

Business Mathematics

Prepared by Dr. Alfaisal Hasan

MTH 112
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Lecture NO 5



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Chapter

Topics

- Chapter 1: Mathematical preliminaries
- Chapter 2: The straight line and applications
- Chapter 3: Functions and Graphs
- Chapter 4: Limits and Continuity
- **Chapter 5: Financial Mathematics**
- Chapter 6: Differentiation
- Chapter 7: Methods of Differentiation
- Chapter 8: Function of several variables
- Chapter 9: Integration and applications
- Chapter 10: Linear algebra and applications

Objectives

- 1: Using and Writing Sequence
- 2: Arithmetic Sequence
- 3: Arithmetic Series

Using and Writing Sequence

The numbers in sequences are called terms.

You can think of a **sequence** as a function whose domain is a set of consecutive integers. If a domain is not specified, it is understood that the domain starts with 1.

3 6 9 12 15

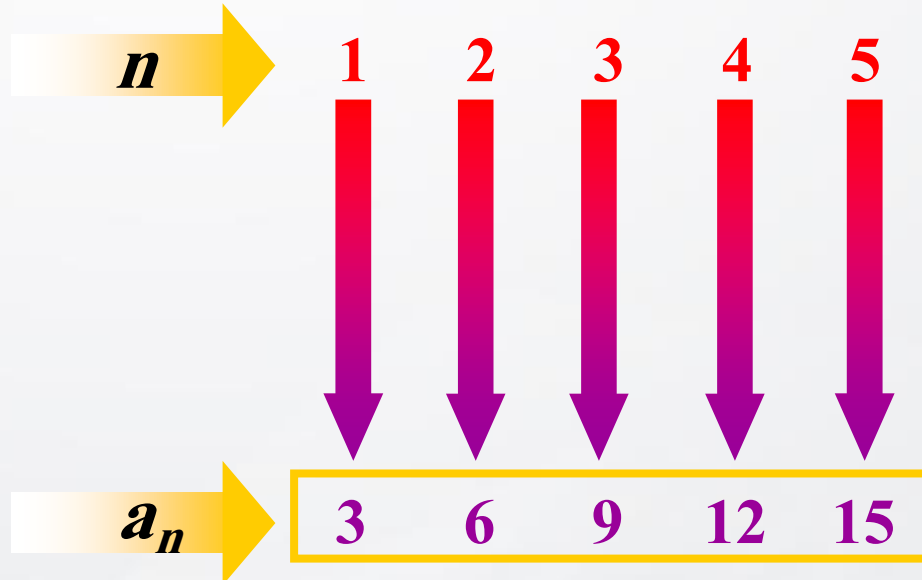
Using and Writing Sequence

Sequence a list of numbers in a specific order.

Term each number in a sequence.

3 6 9 12 15

Using and Writing Sequence



The **domain** gives the **relative position** of each **term**.

The **range** gives the **terms** of the sequence.

This is a **finite sequence** having the rule

$$a_n = 3n,$$

where a_n represents the n th term of the sequence.

Using and Writing Sequence

EXAMPLE

Write the first five terms of the sequence $a_n = 2n + 3$.

Using and Writing Sequence

SOLUTION

$$a_1 = 2(1) + 3 = 5$$

1st term

$$a_2 = 2(2) + 3 = 7$$

2nd term

$$a_3 = 2(3) + 3 = 9$$

3rd term

$$a_4 = 2(4) + 3 = 11$$

4th term

$$a_5 = 2(5) + 3 = 13$$

5th term

Using and Writing Sequence

EXAMPLE

Write the first five terms of the sequence $a_n = \frac{n}{n+1}$.

Using and Writing Sequence

SOLUTION

$$a_1 = \frac{(1)}{(1)+1} = \frac{1}{2}$$

1st term

$$a_2 = \frac{(2)}{(2)+1} = \frac{2}{3}$$

2nd term

$$a_3 = \frac{(3)}{(3)+1} = \frac{3}{4}$$

3rd term

$$a_4 = \frac{(4)}{(4)+1} = \frac{4}{5}$$

4th term

$$a_5 = \frac{(5)}{(5)+1} = \frac{5}{6}$$

5th term

Using and Writing Sequence

EXAMPLE

Write the first five terms of the sequence $a_n = n! - 2$.

Using and Writing Sequence

SOLUTION

$$a_1 = (1)! - 2 = -1$$

1st term

$$a_2 = (2)! - 2 = 0$$

2nd term

$$a_3 = (3)! - 2 = 4$$

3rd term

$$a_4 = (4)! - 2 = 22$$

4th term

$$a_5 = (5)! - 2 = 118$$

5th term

Using and Writing Sequence

EXAMPLE

Write the first five terms of the sequence $f(n) = (-2)^{n-1}$.

