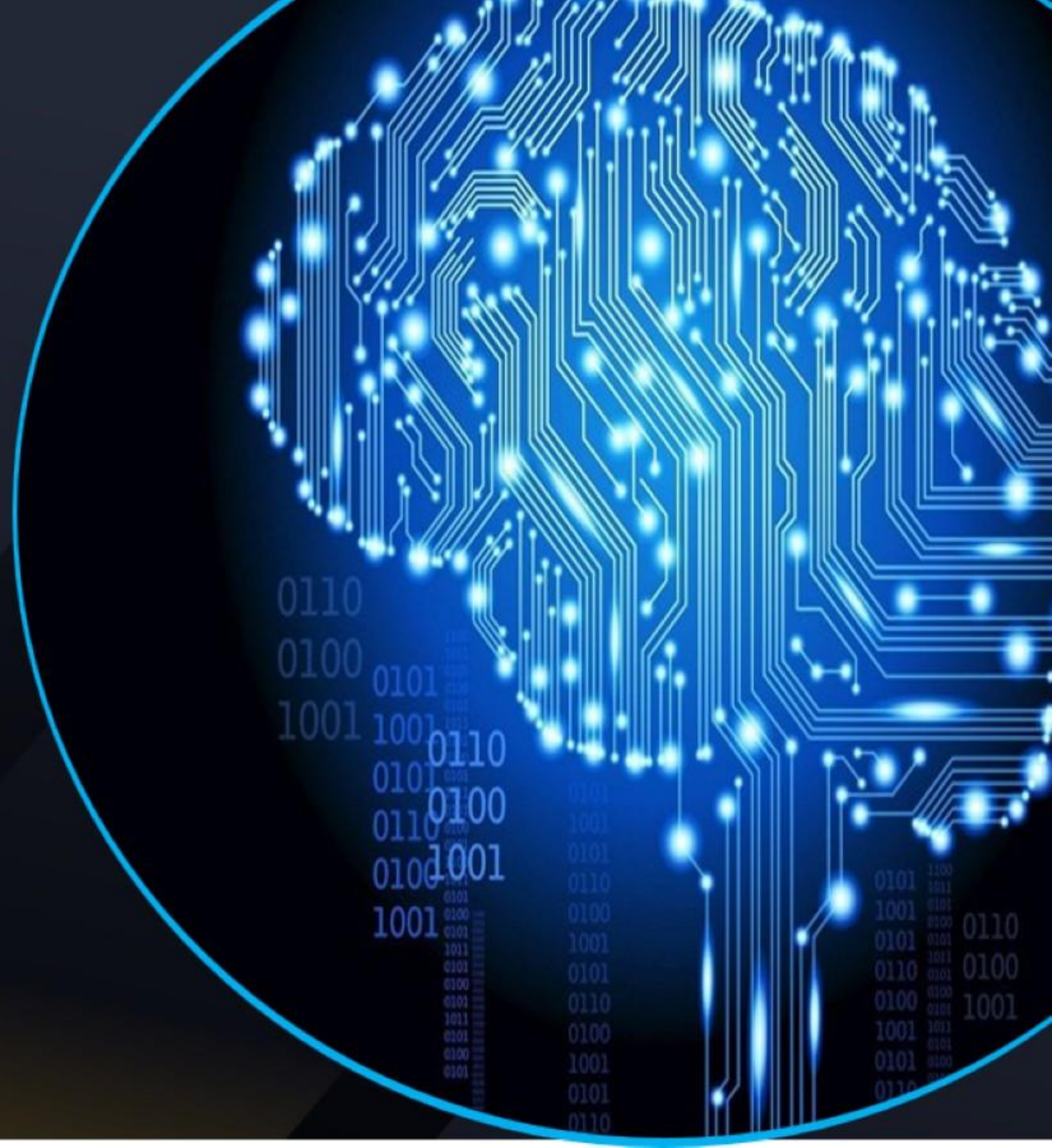


Discrete Mathematics

Dr. Shahira Ahmed

IT241

Lecture 2





Discrete Mathematics

Lecture 2

Relations & Functions

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- Relations and Their Properties
- *Binary* Relations
- Representing Relations
- Equivalence Relations
- Partial Orderings
- Functions

Relations and Their Properties

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Definition: The *Cartesian Product* of two sets A and B , denoted by

$A \times B$ is the set of ordered pairs (a,b) where $a \in A$ and $b \in B$.

$$A \times B = \{(a, b) | a \in A \wedge b \in B\}$$

Example:

$$A = \{a, b\} \quad B = \{1, 2, 3\}$$

$$A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$$

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Example:

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$$A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$$

$$B \times A = \{(1, a), (2, a), (3, a), (1, b), (2, b), (3, b)\}$$

Cartesian Product

Suppose that $A = \{1, 2\}$

$A \times A$

Solution

$\{1, 2\} \times \{1, 2\}$

$\{(1,1), (1,2), (2,1), (2,2)\}$

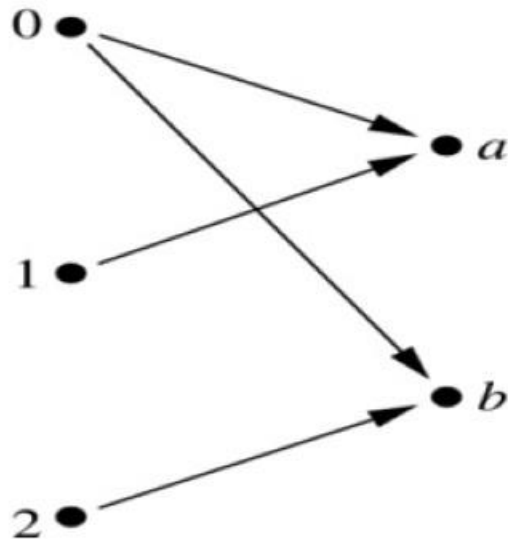


- Relationships between elements of sets occur in many contexts. Every day we deal with relationships such :
 - a business and its telephone number,
 - an employee and his or her salary,
 - a person and a relative.

