

Supply Chain Management

Prepared by Dr. Tarek Mohamed

MGM 660

Lecture 8



Managing economies of scale in a supply chain cycle Inventory

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Course Agenda

Chapter

1. **Balance the appropriate costs to choose the optimal lot size and cycle inventory in a supply chain.**
2. **Understand the impact of quantity discounts on lot size and cycle inventory.**
3. **Devise appropriate discounting schemes for a supply chain.**
4. **Understand the impact of trade promotions on lot size and cycle inventory.**
5. **Identify managerial levers that reduce lot size and cycle inventory in a supply chain without increasing cost.**

Role of Cycle Inventory in a Supply Chain

- **Lot or batch size** is the quantity that a stage of a supply chain either produces or purchases at a time
- Increasing the **lot size** or cycle inventory often decreases the cost incurred by different stages of a supply chain.
- **Cycle inventory** is the average inventory in a supply chain due to either production or purchases in lot sizes that are larger than those demanded by the customer
 - Q: Quantity in a lot or batch size
 - D: Demand per unit time
- **Cycle inventory** exists in a supply chain because different stages exploit economies of scale to lower total cost.

Role of Cycle Inventory in a Supply Chain

$$\text{Cycle inventory} = \frac{\text{lot size}}{2} = \frac{Q}{2}$$

$$\text{Average flow time} = \frac{\text{average inventory}}{\text{average flow rate}}$$

$$\begin{aligned} \text{Average flow time} \\ \text{resulting from} \\ \text{cycle inventory} \end{aligned} = \frac{\text{cycle inventory}}{\text{demand}} = \frac{Q}{2D}$$

Role of Cycle Inventory in a Supply Chain

- **Lower cycle inventory has**
 - Shorter average flow time
 - Lower working capital requirements
 - Lower inventory holding costs
- **Cycle inventory is held to**
 - Take advantage of economies of scale
 - Reduce costs in the supply chain

Role of Cycle Inventory in a Supply Chain

- **Average price paid per unit purchased** is a key cost in the lot-sizing decision

$$\text{Material cost} = C$$

- **Fixed ordering cost** includes all costs that do not vary with the size of the order but are incurred each time an order is placed

$$\text{Fixed ordering cost} = S$$

- **Holding cost** is the cost of carrying one unit in inventory for a specified period of time

$$\text{Holding cost} = H = hC$$

Role of Cycle Inventory in a Supply Chain

- **Primary role** of cycle inventory is to allow different stages to purchase product in lot sizes that minimize the sum of material, ordering, and holding costs
- Ideally, cycle inventory decisions should consider costs across the entire supply chain
- In practice, each stage generally makes its own supply chain decisions
- Increases total cycle inventory and total costs in the supply chain
- A key to reducing cycle inventory is the reduction of lot size

Role of Cycle Inventory in a Supply Chain

- **Economies of scale exploited in three typical situations**

1. A fixed cost is incurred each time an order is placed or produced
2. The supplier offers price discounts based on the quantity purchased per lot
3. The supplier offers short-term price discounts or holds trade promotions

Economies of Scale to Exploit Fixed Costs

- **Lot sizing for a single product (EOQ)**

D = Annual demand of the product

S = Fixed cost incurred per order

C = Cost per unit

H = Holding cost per year as a fraction of product cost

- **Basic assumptions**

- Demand is steady at D units per unit time

- No shortages are allowed

- Replenishment lead time is fixed

Economies of Scale to Exploit Fixed Costs

- **Minimize**
 - Annual material cost
 - Annual ordering cost
 - Annual holding cost

Lot Sizing for a Single Product

Annual material cost = CD

Number of orders per year = $\frac{D}{Q}$

Annual ordering cost = $\frac{c_o D}{Q}$

Annual holding cost = $\frac{c_h Q}{2} = \frac{c_h Q}{2} hC$

Total annual cost, $TC = CD + \frac{c_o D}{Q} + \frac{c_h Q}{2} hC$

Lot Sizing for a Single Product

- **The *economic order quantity (EOQ)***

$$\text{Optimal lot size, } Q^* = \sqrt{\frac{2DS}{hC}}$$

- **The optimal ordering frequency**

$$n^* = \frac{D}{Q^*} = \sqrt{\frac{DhC}{2S}}$$

EOQ Example

Annual demand, $D = 1,000 \times 12 = 12,000$ units

Order cost per lot, $S = \$4,000$

Unit cost per computer, $C = \$500$

Holding cost per year as a fraction of unit cost, $h = 0.2$

$$\text{Optimal order size} = Q^* = \sqrt{\frac{2 \times 12,000 \times 4,000}{0.2 \times 500}} = 980$$