Using and Writing Sequence

SOLUTION

$$f(1) = (-2)^{1-1} = 1$$

1st term

$$f(2) = (-2)^{2-1} = -2$$

2nd term

$$f(3) = (-2)^{3-1} = 4$$

3rd term

$$f(4) = (-2)^{4-1} = -8$$

4th term

$$f(5) = (-2)^{5-1} = 16$$

5th term

Arithmetic Sequence

- **Arithmetic Sequence:** *sequence whose consecutive terms have a common difference.*
- *Example:* 3, 5, 7, 9, 11, 13, ...
- The terms have a common difference of 2.
- The common difference is the number d .
- To find the common difference you use $a_{n+1} - a_n$
- *Example: Is the sequence arithmetic?*
-45, -30, -15, 0, 15, 30
- **Yes,** *the common difference is 15*

Arithmetic Sequence

Find the next four terms of $-9, -2, 5, \dots$

Arithmetic Sequence

$$-2 - (-9) = 7 \text{ and } 5 - (-2) = 7$$

7 is referred to as the common difference (d)

Common Difference (d) – what we ADD to get next term

Next four terms.....12, 19, 26, 33

Arithmetic Sequence

Find the next four terms of 0, 7, 14, ...

Arithmetic Sequence, $d = 7$

21, 28, 35, 42

Arithmetic Sequence

Find the next four terms of $-3x, -2x, -x, \dots$

Arithmetic Sequence, $d = x$

$0, x, 2x, 3x$

Arithmetic Sequence

Find the next four terms of $5k, -k, -7k, \dots$

Arithmetic Sequence, $d = -6k$

$-13k, -19k, -25k, -31k$

Arithmetic Sequence

Find the next four terms of 2, 8, 14, ...

Arithmetic Sequence, $d = 6$

20, 26, 32, 38

Arithmetic Sequence

Find the next four terms of 1, 11, 21, ...

Arithmetic Sequence, $d = 10$

31, 41, 51, 61

Arithmetic Sequence

Find the next four terms of 100, 80, 60, ...

Arithmetic Sequence, $d = -20$

40, 20, 0, -20

Arithmetic Sequence

How do you find any term in this sequence?

To find any term in an arithmetic sequence, use the formula

$$a_n = a_1 + (n - 1)d$$

where d is the common difference.

Arithmetic Sequence

Vocabulary of Sequences (Universal)

$a_1 \rightarrow$ First term

$a_n \rightarrow$ nth term

$n \rightarrow$ number of terms

$S_n \rightarrow$ sum of n terms

$d \rightarrow$ common difference

nth term of arithmetic sequence $\rightarrow a_n = a_1 + (n - 1)d$

sum of n terms of arithmetic sequence $\rightarrow S_n = \frac{n}{2}(a_1 + a_n)$

Arithmetic Sequence

EXAMPLE

Find the 14th term of the arithmetic sequence

4, 7, 10, 13,

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n - 1)d$$

$$\begin{aligned} a_{14} &= 4 + (14 - 1) \\ &= 4 + (13)3 \\ &= 4 + 39 \\ &= 43 \end{aligned}$$

Arithmetic Sequence

EXAMPLE

Find the 12th term of the arithmetic sequence

2, 6, 10, 14,.....

Arithmetic Sequence

2, 6, 10, 14,.....

$$a_n = a_1 + (n-1)d$$

$$a_1 = 2, d = 6 - 2 = 4, n = 12$$

$$a_{12} = 2 + (12-1)4$$

$$a_{12} = 2 + 44$$

$$a_{12} = 46$$

Arithmetic Sequence

EXAMPLE

Find a_{16} if $a_1 = 1.5$ and $d = 0.5$

Arithmetic Sequence

Find a_{16} if $a_1 = 1.5$ and $d = 0.5$

1.5 $a_1 \rightarrow$ First term

a_{16} $a_n \rightarrow$ nth term

16 $n \rightarrow$ number of terms

NA $S_n \rightarrow$ sum of n terms

0.5 $d \rightarrow$ common difference

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n - 1)d$$

$$a_{16} = 1.5 + (16 - 1)(0.5)$$

$$a_{16} = 1.5 + (15)(0.5)$$

$$a_{16} = 1.5 + 7.5$$

$$a_{16} = 9$$

Arithmetic Sequence

EXAMPLE

Find a_{20} if $a_1 = 2.0$ and $d = 3.0$

Arithmetic Sequence

Find a_{20} if $a_1 = 2.0$ and $d = 3.0$

$$a_n = a_1 + (n-1)d$$

$$a_1 = 2, d = 3, n = 20$$

$$a_{20} = 2 + (20-1)3$$

$$a_{20} = 2 + 57$$

$$a_{20} = 59$$

Arithmetic Sequence

EXAMPLE

The table shows typical costs for a construction company to rent a crane for one, two, three, or four months. Assuming that the arithmetic sequence continues, how much would it cost to rent the crane for twelve months?

