

Mathematics

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Lecture 3

جامعة الجميع الذكية

MAT 122

Introduction to Matrix Algebra



جامعة الجميع الذكية
EVERYONE'S SMART UNIVERSITY

Concept of a matrix.

Order and size of a matrix.

Special Definitions in Matrices.

Special types of matrices.

Extra Practice.

A matrix is an ordered rectangular array of numbers, arranged in rows and columns.

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} = (a_{ij})_{mn}$$

A restaurant offers **4 kinds of pizza**
(vegetables, cheese, beef and chicken) with
3 sizes.

Kind	Size		
	small	Medium	Large
Veg.	15	13	9
Chicken	16	18	12
Beef	13	10	8
cheese	18	20	7

If we want to write it in a form of matrix :

A = Name of the matrix	First column	Second column	Third column	
	15	13	9	→ first row
	16	18	12	→ second row
	13	10	8	→ third row
	18	20	17	→ fourth row
	4 * 3 size of the matrix			
Size of matrix = m x n , m → row , n → column				

The **size** or **order** of a matrix is described by its number of rows and the number of columns.

If a matrix, A , has m rows and n columns then A is described as an $m \times n$ matrix.

A matrix consisting of m horizontal rows and n vertical columns is called an **$m \times n$ matrix** or a **matrix of size $m \times n$** .

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{12} & \dots & a_{2n} \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

For the entry a_{ij} , we call i the row subscript and j the column subscript.

Find the size for the following matrices:

A- $\begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$

B- $\begin{bmatrix} 1 & -6 \\ 5 & 1 \\ 9 & 4 \end{bmatrix}$

C- $\begin{bmatrix} 7 \end{bmatrix}$

D- $\begin{bmatrix} 1 & 3 & 7 & -2 & 4 \\ 9 & 11 & 5 & 6 & 8 \\ 6 & -2 & -1 & 1 & 1 \end{bmatrix}$

Find the size for the following matrices:

a. The matrix $\begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$ has size 1×3 .

b. The matrix $\begin{bmatrix} 1 & -6 \\ 5 & 1 \\ 9 & 4 \end{bmatrix}$ has size 3×2 .

Find the size for the following matrices:

C- The matrix $\begin{bmatrix} 7 \end{bmatrix}$ has size 1×1 .

d. The matrix $\begin{bmatrix} 1 & 3 & 7 & -2 & 4 \\ 9 & 11 & 5 & 6 & 8 \\ 6 & -2 & -1 & 1 & 1 \end{bmatrix}$ has size 3×5

- Elements of a matrix
- Row Matrix
- Column Matrix
- Matrices of the same order
- Equal Matrices

The numbers in a matrix are called its **elements**. The element in the i^{th} row and j^{th} column of a matrix is generally denoted by a^{ij} . A matrix with m rows and n columns is written $(a_{ij})_{mn}$ or (a_{ij}) .

A matrix with just one row is called a row matrix (or row vector).

$$A = [a_1 a_2 \dots, a_n] = \{a_j\}_{(1 \times n)}$$

A matrix with just one column is called a **column matrix**.

$$A = \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_m \end{bmatrix} = \{a_i\}_{(m \times 1)}$$

Two matrices which have the Same number of rows and columns are said to be matrices of the **same order**.

Two matrices A and B are said to be equal if, and only if, each element a_{ij} of A is equal to the corresponding element b_{ij} of B .

Two matrices $A = (a_{ij})$ and $B = (b_{ij})$ are said to be equal if, and only if, each element a_{ij} of A is equal to the corresponding element b_{ij} of B .

In symbolic form this reads:

$$A=B \Leftrightarrow a_{ij} = b_{ij} \text{ for all } i \text{ and } j$$

From this it follows that equal matrices are of the same order but matrices of the same order are not necessarily equal.

- Null Matrix
- Identity Matrix
- Square Matrix
- Transpose Matrix

Any matrix all of whose elements are zero is called a null or zero matrix

Any matrix, all of whose elements are zero, is called a **null** or **zero** matrix and is denoted by O .

$$[0], \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Square matrix with ones on the diagonal and zeros elsewhere.

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Same number of rows and columns

$$B = \begin{bmatrix} 5 & 4 & 7 \\ 3 & 6 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

Rows become columns and columns become rows

$$A^T = \begin{bmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & a_{m2} \\ \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n} & \dots & a_{mn} \end{bmatrix}$$

The transpose of an $m \times n$ matrix A , denoted A^T , is the $n \times m$ matrix.

Let's Practice



If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Find A^T .



Transpose of matrix A

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

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Example 2

$$\text{If } A = \begin{bmatrix} 2 & -2 & 5 \\ -3 & 4 & 1 \\ -7 & 2 & 5 \end{bmatrix}$$

Find A^T



$$A^T = \begin{bmatrix} 2 & -3 & -7 \\ -2 & 4 & 2 \\ 5 & 2 & 5 \end{bmatrix}$$

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Find A^T



Matrix A is 2×3 , so A^T is 3×2

→ First row of A becomes the first column of A^T

→ Second row of A becomes the second column of A^T

$$\therefore A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

Examples:

If $A = \begin{pmatrix} -2 & 0 & \frac{1}{2} \\ 1 & 6 & -4 \end{pmatrix}$

Find a) The size of the matrix

a) a_{22} , a_{12} and a_{23}

Solution

a) size = 2 x 3

b) $a_{22} = 6$, $a^{12} = 0$ $a_{23} = -4$

$$\text{If } B = \begin{pmatrix} 4 & 5 \\ -1 & 8 \end{pmatrix}$$

Find a) the size

a) b_{11} & b_{22}

Solution

a) the size = 2×2

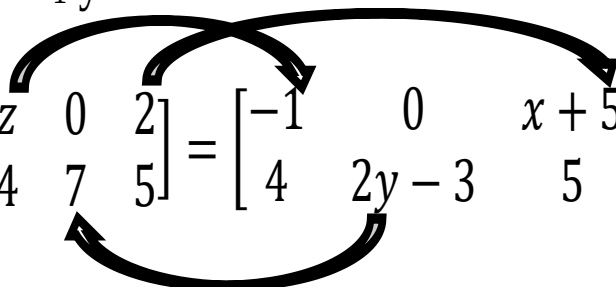
b) $b_n = 4$

$b_{22} = 8$



Example

Find x , y and z

$$\text{If } \begin{bmatrix} z & 0 & 2 \\ 4 & 7 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & x+5 \\ 4 & 2y-3 & 5 \end{bmatrix}$$


$$Z = -1$$

$$X + 5 = 2 \rightarrow x = 2 - 5 = -3$$

$$2y - 3 = 7 \rightarrow 2y = 7 + 3 \rightarrow 2y = 10$$

$$\rightarrow y = \frac{10}{2} = 5$$



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