Definition: A *binary relation* R from a set A to a set B is a subset $R \subseteq A \times B$.

Example:

- Let $A = \{0,1,2\}$ and $B = \{a,b\}$
- $\{(0, a), (0, b), (1,a) , (2, b)\}$ is a relation from A to B .
- We can represent relations from a set A to a set B graphically or using a table:



R	a	b
0	×	×
1	×	
2		×

Definition: A binary relation R on a set A is a subset of $A \times A$ or a relation from A to A .

Example:

- Suppose that $A = \{a, b, c\}$. Then $R = \{(a, a), (a, b), (a, c)\}$ is a relation on A .
- Let $A = \{1, 2, 3, 4\}$. The ordered pairs in the relation $R = \{(a, b) \mid a \text{ divides } b\}$ are $(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3),$ and $(4, 4)$.

Consider these relations on the set of integers:

$$R_1 = \{(a,b) \mid a \leq b\},$$

$$R_4 = \{(a,b) \mid a = b\},$$

$$R_2 = \{(a,b) \mid a > b\},$$

$$R_5 = \{(a,b) \mid a = b + 1\},$$

$$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\},$$

$$R_6 = \{(a,b) \mid a + b \leq 3\}.$$

Note that these relations are on an infinite set and each of these relations is an infinite set.

Consider these relations on the set of integers:

$$R_1 = \{(a,b) \mid a \leq b\},$$

$$R_2 = \{(a,b) \mid a > b\},$$

$$R_4 = \{(a,b) \mid a = b\},$$

$$R_5 = \{(a,b) \mid a = b + 1\},$$

$$R_6 = \{(a,b) \mid a + b \leq 3\}.$$

Which of these relations contain each of the pairs

- (1,1)
- (1, 2)
- (2, 1)
- (1, -1)
- (2, 2)

Consider these relations on the set of integers:

$$R_1 = \{(a,b) \mid a \leq b\},$$

$$R_2 = \{(a,b) \mid a > b\},$$

$$R_4 = \{(a,b) \mid a = b\},$$

$$R_5 = \{(a,b) \mid a = b + 1\},$$

$$R_6 = \{(a,b) \mid a + b \leq 3\}.$$

Which of these relations contain each of the pairs

- $(1,1)$ in R_1 , R_4 and R_6
- $(2, 1)$ in R_2 , R_5 , and R_6

Consider these relations on the set of integers:

$$R_1 = \{(a,b) \mid a \leq b\},$$

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$$R_5 = \{(a,b) \mid a = b + 1\},$$

$$R_6 = \{(a,b) \mid a + b \leq 3\}.$$

Which of these relations contain each of the pairs

- $(1,1)$ in R_1 , R_4 and R_6
- $(1, 2)$ in R_1 and R_6 :
- $(2, 1)$ in R_2 , R_5 , and R_6
- $(1, -1)$ in R_2 , and R_6
- $(2, 2)$ in R_1 and R_4

Definition: R is *reflexive* iff $(a,a) \in R$ for every element $a \in A$. Written symbolically, R is reflexive if and only if

$$\forall x[x \in U \longrightarrow (x,x) \in R]$$

If $A = \emptyset$ then the empty relation is reflexive vacuously.
That is the empty relation on an empty set is reflexive!

The following relations are reflexive?

- 1. Set $A=\{a,b,c\}$, $R_1=\{(a,a), (b,b), (c,c),(a,c),(b,c)\}$**
 - 2. Set $B=\{1,2,3\}$, $R_2=\{(1,1), (1,2), (2,2),(3,3),(2,1)\}$**
 - 3. Set $C=\{1,2,3,4\}$, $R_3=\{(1,1), (1,2), (2,2),(3,3),(3,1)\}$**
- Solution 1,2**

The following relations on the integers are reflexive?

If $A = \emptyset$ then the empty relation is reflexive vacuously.
That is the empty relation on an empty set is reflexive!

The following relations on the integers are reflexive?

$$R_1 = \{(a,b) \mid a \leq b\},$$

$$R_2 = \{(a,b) \mid a > b\}$$

$$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\},$$

$$R_4 = \{(a,b) \mid a = b\}.$$

$$R_5 = \{(a,b) \mid a = b + 1\}$$

$$R_6 = \{(a,b) \mid a + b \leq 3\}.$$

The following relations on the integers are reflexive?

$$R_1 = \{(a,b) \mid a \leq b\}, \quad \rightarrow \text{True}$$

$$R_2 = \{(a,b) \mid a > b\} \text{ (note that } 3 \not> 3), \rightarrow \text{False}$$

$$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\}, \rightarrow \text{True}$$

$$R_4 = \{(a,b) \mid a = b\}. \rightarrow \text{True}$$

$$R_5 = \{(a,b) \mid a = b + 1\} \text{ (note that } 3 \neq 3 + 1), \rightarrow \text{False}$$

$$R_6 = \{(a,b) \mid a + b \leq 3\} \text{ (note that } 4 + 4 \not\leq 3). \text{False}$$

Definition: R is *symmetric* iff $(b,a) \in R$ whenever $(a,b) \in R$ for all $a,b \in A$.

