

Business Mathematics

Prepared by Dr. Faisal Hasan

Lecture NO 3

جامعة الجميع الذكية

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التعاوني ويقدم العلم مقابل العمل لتنمية وبناء المجتمعات.

MTH 112

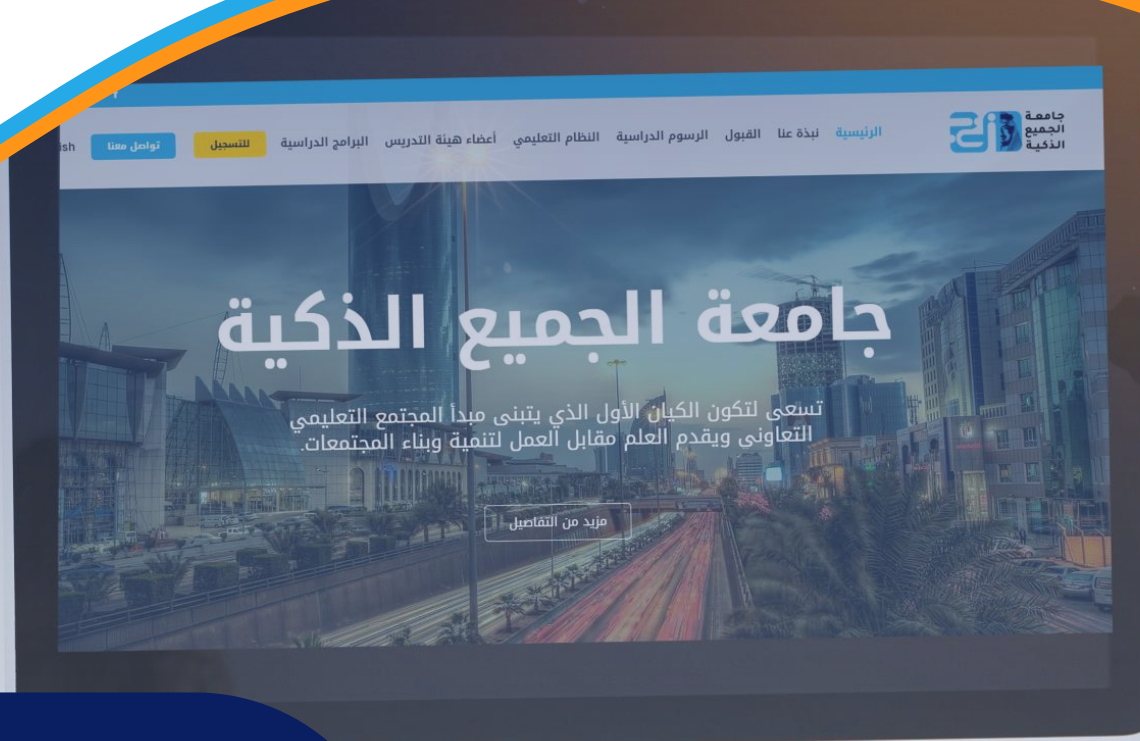
Business Mathematics



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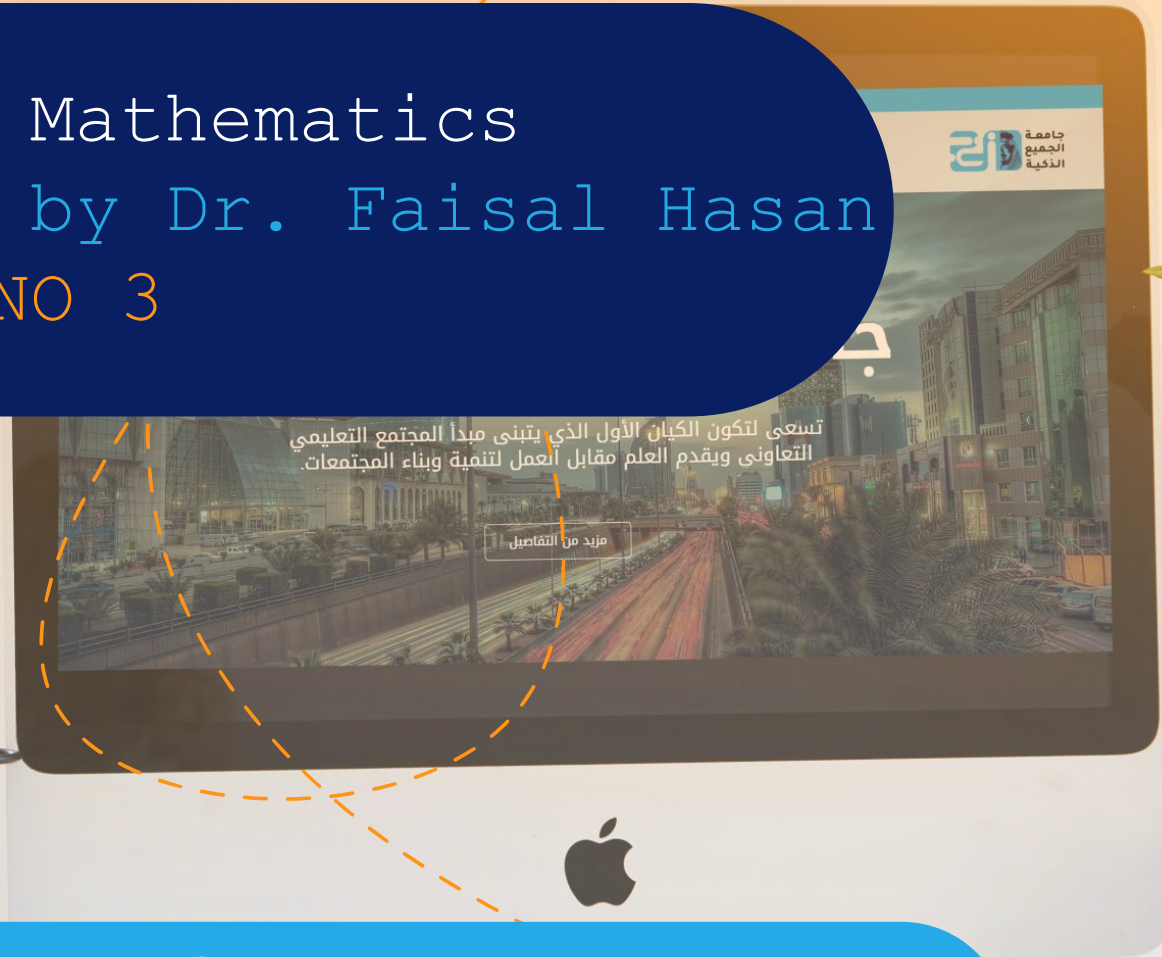
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MTH 112

Business Mathematics

Topics

- Chapter 1: Mathematical preliminaries
- Chapter 2: The straight line and applications
- Chapter 3: Functions and Graphs
- Chapter 4: Limits and Continuity
- Chapter 5: Financial Mathematics
- Chapter 6: Differentiation
- Chapter 7: Methods of Differentiation
- Chapter 8: Function of several variables
- Chapter 9: Integration and applications
- Chapter 10: Linear algebra and applications

Objectives

- To use the properties of exponents to:
 - Simplify exponential expressions.
 - Solve exponential equations.
