straight line and Applications

Example

- ◆ Find the equation of the line L_1 that passes through the point $(3, 1)$ and is perpendicular to the line L_2 described by

$$y - 3 = 2(x - 1)$$

Solution

- ◆ L_2 is described in **point-slope form**, so its **slope** is $m_2 = 2$.
- ◆ Since the lines are **perpendicular**, the **slope** of L_1 must be

$$m_1 = -1/2$$

- ◆ Using the **point-slope form** of the equation for L_1 we obtain

$$y - 1 = -\frac{1}{2}(x - 3)$$

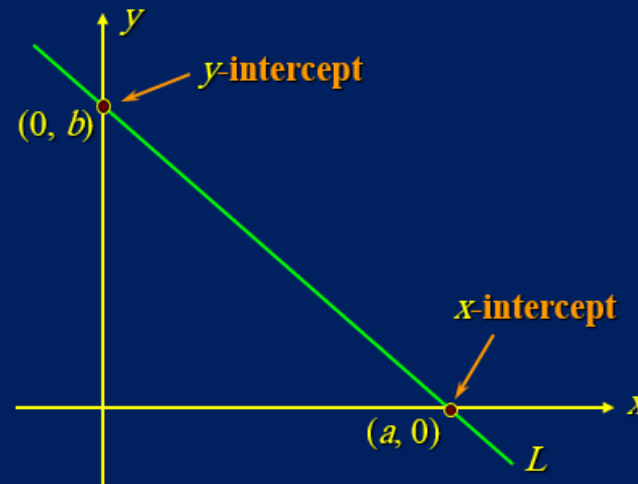
$$2y - 2 = -x + 3$$

$$x + 2y - 5 = 0$$

The Straight line and Applications

Crossing the Axis

- ◆ A straight line L that is **neither horizontal nor vertical** cuts the x -axis and the y -axis at, say, points $(a, 0)$ and $(0, b)$, respectively.
- ◆ The numbers a and b are called the **x -intercept** and **y -intercept**, respectively, of L .



The Straight line and Applications

Slope-Intercept Form

- ◆ An equation of the line that has **slope** m and **intersects** the y -axis at the **point** $(0, b)$ is given by

$$y = mx + b$$

The Straight line and Applications

Examples

- ◆ Find the equation of the line that has **slope 3** and **y-intercept** of -4 .

Solution

- ◆ We substitute $m = 3$ and $b = -4$ into $y = mx + b$ and get

$$y = 3x - 4$$

Find the equation of line with slope 5 and y-intercept 10

$$y = mx + b$$

$$y = 5x + 10$$

The Straight line and Applications

Examples

- ◆ Determine the **slope** and **y-intercept** of the line whose equation is $3x - 4y = 8$.

Solution

- ◆ Rewrite the given equation in the **slope-intercept form**.

$$3x - 4y = 8$$

$$-4y = 8 - 3x$$

$$y = \frac{3}{4}x - 2$$

- ◆ Comparing to $y = mx + b$, we find that $m = \frac{3}{4}$ and $b = -2$.
- ◆ So, the **slope** is $\frac{3}{4}$ and the **y-intercept** is -2 .

The Straight line and Applications

Applied Example

- ◆ Suppose an art object **purchased** for \$50,000 is expected to **appreciate in value** at a **constant rate** of \$5000 per year for the next 5 years.
- ◆ Write an **equation** predicting the value of the art object for any given year.
- ◆ What will be its **value** 3 years after the purchase?

The Straight line and Applications

Solution

- ◆ Let $x = \text{time (in years) since the object was purchased}$
 $y = \text{value of object (in dollars)}$
- ◆ Then, $y = 50,000$ when $x = 0$, so the **y -intercept** is $b = 50,000$.
- ◆ Every year the value rises by 5000, so the **slope** is $m = 5000$.
- ◆ Thus, the equation must be $y = 5000x + 50,000$.
- ◆ After 3 years the **value of the object** will be \$65,000:

$$y = 5000(3) + 50,000 = 65,000$$

The Straight line and Applications

General Form of a Linear Equation

◆ The equation

$$Ax + By + C = 0$$

where A , B , and C are **constants** and A and B are not both zero, is called the **general form of a linear equation** in the **variables** x and y .

The Straight line and Applications

General Form of a Linear Equation

- ◆ An equation of a **straight line** is a **linear equation**; conversely, every **linear equation** represents a **straight line**.

The Straight line and Applications

Equation of Straight Lines

Vertical line: $x = a$

Horizontal line: $y = b$

Point-slope form: $y - y_1 = m(x - x_1)$

Slope-intercept form: $y = mx + b$

General Form: $Ax + By + C = 0$

Un Solved Examples

What the Slope of line that perpendicular to line with slope = 4

Un Solved Examples

What the Slope of line that parallel to line
with slope = 6

Find the slope of line with equation

$$(y - 4) = 5(x - 3)$$

Un Solved Examples

The line with equation $(y - 4) = 5(x - 3)$
is passing through point

Un Solved Examples

What the Slope of line that passing through two points $(1,4)$ and $(2,9)$

Chapter

Un Solved Examples

Find the equation of line with slope 3 and y-intercept 7



Thank You