Written symbolically, R is symmetric if and only if

$$\forall x \forall y [(x,y) \in R \longrightarrow (y,x) \in R]$$

The following relations are Symmetric?

- 1. Set $A=\{a,b,c\}$, $R_1=\{(a,a), (b,b), (c,c),(a,c),(b,c)\}$**
 - 2. Set $B=\{1,2,3\}$, $R_2=\{(1,1), (1,2), (2,2),(3,3),(2,1)\}$**
 - 3. Set $C=\{1,2,3,4\}$, $R_3=\{(1,1), (1,2), (2,2),(3,3),(3,1)\}$**
- Solution 2**

Example: The following relations on the integers are symmetric?

$$R_1 = \{(a,b) \mid a \leq b\}$$

$$R_2 = \{(a,b) \mid a > b\}$$

$$R_4 = \{(a,b) \mid a = b\},$$

$$R_5 = \{(a,b) \mid a = b + 1\}$$

$$R_6 = \{(a,b) \mid a + b \leq 3\}.$$

The following relations on the integers are symmetric?

$R_1 = \{(a,b) \mid a \leq b\}$ (note that $3 \leq 4$, but $4 \not\leq 3$), $\rightarrow$ False

$R_2 = \{(a,b) \mid a > b\}$ (note that $4 > 3$, but $3 \not> 4$), $\rightarrow$ False

$R_4 = \{(a,b) \mid a = b\}$, $\rightarrow$ True

$R_5 = \{(a,b) \mid a = b + 1\}$ (note that $4 = 3 + 1$, but $3 \neq 4 + 1$). $\rightarrow$ False

$R_6 = \{(a,b) \mid a + b \leq 3\}$. $\rightarrow$ True

The following relations are antisymmetric?

- 1. Set $A=\{a,b,c\}$, $R_1=\{(a,a), (b,b), (c,c),(a,c),(b,c)\}$**
 - 2. Set $B=\{1,2,3\}$, $R_2=\{(1,1), (1,2), (2,2),(3,3),(2,1)\}$**
 - 3. Set $C=\{1,2,3,4\}$, $R_3=\{(1,1), (1,2), (2,2),(3,3),(3,1)\}$**
- Solution 1,3**

Definition: such that for all $a, b \in A$ if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ is called *antisymmetric*. Written symbolically, R is antisymmetric if and only if

$$\forall x \forall y [(x, y) \in R \wedge (y, x) \in R \rightarrow x = y]$$

- **Example:** The following relations on the integers are antisymmetric?

$$R_1 = \{(a,b) \mid a \leq b\},$$

$$R_2 = \{(a,b) \mid a > b\},$$

$$R_4 = \{(a,b) \mid a = b\},$$

$$R_5 = \{(a,b) \mid a = b + 1\}.$$

$$R_6 = \{(a,b) \mid a + b \leq 3\}$$

- **Example:** The following relations on the integers are antisymmetric?

$$R_1 = \{(a,b) \mid a \leq b\}, \rightarrow \text{True}$$

$$R_2 = \{(a,b) \mid a > b\}, \rightarrow \text{True}$$

$$R_4 = \{(a,b) \mid a = b\}, \rightarrow \text{True}$$

$$R_5 = \{(a,b) \mid a = b + 1\}, \rightarrow \text{True}$$

$$R_6 = \{(a,b) \mid a + b \leq 3\} \rightarrow \text{False}$$

Definition: A relation R on a set A is called transitive if whenever $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$, for all $a,b,c \in A$. Written symbolically, R is transitive if and only if

$$\forall x \forall y \forall z [(x,y) \in R \wedge (y,z) \in R \longrightarrow (x,z) \in R]$$

- The following relations on the integers are transitive?

$$R_1 = \{(a,b) \mid a \leq b\},$$

$$R_2 = \{(a,b) \mid a > b\},$$

$$R_4 = \{(a,b) \mid a = b\}.$$

$$R_5 = \{(a,b) \mid a = b + 1\}$$

$$R_6 = \{(a,b) \mid a + b \leq 3\}$$

- The following relations on the integers are transitive?

$$R_1 = \{(a,b) \mid a \leq b\}, \rightarrow \text{True}$$

$$R_2 = \{(a,b) \mid a > b\}, \rightarrow \text{True}$$

$$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\}, \rightarrow \text{True}$$

$$R_5 = \{(a,b) \mid a = b + 1\} \rightarrow \text{False (note that both } (3,2) \text{ and } (4,3) \text{ belong to } R_5, \text{ but not } (3,3)),$$

$$R_6 = \{(a,b) \mid a + b \leq 3\} \rightarrow \text{False (note that both } (2,1) \text{ and } (1,2) \text{ belong to } R_6, \text{ but not } (2,2)).$$