- To sketch graphs of exponential functions.

- A polynomial function has the basic form: $f(x) = x^3$
- An exponential function has the basic form: $f(x) = 3^x$
- An exponential function has the variable in the exponent, not in the base.
- General Form of an Exponential Function:
$$f(x) = N^x, N > 0$$

Properties of Exponents

$$A^X \cdot A^Y = A^{X+Y}$$

$$\frac{A^X}{A^Y} = A^{X-Y}$$

$$(A^X)^Y = A^{XY}$$

$$A^{-X} = \frac{1}{A^X}$$

$$(AB)^X = A^X B^X$$

$$\frac{1}{A^{-X}} = A^X$$

$$\left(\frac{A}{B}\right)^X = \frac{A^X}{B^X}$$

$$A^{x/y} = \sqrt[y]{A^x} = \left(\sqrt[y]{A}\right)^x$$



■ Simplify: $2^2 \cdot 2^3 = 2^5 = 32$

$$2^2 \cdot 2^{-6} = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

$$(2^3)^2 = 2^6 = 64$$



■ Simplify: $\left(\frac{2}{3}\right)^{-3} = \frac{2^{-3}}{3^{-3}} = \frac{3^3}{2^3} = \frac{27}{8}$

$$\frac{3^7}{3^9} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$(2^{1/2})(8^{1/2}) = (2 \cdot 8)^{1/2} = 16^{1/2} = \sqrt{16} = 4$$