Months	Cost (\$)
1	75,000
2	90,000
3	105,000
4	120,000

Arithmetic Sequence

SOLUTION

Months	Cost (\$)
1	75,000
2	90,000
3	105,000
4	120,000

75,000 $a_1 \rightarrow$ First term

a_{12} $a_n \rightarrow$ nth term

12 $n \rightarrow$ number of terms

NA $S_n \rightarrow$ sum of n terms

15,000 $d \rightarrow$ common difference

Arithmetic Sequence

$$a_n = a_1 + (n - 1)d$$

$$a_{12} = 75,000 + (12 - 1)(15,000)$$

$$a_{12} = 75,000 + (11)(15,000)$$

$$a_{12} = 75,000 + 165,000$$

$$a_{12} = \$240,000$$

Arithmetic Sequence

EXAMPLE

In the arithmetic sequence 4,7,10,13,..., which term has a value of 301?

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n - 1)d$$

$$301 = 4 + (n - 1)3$$

$$301 = 4 + 3n - 3$$

$$301 = 1 + 3n$$

$$300 = 3n$$

$$100 = n$$

Arithmetic Sequence

EXAMPLE

Find n if $a_n = 633$, $a_1 = 9$, and $d = 24$

Arithmetic Sequence

Find n if $a_n = 633$, $a_1 = 9$, and $d = 24$

9 $a_1 \rightarrow$ First term

633 $a_n \rightarrow$ nth term

n $n \rightarrow$ number of terms

NA $S_n \rightarrow$ sum of n terms

24 $d \rightarrow$ common difference

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n-1)d$$

$$633 = 9 + (n-1)(24)$$

$$633 = 9 + 24n - 24$$

$$633 = 24n - 15$$

$$648 = 24n$$

$$n = 27$$

Arithmetic Sequence

EXAMPLE

Given an arithmetic sequence with

$$a_{15} = 38 \text{ and } d = -3, \text{ find } a_1.$$

Arithmetic Sequence

Given an arithmetic sequence with

$$a_{15} = 38 \text{ and } d = -3, \text{ find } a_1.$$

a_1 $a_1 \rightarrow$ First term

38 $a_n \rightarrow$ nth term

15 $n \rightarrow$ number of terms

NA $S_n \rightarrow$ sum of n terms

-3 $d \rightarrow$ common difference

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n - 1)d$$

$$38 = a_1 + (15 - 1)(-3)$$

$$38 = a_1 + (14)(-3)$$

$$38 = a_1 - 42$$

$$a_1 = 80$$

Arithmetic Sequence

EXAMPLE

Find d if $a_1 = -6$ and $a_{29} = 20$

Arithmetic Sequence

Find d if $a_1 = -6$ and $a_{29} = 20$

-6 $a_1 \rightarrow$ First term

20 $a_n \rightarrow$ nth term

29 $n \rightarrow$ number of terms

NA $S_n \rightarrow$ sum of n terms

d $d \rightarrow$ common difference

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n - 1)d$$

$$20 = -6 + (29 - 1)(d)$$

$$20 = -6 + (28)(d)$$

$$26 = 28d$$

$$d = \frac{26}{28}$$

Arithmetic Sequence

EXAMPLE

Write an equation for the n^{th} term of the arithmetic sequence
8, 17, 26, 35, ...

Arithmetic Sequence

Write an equation for the n^{th} term of the arithmetic sequence
8, 17, 26, 35, ...

8 $a_1 \rightarrow$ First term

9 $d \rightarrow$ common difference

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n - 1)d$$

$$a_n = 8 + (n - 1)(9)$$

$$a_n = 8 + 9n - 9$$

$$a_n = 9n - 1$$

Arithmetic Sequence

Arithmetic Mean: The terms between any two nonconsecutive terms of an arithmetic sequence.