- Given two relations R_1 and R_2 , we can combine them using basic set operations to form new relations such as $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 - R_2$, and $R_2 - R_1$.
- Example:** Let $A = \{1,2,3\}$ and $B = \{1,2,3,4\}$.
 - The relations $R_1 = \{(1,1),(2,2),(3,3)\}$ and
 - $R_2 = \{(1,1),(1,2),(1,3),(1,4)\}$
 - can be combined using basic set operations to form new relations:

$$R_1 \cup R_2 = \{(1,1),(1,2),(1,3),(1,4),(2,2),(3,3)\}$$

$$R_1 \cap R_2 = \{(1,1)\}$$

$$R_1 - R_2 = \{(2,2),(3,3)\}$$

$$R_2 - R_1 = \{(1,2),(1,3),(1,4)\}$$

Representing Relations

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- The relation R is represented by the matrix
- $M_R = [m_{ij}]$, where

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R, \\ 0 & \text{if } (a_i, b_j) \notin R. \end{cases}$$

Example 1: Suppose that $A = \{1,2,3\}$ and $B = \{1,2\}$. Let R be the relation from A to B containing (a,b) if $a \in A$, $b \in B$, and $a > b$. What is the matrix representing R (assuming the ordering of elements is the same as the increasing numerical order)?

Solution: Because $R = \{(2,1), (3,1), (3,2)\}$, the matrix is

$$M_R = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

Example 2: Let $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3, b_4, b_5\}$. Which ordered pairs are in the relation R represented by the matrix

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}?$$

Solution: Because R consists of those ordered pairs (a_i, b_j) with $m_{ij} = 1$, it follows that:

$$R = \{(a_1, b_2), (a_2, b_1), (a_2, b_3), (a_2, b_4), (a_3, b_1), (a_3, b_3), (a_3, b_5)\}.$$

- If R is a reflexive relation, all the elements on the main diagonal of M_R are equal to 1.

$$\begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix}$$

- R is a symmetric relation, if and only if $m_{ij} = 1$ whenever $m_{ji} = 1$. R is an antisymmetric relation, if and only if $m_{ij} = 0$ or $m_{ji} = 0$ when $i \neq j$.

A 3x3 matrix diagram illustrating a symmetric relation. The main diagonal elements are 1, 1, and 1. The off-diagonal elements are 0, 0, and 0. The matrix is symmetric about the main diagonal, with blue lines connecting the 1s and 0s across the diagonal.

(a) Symmetric

A 3x3 matrix diagram illustrating an antisymmetric relation. The main diagonal elements are 1, 0, and 0. The off-diagonal elements are 0, 0, and 1. The matrix is antisymmetric about the main diagonal, with blue lines connecting the 1s and 0s across the diagonal.

(b) Antisymmetric

Example of a Relation on a Set

Suppose that the relation R on a set is represented by the matrix

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

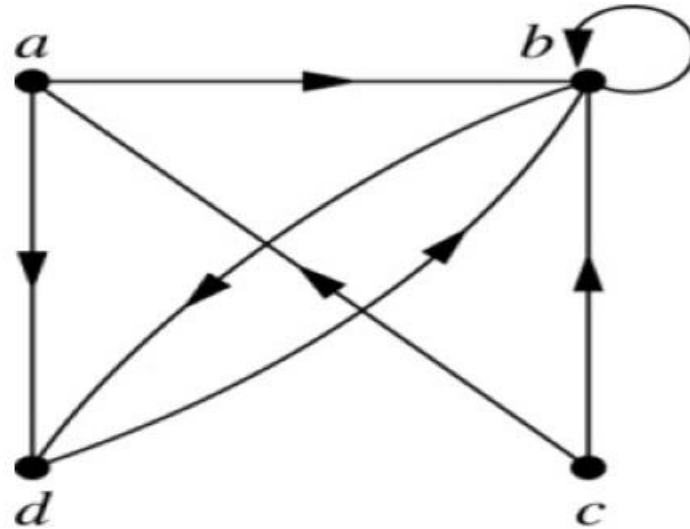
Is R reflexive, symmetric, and/or antisymmetric?

Solution:

- Because all the diagonal elements are equal to 1, R is reflexive.
- Because M_R is symmetric.
- R is symmetric and not antisymmetric because both $m_{1,2}$ and $m_{2,1}$ are 1.

Representing Relations Using Digraphs

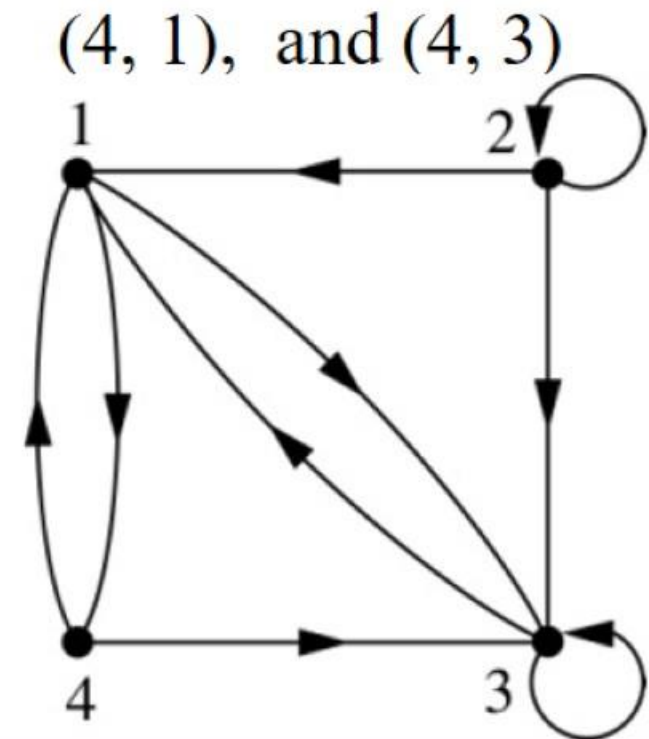
A drawing of the directed graph with **vertices** a , b , c , and d , and **edges** (a, b) , (a, d) , (b, b) , (b, d) , (c, a) , (c, b) , and (d, b) is shown here.