■ Solve: $5^x = 125$

$$5^x = 5^3$$

$$x = 3$$



■ Solve: $7^{(x-1)} = 7^{-1/2}$

$$x - 1 = -\frac{1}{2}$$
$$x = \frac{1}{2}$$

■ Solve: $8^x = 2$

$$(2^3)^x = 2^1$$

$$2^{3x} = 2^1$$

$$3x = 1$$

$$x = \frac{1}{3}$$



■ Solve: $8^x = 4$

$$(2^3)^x = 2^2$$

$$2^{3x} = 2^2$$

$$3x = 2$$

$$x = \frac{2}{3}$$



■ Solve: $\left(\frac{1}{3}\right)^x = 27$

$$\left(3^{-1}\right)^x = 27$$

$$3^{-x} = 3^3$$

$$-x = 3$$

$$x = -3$$



■ Solve: $x^{1/3} = 27$

$$\left(x^{1/3}\right)^3 = 27^3$$

$$x = 19,683$$

Not considered an
exponential equation,
because the variable
is now in the base.



■ Solve:

Not considered an exponential equation, because the variable is in the base.

$$x^{3/4} = 8$$

$$\left(x^{3/4}\right)^{4/3} = 8^{4/3}$$

$$x = \left(\sqrt[3]{8}\right)^4$$

$$x = (2)^4$$

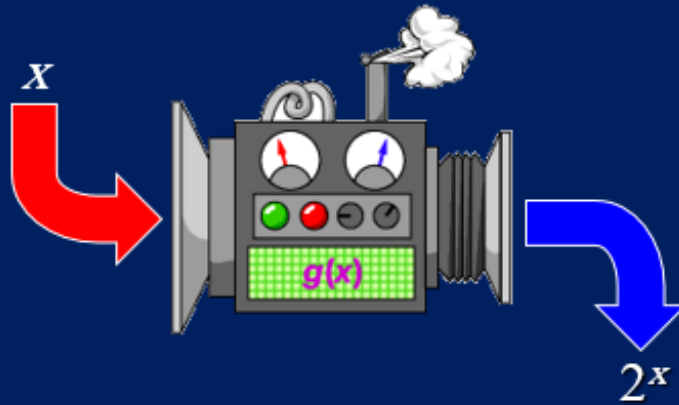
$$x = 16$$



■ General Form of an Exponential Function:

$$f(x) = N^x, N > 0$$

$$g(x) = 2^x$$



$$g(3) = 8$$

$$g(2) = 4$$

$$g(1) = 2$$

$$g(0) = 1$$

$$g(-1) = 2^{-1} = \frac{1}{2}$$

$$g(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

Chapter 03

Exponential Functions

$$g(x) = 2^x$$

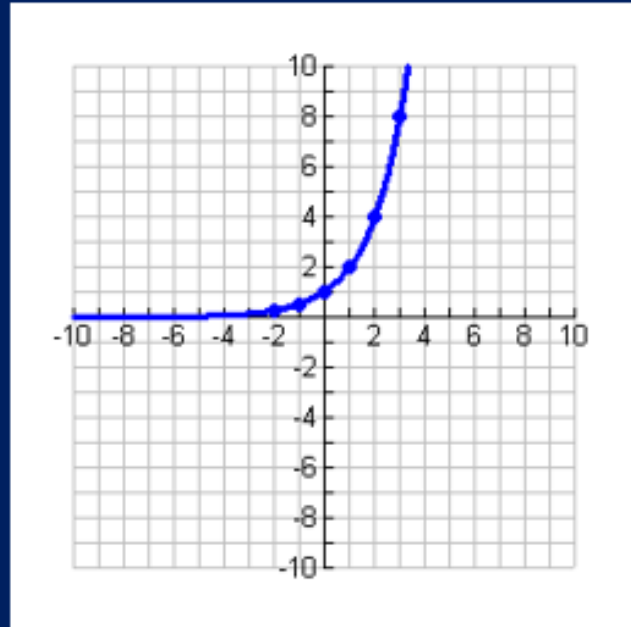
x	$g(x)$
2	4
1	2
0	1
-1	$\frac{1}{2}$
-2	$\frac{1}{4}$