EOQ Example

$$\text{Cycle inventory} = \frac{Q^*}{2} = \frac{980}{2} = 490$$

$$\text{Number of orders per year} = \frac{D}{Q^*} = 12.24$$

$$\text{Annual ordering and holding cost} = \frac{D}{Q^*}S + \frac{Q^*}{2}hC = 97,980$$

$$\text{Average flow time} = \frac{Q^*}{2D} = \frac{490}{12,000} = 0.041 = 0.49 \text{ month}$$

EOQ Example

- Lot size reduced to $Q = 200$ units

$$\text{Annual inventory-related costs} = \frac{D}{Q^*} S + \frac{Q^*}{2} hC = 250,000$$

Lot Size and Ordering Cost

- If the lot size $Q^* = 200$, how much should the ordering cost be reduced?

Desired lot size, $Q^* = 200$

Annual demand, $D = 1,000 \times 12 = 12,000$ units

Unit cost per computer, $C = \$500$

Holding cost per year as a fraction of inventory value, $h = 0.2$

$$S = \frac{hC(Q^*)^2}{2D} = \frac{0.2 \times 500 \times 200^2}{2 \times 12,000} = 166.7$$

EOQ Example

Inputs			
EOQ		Quantity Discount	
Annual Demand (units)	500	Discount Quantity	Unit Cost
Ordering Cost	\$ 10.00	275	\$ 4.60
Annual Holding Cost (%)	25%		
Unit Cost	\$ 5.00		
Number of days (week)	250		
Lead Time (# days, week)	2		
Results			
EOQ		Quantity Discount	
Annual Holding Cost	\$ 1.25	Annual Holding Cost	\$ 1.15
EOQ (units)	89	Annual Holding Cost	\$ 158.13
# of Orders/Year	5.59	Annual Ordering Cost	\$ 18.18
Annual Ordering Cost	\$ 55.90	Total Costs (\$)	\$ 176.31
Annual Holding Cost	\$ 55.90	Total Purchase Price	\$ 2,300.00
Total Costs (\$)	\$ 111.80	Purchase + Inventory	\$ 2,476.31
Total Purchase Price	\$2,500.00		
Purchase + Inventory	\$2,611.80		
Reorder point (units)	4		

EOQ Example

Input			Results			
Item	Annual Demand	Unit Cost	Annual Dollar Usage	% Annual Dollar	Cumulative % Annual Dollar Usage	Classification
1	5000	\$ 30.00	\$ 150,000	48.91%	48.91%	A
2	200	\$450.00	\$ 90,000	29.34%	78.25%	A
3	2000	\$ 10.00	\$ 20,000	6.52%	84.77%	B
4	800	\$ 20.00	\$ 16,000	5.22%	89.99%	B
5	1000	\$ 10.00	\$ 10,000	3.26%	93.25%	B
6	1200	\$ 5.00	\$ 6,000	1.96%	95.21%	C
7	1300	\$ 4.00	\$ 5,200	1.70%	96.90%	C
8	2500	\$ 2.00	\$ 5,000	1.63%	98.53%	C
9	3500	\$ 1.00	\$ 3,500	1.14%	99.67%	C
10	500	\$ 2.00	\$ 1,000	0.33%	100.00%	C
		Total	\$ 306,700			

Production Lot Sizing

- The entire lot does not arrive at the same time
- Production occurs at a specified rate P
- Inventory builds up at a rate of $P - D$

$$Q^P = \sqrt{\frac{2DS}{(1 - D/P)hC}}$$

Annual setup cost

$$\frac{DS}{Q^P} \div S$$

Annual holding cost

$$(1 - D/P) \frac{Q^P}{2} \div hC$$

Aggregating Multiple Products in a Single Order

- **Aggregating** across products, retailers, or suppliers in a single order allows for a reduction in lot size for individual products because fixed ordering and transportation costs are now spread across multiple products, retailers, or suppliers.
- **Savings in transportation costs**
 - Reduces fixed cost for each product
 - Lot size for each product can be reduced
 - Cycle inventory is reduced
- **Single delivery from multiple suppliers or single truck delivering to multiple retailers**
- **Receiving and loading costs reduced**

Lot Sizing with Multiple Products or Customers

- Ordering, transportation, and receiving costs grow with the variety of products or pickup points
- Reduction of fixed cost may be achieved by aggregating lots across multiple products, customers, or suppliers.
- **Lot sizes and ordering policy that minimize total cost**

D_i : Annual demand for product i

S : Order cost incurred each time an order is placed, independent of the variety of products in the order

s_i : Additional order cost incurred if product i is included in the order

- **Three approaches**

1. Each product manager orders his or her model independently
2. The product managers jointly order every product in each lot
3. Product managers order jointly but not every order contains every product; that is, each lot contains a selected subset of the products