Examples of Digraphs Representing Relations

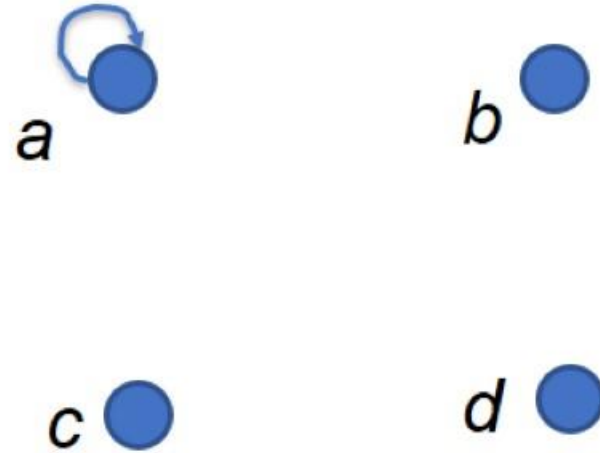
What are the ordered pairs in the relation represented by this directed graph?

Solution: The ordered pairs in the relation are $(1, 3)$, $(1, 4)$, $(2, 1)$, $(2, 2)$, $(2, 3)$, $(3, 1)$, $(3, 3)$,



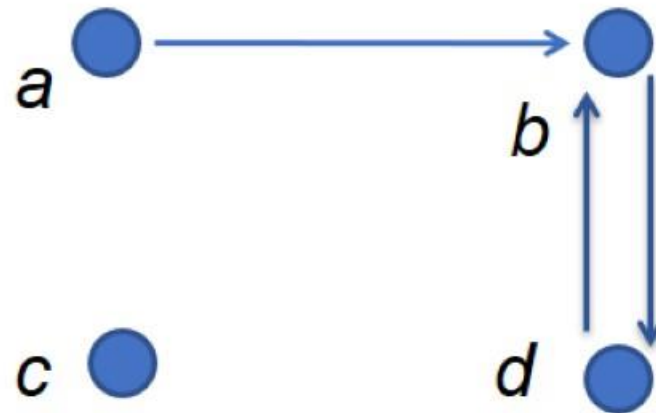
- *Reflexivity*: A loop must be present at all vertices in the graph.
- *Symmetry*: If (x,y) is an edge, then so is (y,x) .
- *Antisymmetry*: If (x,y) with $x \neq y$ is an edge, then (y,x) is not an edge.
- *Transitivity*: If (x,y) and (y,z) are edges, then so is (x,z) .

- Determining which Properties a Relation has from its Digraph



- Reflexive?* No, not every vertex has a loop
- Symmetric?* Yes (trivially), there is no edge from one vertex to another
- Antisymmetric?* Yes (trivially), there is no edge from one vertex to another
- Transitive?* Yes, (trivially) since there is no edge from one vertex to another

- Determining which Properties a Relation has from its



- Reflexive?* No, there are no loops
- Symmetric?* No, there is an edge from a to b , but not from b to a
- Antisymmetric?* No, there is an edge from d to b and b to d
- Transitive?* No, there are edges from a to b and from b to d , but there is no edge from a to d

Definition 1: A relation on a set A is called an *equivalence relation* if it is

- reflexive,
- symmetric,
- transitive.

Show that the “divides” relation on the set of positive integers is not an equivalence relation.

Solution

- *Reflexivity*: $a \mid a$ for all a .
- *Not Symmetric*: For example, $2 \mid 4$, but $4 \nmid 2$. Hence, the relation is not symmetric.
- *Transitivity*: the relation is transitive.

Definition : A relation R on a set S is called a *partial ordering*, or *partial order*

- Reflexive
- antisymmetric
- transitive.

“greater than or equal” relation ($\geq$) is a partial ordering on the set of integers”