Chapter 03

Exponential Functions

$$g(x) = 2^x$$



Chapter 03

Exponential Functions

$$h(x) = 3^x$$

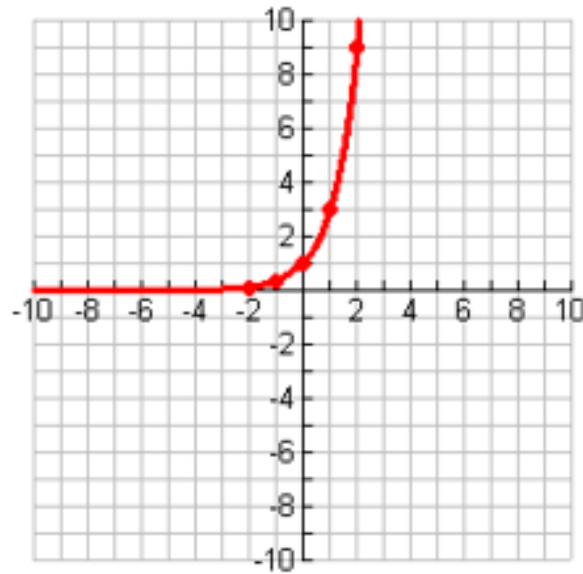
x	$h(x)$
2	9
1	3
0	1
-1	$\frac{1}{3}$
-2	$\frac{1}{9}$



Chapter 03

Exponential Functions

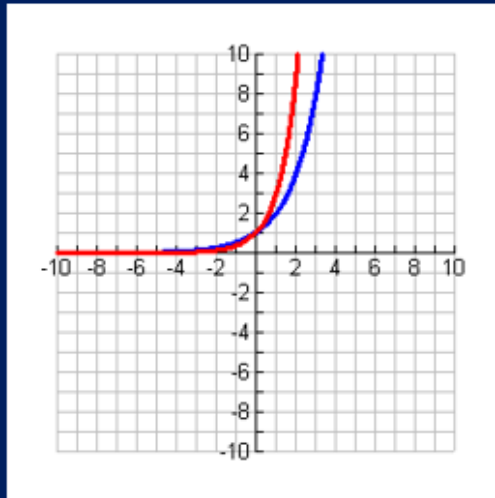
$$h(x) = 3^x$$



Chapter 03

Exponential Functions

$$g(x) = 2^x \text{ (blue)}$$



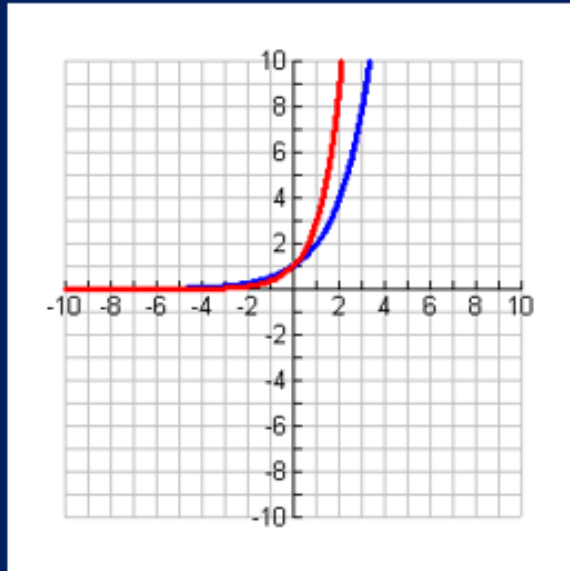
$$h(x) = 3^x \text{ (red)}$$

- Exponential functions with positive bases greater than 1 have graphs that are increasing.
- The function never crosses the x -axis because there is nothing we can plug in for x that will yield a zero answer.
- The x -axis is a left horizontal asymptote.