Multiple Products Ordered and Delivered Independently

Demand

$$D_L = 12,000/\text{yr}, D_M = 1,200/\text{yr}, D_H = 120/\text{yr}$$

Common order cost

$$S = \$4,000$$

Product-specific order cost

$$s_L = \$1,000, s_M = \$1,000, s_H = \$1,000$$

Holding cost

$$h = 0.2$$

Unit cost

$$C_L = \$500, C_M = \$500, C_H = \$500$$

Multiple Products Ordered and Delivered Independently

	Litepro	Medpro	Heavypro
Demand per year	12,000	1,200	120
Fixed cost/order	\$5,000	\$5,000	\$5,000
Optimal order size	1,095	346	110
Cycle inventory	548	173	55
Annual holding cost	\$54,772	\$17,321	\$5,477
Order frequency	11.0/year	3.5/year	1.1/year
Annual ordering cost	\$54,772	\$17,321	\$5,477
Average flow time	2.4 weeks	7.5 weeks	23.7 weeks
Annual cost	\$109,544	\$34,642	\$10,954

- Total annual cost = \$155,140

Economies of Scale to Exploit Quantity Discounts

- **Lot size-based discount** – discounts based on quantity ordered in a single lot
- **Volume based discount** – discount is based on total quantity purchased over a given period
- **Two common schemes**
 - All-unit quantity discounts
 - Marginal unit quantity discount or multi-block tariffs
- For commodity products where price is set by the market, manufacturers can use lot size-based quantity discounts to achieve coordination in the supply chain and decrease supply chain cost.

- **Two basic questions**

1. What is the optimal purchasing decision for a buyer seeking to maximize profits? How does this decision affect the supply chain in terms of lot sizes, cycle inventories, and flow times?
2. Under what conditions should a supplier offer quantity discounts? What are appropriate pricing schedules that a supplier seeking to maximize profits should offer?

All-Unit Quantity Discounts

- Pricing schedule has specified quantity break points $q_0, q_1, \dots, q_r$, where $q_0 = 0$
- If an order is placed that is at least as large as q_i but smaller than q_{i+1} , then each unit has an average unit cost of C_i
- Unit cost generally decreases as the quantity increases, i.e., $C_0 > C_1 > \dots > C_r$
- Objective is to decide on a lot size that will minimize the sum of material, order, and holding costs

All-Unit Quantity Discounts

Step 1: Evaluate the optimal lot size for each price $C_i, 0 \leq i \leq r$ as follows

$$Q_i = \sqrt{\frac{2DS}{hC_i}}$$

All-Unit Quantity Discounts

Step 2: We next select the order quantity Q^*_i for each price C_i

1. $q_i \leq Q_i < q_{i+1}$
2. $Q_i < q_i$
3. $Q_i \geq q_{i+1}$

- Case 3 can be ignored as it is considered for Q_{i+1}
- For Case 1 if $q_i \leq Q_i < q_{i+1}$ then set $Q^*_i = Q_i$
- If $Q_i < q_i$ then a discount is not possible
- Set $Q^*_i = q_i$ to qualify for the discounted price of C_i

All-Unit Quantity Discounts

Step 3: Calculate the total annual cost of ordering Q_i^* units

$$\text{Total annual cost, } TC_i = \frac{D}{Q_i^*} S + \frac{Q_i^*}{2} h C_i + DC_i$$

All-Unit Quantity Discounts

Step 4: Select Q^*_i with the lowest total cost TC_i

- Cutoff price

$$C^* = \frac{1}{D} \left(\frac{DS}{q_r} + \frac{h}{2} q_r C_r - \sqrt{2hDSC_r} \right)$$

All-Unit Quantity Discount Example

Order Quantity	Unit Price
0–4,999	\$3.00
5,000–9,999	\$2.96
10,000 or more	\$2.92

$$q_0 = 0, q_1 = 5,000, q_2 = 10,000$$

$$C_0 = \$3.00, C_1 = \$2.96, C_2 = \$2.92$$

$$D = 120,000/\text{year}, S = \$100/\text{lot}, h = 0.2$$

Why Quantity Discounts?

- **Quantity discounts can increase the supply chain surplus for the following two main reasons**
 1. Improved coordination to increase total supply chain profits
 2. Extraction of surplus through price discrimination

Two-Part Tariff

- Manufacturer charges its entire profit as an up-front franchise fee ff
- Sells to the retailer at cost
- Retail pricing decision is based on maximizing its profits
- Effectively maximizes the coordinated supply chain profit
- For products where the firm has market power, two-part tariffs can be used to achieve coordination in the supply chain and maximize supply chain profits.