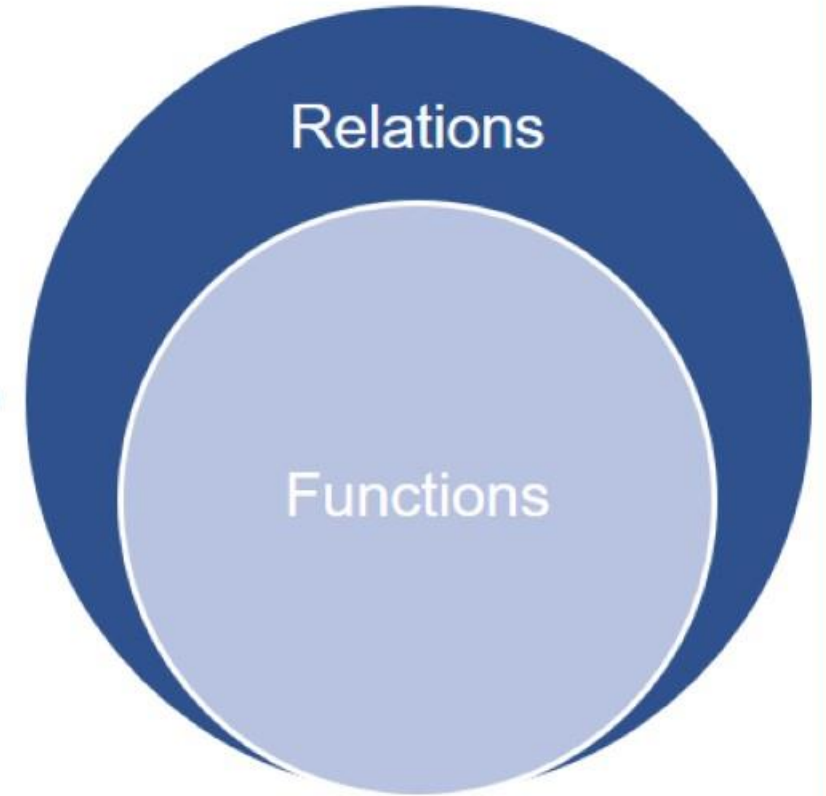
- *Reflexivity*: $a \geq a$ for every integer a .
- *Antisymmetry*: If $a \geq b$ and $b \geq a$, then $a = b$.
- *Transitivity*: If $a \geq b$ and $b \geq c$, then $a \geq c$.

The divisibility relation ($/$) is a partial ordering on the set of integers.

Solution

- *Reflexivity*: $a \mid a$ for all integers a .
- *Antisymmetry*: If a and b are positive integers with $a \mid b$ and $b \mid a$, then $a = b$.
- *Transitivity*: Suppose that a divides b and b divides $c \rightarrow$ so a divides c . Therefore, the relation is transitive.

- A function is a relation where exactly one element of B is related to each element of A .
- Example $R=\{(2, 1),(3,6),(2,3),(5, 8)\}$ is a function?
- Solution False



R from A to B $\subseteq$ *Cartesian Product*

- a) $A \times A$
- b) $A \times B$
- c) $B \times A$
- d) $B \times B$

Solution b

R from B to A $\subseteq$ *Cartesian Product*

- a) $A \times A$
- b) $A \times B$
- c) $B \times A$
- d) $B \times B$

Solution C

R on a set $A \subseteq \text{Cartesian Product}$

- a) $A \times A$
- b) $A \times B$
- c) $B \times A$
- d) $B \times B$

Solution a

List the ordered pairs in the relation R from

$A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$, where $(a, b) \in R$

if and only if

- a) $a = b$.
- b) $a + b = 4$.
- c) $a > b$.

List the ordered pairs in the relation R from

$A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$, where $(a, b) \in R$

if and only if

➤ a) $a = b$.

➤ b) $a + b = 4$.

➤ c) $a > b$.

a) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$

b) $\{(1, 3), (2, 2), (3, 1), (4, 0)\}$

c) $\{(1, 0), (2, 0), (2, 1), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4, 2), (4, 3)\}$

- Find The property of Relation
 - a) $(a, a) \in R$, for every $a \in A$
 - b) $(a, b) \in R$, for every $a \in A \mid a=b$.
 - c) $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$
 - d) $(a, b) \in R$, then $(b, a) \in R$
 - e) $a, b \in A$ if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$

- Find The property of Relation
 - a) $(a, a) \in R$, for every $a \in A$
 - b) $(a, b) \in R$, for every $a, b \in A \mid a=b$.
 - c) $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$
 - d) $(a, b) \in R$, then $(b, a) \in R$
 - e) $a, b \in A$ if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$

Solution

- a) *Reflexive*
- b) *Reflexive*
- c) *Transitive*
- d) *Symmetric*
- e) *AntiSymmetric*

For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

- a) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
- b) $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
- c) $\{(2, 4), (4, 2)\}$
- d) $\{(1, 2), (2, 3), (3, 4)\}$
- e) $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$
- f) $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

- a) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
- b) $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
- c) $\{(2, 4), (4, 2)\}$
- d) $\{(1, 2), (2, 3), (3, 4)\}$
- e) $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$
- f) $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

- a) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
- b) $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
- c) $\{(2, 4), (4, 2)\}$
- d) $\{(1, 2), (2, 3), (3, 4)\}$
- e) $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$
- f) $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

- a) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
- b) $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
- c) $\{(2, 4), (4, 2)\}$
- d) $\{(1, 2), (2, 3), (3, 4)\}$
- e) $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$
- f) $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

- a) Transitive
- b) Reflexive, symmetric, transitive
- c) Symmetric
- d) Antisymmetric
- e) Reflexive, symmetric, antisymmetric, transitive
- f) None of these properties

- Suppose that $A = \{1, 2, 3\}$ and $B = \{1, 2\}$. Let R be the relation from A to B containing (a, b) if $a \in A$, $b \in B$, and $a > b$. What is the matrix representing R ?