Chapter 03

Exponential Functions

$$g(x) = 2^x \text{ (blue)}$$



$$h(x) = 3^x \text{ (red)}$$

- A smaller base means the graph rises more gradually.
- A larger base means the graph rises more quickly.
- Exponential functions will not have negative bases.

The Number e

A base often associated with exponential functions is:

$$e \approx 2.71828169$$

The Number e

- Euler's number
- Leonhard Euler
(pronounced "oiler")
- Swiss mathematician
and physicist

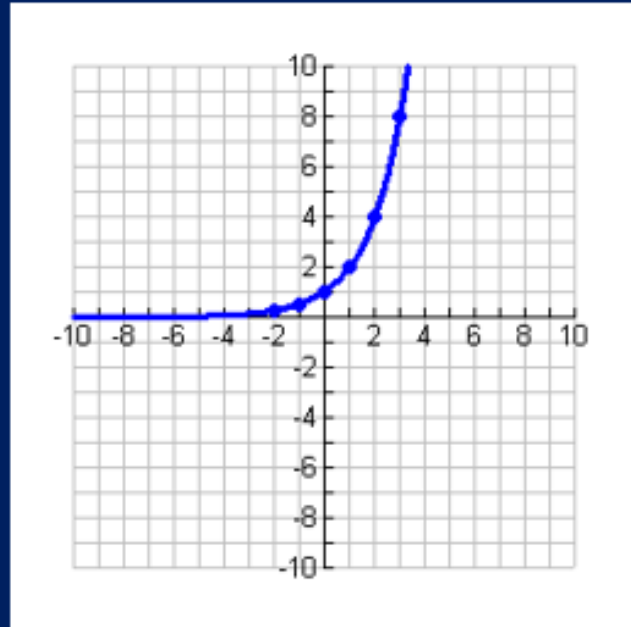


Chapter 03

Exponential Functions

Remember

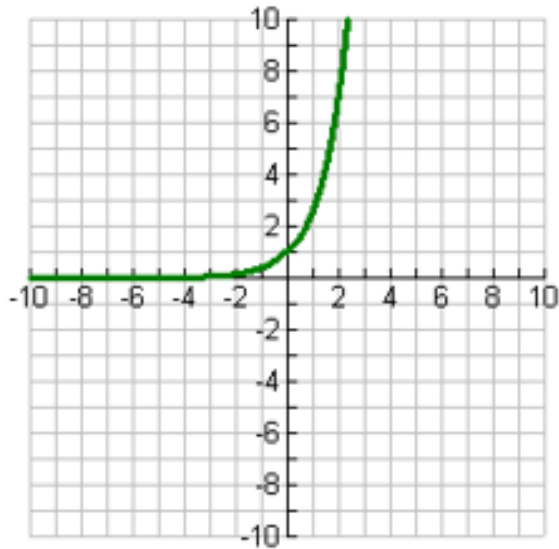
$$g(x) = 2^x$$



Chapter 03

Exponential Functions

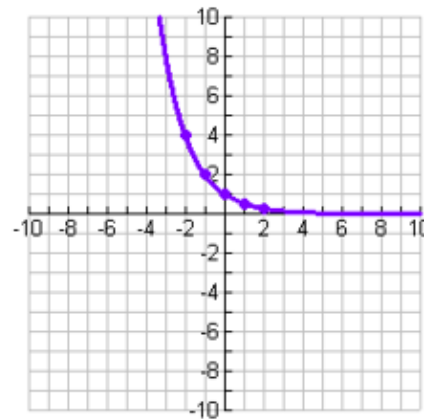
$$f(x) = e^x$$



Chapter 03

Exponential Functions

$$j(x) = \left(\frac{1}{2}\right)^x$$



- Exponential functions with positive bases less than 1 have graphs that are decreasing.

Why study exponential functions?