Volume-Based Quantity Discounts

- A discount is volume-based if the discount is based on the total quantity purchased over a given period, regardless of the number of lots purchased over that period
- Design a volume-based discount scheme that gets the retailer to purchase and sell the quantity sold when the two stages coordinate their actions

- Quantity discounts play a role in supply chain coordination and improved supply chain profits
- Discount schemes that are optimal are volume based and not lot size based unless the manufacturer has large fixed costs associated with each lot
- Even in the presence of large fixed costs for the manufacturer, a two-part tariff or volume-based discount, with the manufacturer passing on some of the fixed cost to the retailer, optimally coordinates the supply chain and maximizes profits

Lessons from Discounting Schemes

- Lot size–based discounts tend to raise the cycle inventory in the supply chain
- Volume-based discounts are compatible with small lots that reduce cycle inventory
- Retailers will tend to increase the size of the lot toward the end of the evaluation period, the *hockey stick phenomenon*
- With multiple retailers with different demand curves optimal discount continues to be volume based with the average price charged to the retailers decreasing as the rate of purchase increases

Price Discrimination to Maximize Supplier Profits

- Firm charges differential prices to maximize profits
- Setting a fixed price for all units does not maximize profits for the manufacturers
- Manufacturer can obtain maximum profits by pricing each unit differently based on customers' marginal evaluation at each quantity
- Quantity discounts are one mechanism for price discrimination because customers pay different prices based on the quantity purchased

Short-Term Discounting: Trade Promotions

- Trade promotions are price discounts for a limited period of time
- **Key goals**
 1. Induce retailers to use price discounts, displays, or advertising to spur sales
 2. Shift inventory from the manufacturer to the retailer and the customer
 3. Defend a brand against competition

Short-Term Discounting: Trade Promotions

- Impact on the behavior of the retailer and supply chain performance
- **Retailer has two primary options**
 1. Pass through some or all of the promotion to customers to spur sales
 2. Pass through very little of the promotion to customers but purchase in greater quantity during the promotion period to exploit the temporary reduction in price (*forward buy*)

Forward Buy

- Costs to be considered – material cost, holding cost, and order cost
- **Three assumptions**
 1. The discount is offered once, with no future discounts
 2. The retailer takes no action to influence customer demand
 3. Analyze a period over which the demand is an integer multiple of Q^*
- Although a forward buy is often the retailer's appropriate response and increases their own profits, it usually increases demand variability with a resulting increase in inventory and flow times within the supply chain.

Forward Buy

- Optimal order quantity

$$Q^d = \frac{dD}{(C-d)h} + \frac{CQ^*}{C-d}$$

- Retailers are often aware of the timing of the next promotion

$$\text{Forward buy} = Q^d - Q^*$$

- Trade promotions generally increase cycle inventory in a supply chain and hurt performance
- Trade promotions lead to a significant increase in lot size and cycle inventory because of forward buying by the retailer.
- **Counter measures**
 - EDLP (every day low pricing)
 - Discount applies to items sold to customers (sell-through) not the quantity purchased by the retailer (sell-in)
 - Scan based promotions

Managing Multi-Echelon Cycle Inventory

- Multi-echelon supply chains have multiple stages with possibly many players at each stage
- Lack of coordination in lot sizing decisions across the supply chain results in high costs and more cycle inventory than required
- The goal is to decrease total costs by coordinating orders across the supply chain

Integer Replenishment Policy

- Divide all parties within a stage into groups such that all parties within a group order from the same supplier and have the same reorder interval
- Set reorder intervals across stages such that the receipt of a replenishment order at any stage is synchronized with the shipment of a replenishment order to at least one of its customers
- For customers with a longer reorder interval than the supplier, make the customer's reorder interval an integer multiple of the supplier's interval and synchronize replenishment at the two stages to facilitate cross-docking

Integer Replenishment Policy

- For customers with a shorter reorder interval than the supplier, make the supplier's reorder interval an integer multiple of the customer's interval and synchronize replenishment at the two stages to facilitate cross-docking
- The relative frequency of reordering depends on the setup cost, holding cost, and demand at different parties
- These policies make the most sense for supply chains in which cycle inventories are large and demand is relatively predictable

Summary of Learning Objectives

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1. **Balance the appropriate costs to choose the optimal lot size and cycle inventory in a supply chain.**
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- Lysons, K., & Farrington, B. (2020). *Procurement and supply chain management*. Pearson UK.
- Chopra, S., & Meindl, P. (2016). Global Edition. Pearson Education Limited. **Chapter 11**.
- Morris, P. W., & Pinto, J. K. (Eds.). (2007). The Wiley guide to project technology, supply chain, and procurement management (Vol. 4). John Wiley & Sons.
- Schwalbe, K. (2015). Information technology project management. Cengage Learning.
- Supply Chain Council



Thank You