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- Solution

$R = \{(2, 1), (3, 1), (3, 2)\}$, the matrix for R is

$$\mathbf{M}_R = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

- Let $A = \{a1, a2, a3\}$ and $B = \{b1, b2, b3, b4, b5\}$. Which ordered pairs are in the relation R represented by the matrix. Are the following pair in the relation

- a) $(a1, b2)$
- b) $(a2, b1)$
- c) $(a3, b3)$
- d) $(a1, b1)$

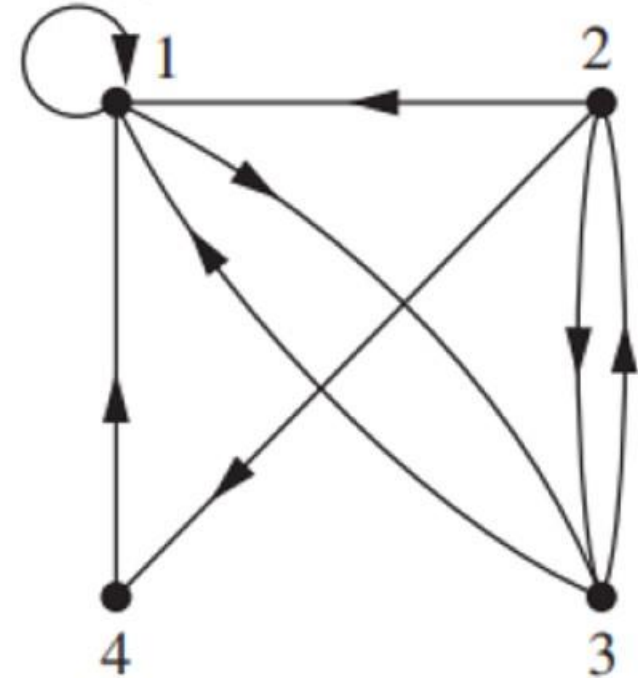
$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} ?$$

Solution

- a) True
- b) True
- c) True
- d) False

List the ordered pairs in R?

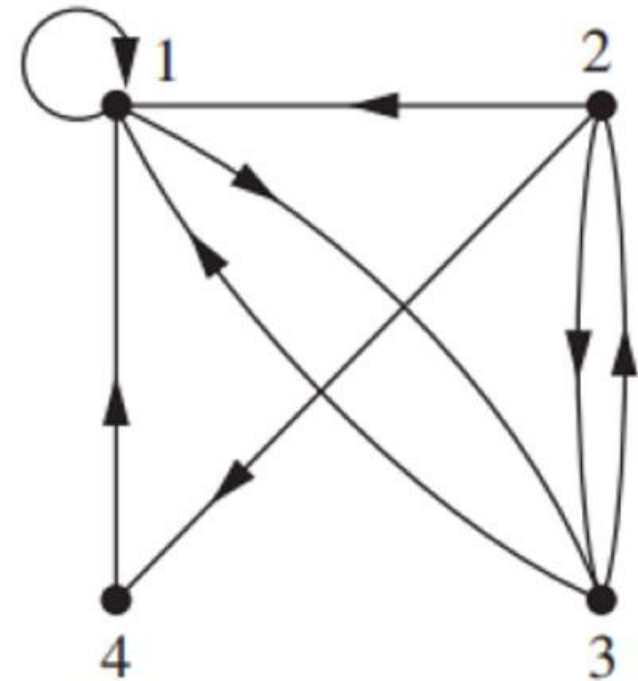
$\{(1, 1), (1, 3), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (4, 1)\}$



Are the Following the ordered pairs in R?

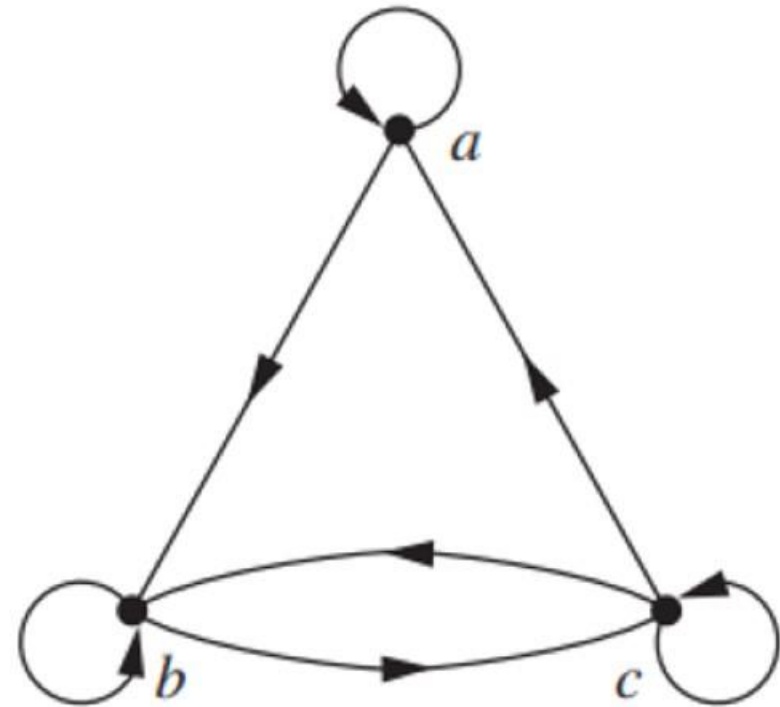
- a) (1, 1)
- b) (1, 3)
- c) (2, 1)
- d) (4, 3)
- e) (4,4)
- f) (4,2)