- Exponential functions are used in our real world to measure growth, interest, and decay.
- Growth obeys exponential functions.
- Ex: rumors, human population, bacteria, computer technology, nuclear chain reactions, compound interest
- Decay obeys exponential functions.
- Ex: Carbon-14 dating, half-life, Newton's Law of Cooling



You can use exponential functions to model the way certain populations increase or decrease over time.

Increasing populations can be modeled by exponential growth function.

Can you tell by looking at an equation whether the function models exponential growth or decay?

$$y = ab^x$$

When $b > 1$, the function models exponential growth,
and b is the **growth factor**.

When $0 < b < 1$,
the function models exponential **decay**,
and b is the **decay factor**.
It also called negative growth.

Now tell me just by looking at the equation if it represent exponential growth or decay.

$$y = 15(1.45)^x \quad \text{Exponential Growth}$$

$$y = 7.3(0.8)^x \quad \text{Exponential Decay}$$

For $x > 0$, $b > 0$ and b not equal to 1 the logarithm of x with base b is defined by the following:

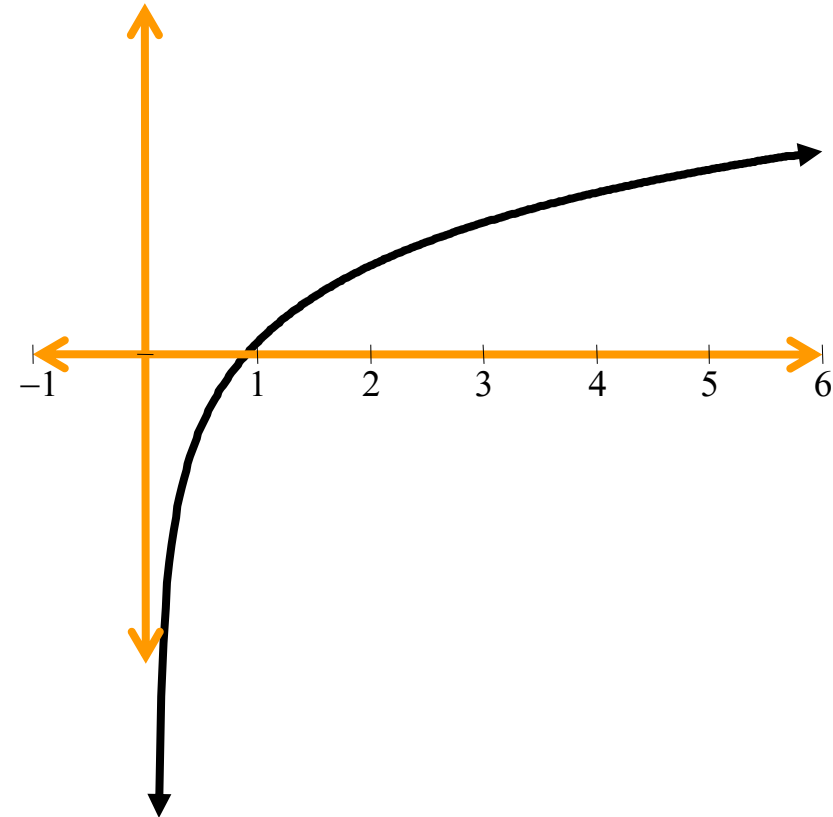
$$\log_b x = y \Leftrightarrow x = b^y$$

Properties of Logarithmic Function

- Domain: $\{x > 0\}$
- Range: all real numbers
- x intercept: (1,0)
- No y intercept
- Approaches y axis as vertical asymptote
- Base determines shape.

Shape of logarithmic graphs

- For $b > 1$, the graph rises from left to right.



Common Logarithmic Function

The logarithmic function with
base 10

$$\log_{10} x = y \Leftrightarrow \log x = y$$

Natural logarithmic function
The logarithmic function with a
base of e

$$\log_e x = y \Leftrightarrow \ln x = y$$

Calculator Keys

- [LOG]
- [LN]



$$\log_b 1 = 0$$

$$\log_b b = 1$$

$$\log_b x = \log_b y \Leftrightarrow x = y$$



For $x > 0$, $y > 0$, $b > 0$ and $b \neq 1$
Product rule of Logarithms

$$\log_b (xy) = \log_b x + \log_b y$$

For $x > 0$, $y > 0$, $b > 0$ and $b \neq 1$
Quotient rule for Logarithms

$$\log_b \left(\frac{x}{y} \right) = \log_b x - \log_b y$$

For $x > 0$, $y > 0$, $b > 0$ and b not 1
Power rule for Logarithms

$$\log_b x^r = r (\log_b x)$$



- Apply the product, quotient, and power properties of logarithms.
- Combine and Expand logarithmic expressions

$$\log_b (xy) = \log_b x + \log_b y$$

$$\log_b \left(\frac{x}{y} \right) = \log_b x - \log_b y$$

$$\log_b x^r = r (\log_b x)$$

$$\ln(xy) = \ln x + \ln y$$

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

$$\ln x^r = r(\ln x)$$



- A **logarithm**, or **log**, is defined in terms of an exponential.

If $b^x = a$, then $\log_b a = x$

If $5^2 = 25$, then $\log_5 25 = 2$

Chapter 03

Exponential and Logarithmic Functions

Solve each logarithmic equation.

$$\ln 3x = 2$$



Chapter 03

Exponential and Logarithmic Functions

$$\ln 3x = 2$$

Write original equation.

$$e^{\ln 3x} = e^2$$

Exponentiate each side.



Chapter 03

Exponential and Logarithmic Functions

$$3x = e^2$$

Inverse Property

$$x = \frac{1}{3}e^2$$

Multiply each side by $\frac{1}{3}$.

$$x \approx 2.46$$

Use a calculator.

The solution is $x = \frac{1}{3}e^2$
 ≈ 2.46

Check this in the original equation.



Solve the equation

$$\log_5 x - 2\log_5 7 = \log_5 125$$

$$\log_5 x - 2\log_5 7 = \log_5 125$$

$$\log_5 x - \log_5 7^2 = \log_5 125$$

$$\log_5 x - \log_5 49 = \log_5 125$$

$$\log_5 \frac{x}{49} = \log_5 125$$

$$\frac{x}{49} = 125$$

$$x = 125 \times 49 = 6125$$

Solve the equation

$$\log_2 x + 2\log_2 5 = 2\log_2 (x - 6)$$

$$\log_2 x + 2\log_2 5 = 2\log_2 (x - 6)$$

$$\log_2 x + \log_2 5^2 = \log_2 (x - 6)^2$$

$$\log_2 x + \log_2 25 = \log_2 (x - 6)^2$$

$$\log_2(25x) = \log_2(x - 6)^2$$

$$25x = (x - 6)^2$$

$$25x = x^2 - 12x + 36$$

$$0 = x^2 - 37x + 36$$

$$(x - 36)(x - 1) = 0$$

$$x - 36 = 0 \quad x - 1 = 0$$

$$x = 36$$

$$\cancel{x = 1}$$

Reject solutions
that would mean
taking the log of
a negative number

Given that $f(x) = 5^x$

Find the value of $f(-2)$

Given that $f(x) = 5^x$

Find the value of $f(0)$



Given that $f(x) = 5^x$

Find the value of $f(1)$

Given that $f(x) = 5^x$

Find the value of $f(-1)$

Given that $f(x) = \left(\frac{1}{4}\right)^x$

Find the value of $f(-1)$



Given that $f(x) = \left(\frac{1}{4}\right)^x$

Find the value of $f(0)$



Given that exponential form $(16)^{\frac{1}{2}} = 4$

Find the logarithmic form

Given that logarithmic form $\log_8 16 = \frac{3}{4}$

Find the exponential form

Given that logarithmic form $\log 100 = 2$

Find the exponential form

Given the function $y = 28(0.3)^x$

Is exponential decay or growth



Given the function $y = 2(4.3)^x$

Is exponential decay or growth

Given the expression

$$\ln \frac{y}{x^3}$$

Simplify it



Solve the equation $\ln\left(\frac{x}{10}\right) = \ln(15)$