Solution a,b,c

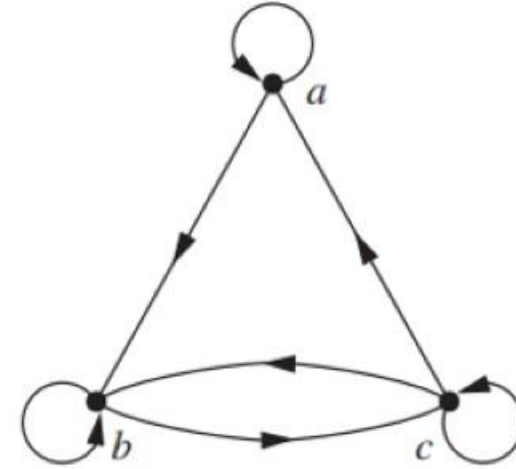


Quiz

- Reflexive?
- Symmetric?
- antisymmetric?
- Transitive?



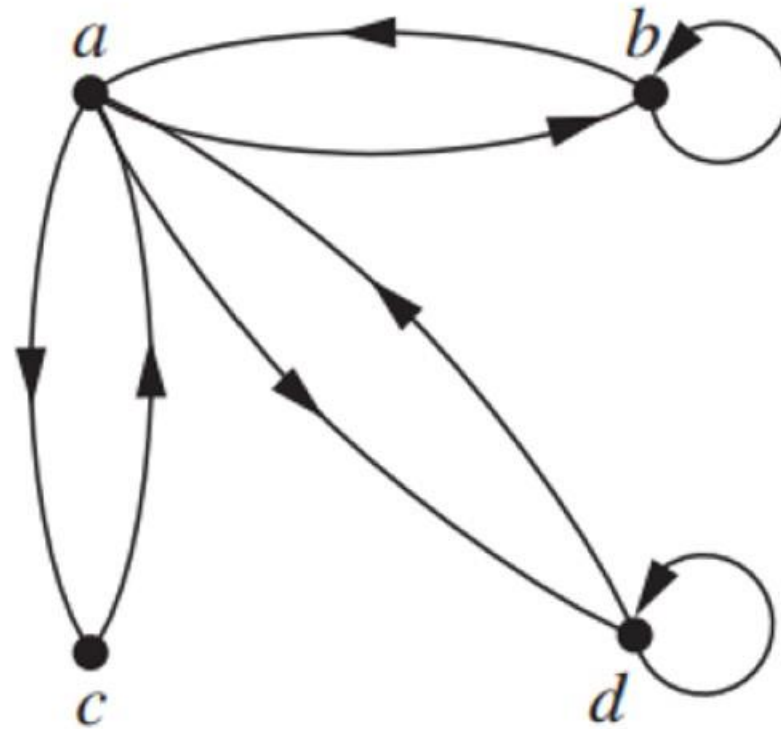
- Reflexive?
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- antisymmetric?
- Transitive?



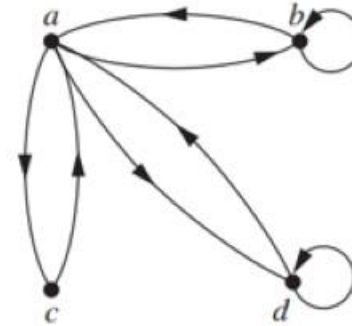
- Because there are loops at every vertex of the directed graph of $S1$, it is reflexive. The relation $S1$ is neither symmetric nor antisymmetric because there is an edge from a to b but not one from b to a , but there are edges in both directions connecting b and c . Finally, $S1$ is not transitive because there is an edge from a to b and an edge from b to c , but no edge from a to c .

Quiz

- Reflexive?
- Symmetric?
- antisymmetric?
- Transitive?



- Reflexive?
- Symmetric?
- antisymmetric?
- Transitive?



Because loops are not present at all the vertices of the directed graph of S_2 , this relation is not reflexive. It is symmetric and not antisymmetric, because every edge between distinct vertices is accompanied by an edge in the opposite direction. It is also not hard to see from the directed graph that S_2 is not transitive, because (c, a) and (a, b) belong to S_2 , but (c, b) does not belong to S_2 .

True or False

- 1) A relation R is reflexive if the matrix diagonal elements are all 0
- 2) The binary relation R equal on the set of all integers is Symmetric
- 3) The binary relation R equal on the set of all integers is Reflexive
- 4) The binary relation R not equal on the set of all integers is Symmetric
- 5) The binary relation R not equal on the set of all integers is Reflexive

True or False

- 1) A relation R is reflexive if the matrix diagonal elements are all 0
- 2) The binary relation R equal on the set of all integers is Symmetric
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- 5) The binary relation R not equal on the set of all integers is Reflexive

Solution False,True,True,True,False

True or false

1. A relation on a set A is called an *equivalence relation* if it is Reflexive, symmetric, antisymmetric
2. A relation R on a set S is called a *partial ordering*, or *partial order* if it is Reflexive, antisymmetric, transitive.

Solution

False

True

- Relations
 - Reflexive
 - Symmetric
 - Antisymmetric
 - Transitive
- Representation of Relations
- Equivalence
- Partial ordering
- Function



Thank You