Ex. 19, **30**, 41, 52, 63, **74**, 85, 96

41, 52, 63 are the Arithmetic Mean between 30 and 74

Arithmetic Sequence

EXAMPLE

Find two arithmetic means between -4 and 5

$-4, \underline{\quad}, \underline{\quad}, 5$

Arithmetic Sequence

Find two arithmetic means between -4 and 5

$-4, \underline{\quad}, \underline{\quad}, 5$

- -4 $a_1 \rightarrow$ First term
- 5 $a_n \rightarrow$ nth term
- 4 $n \rightarrow$ number of terms
- NA $S_n \rightarrow$ sum of n terms
- d $d \rightarrow$ common difference

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n - 1)d$$

$$5 = -4 + (4 - 1)(d)$$

$$5 = -4 + (3)(d)$$

$$9 = (3)(d)$$

$$d = 3$$

The two arithmetic means are -1 and 2 , since $-4, -1, 2, 5$ forms an arithmetic sequence

Arithmetic Sequence

EXAMPLE

Find three arithmetic means between 21 and 45

21, _____, _____, _____, 45

Arithmetic Sequence

Find three arithmetic means between 21 and 45

21, _____, _____, _____, 45

21 $a_1 \rightarrow$ First term

45 $a_n \rightarrow$ nth term

5 $n \rightarrow$ number of terms

NA $S_n \rightarrow$ sum of n terms

d $d \rightarrow$ common difference

Arithmetic Sequence

SOLUTION

$$a_n = a_1 + (n - 1)d$$

$$45 = 21 + (5 - 1)(d)$$

$$45 = 21 + (4)(d)$$

$$24 = (4)(d)$$

$$d = 6$$

The three arithmetic means are 27, 33, and 39
since 21, 27, 33, 39, 45 forms an arithmetic sequence

Arithmetic Series

Arithmetic Series: *The sum of terms in an arithmetic sequence.*

Arithmetic Sequence

VS

Arithmetic Series

3, 5, 7, 9, 11, 13

$3 + 5 + 7 + 9 + 11 + 13$

Arithmetic Series

Vocabulary of Sequences (Universal)

$a_1 \rightarrow$ First term

$a_n \rightarrow$ nth term

$n \rightarrow$ number of terms

$S_n \rightarrow$ sum of n terms

$d \rightarrow$ common difference

nth term of arithmetic sequence $\rightarrow a_n = a_1 + (n - 1)d$

Sum of n terms of a **FINITE** arithmetic sequence $\rightarrow S_n = \frac{n}{2}(a_1 + a_n)$

Arithmetic Series

EXAMPLE

Find the sum of the first 63 terms of the arithmetic sequence $-19, -13, -7, \dots$

Arithmetic Series

Find the sum of the first 63 terms of the arithmetic sequence -19, -13, -7,...

-19 $a_1 \rightarrow$ First term

?? $a_n \rightarrow$ nth term

63 $n \rightarrow$ number of terms

S_n $S_n \rightarrow$ sum of n terms

6 $d \rightarrow$ common difference

Arithmetic Series

$$a_n = a_1 + (n - 1)d$$

$$a_{63} = -19 + (63 - 1)(6)$$

$$a_{63} = 353$$

Arithmetic Series

Find the sum of the first 63 terms of the arithmetic sequence -19, -13, -7,...

-19 $a_1 \rightarrow$ First term

353 ?? $a_n \rightarrow$ nth term

63 $n \rightarrow$ number of terms

S_n $S_n \rightarrow$ sum of n terms

6 $d \rightarrow$ common difference

Arithmetic Series

SOLUTION

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{63} = \frac{63}{2}(-19 + 353)$$

$$S_{63} = 10521$$

Arithmetic Series

EXAMPLE

Find the sum of the first 55 terms of the arithmetic sequence 1, 5, 9,...

$$a_1 = 1, d = 4, n = 55$$

$$a_n = a_1 + (n-1)d$$

$$a_1 = 1, d = 4, n = 55$$

$$a_{55} = 1 + (55-1)4$$

$$a_{55} = 1 + 216$$

$$a_{55} = 217$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$a_1 = 1, a_{55} = 217, n = 55$$

$$S_{55} = \frac{55}{2}(1 + 217)$$

$$S_{55} = 5995$$

Arithmetic Series

EXAMPLE

Find the first 3 terms for an arithmetic series in which $a_1 = 9$, $a_n = 105$, $S_n = 741$.

Arithmetic Series

Find the first 3 terms for an arithmetic series in which $a_1 = 9$, $a_n = 105$, $S_n = 741$.

9 $a_1 \rightarrow$ First term

105 $a_n \rightarrow$ nth term

?? $n \rightarrow$ number of terms

741 $S_n \rightarrow$ sum of n terms

?? $d \rightarrow$ common difference

Arithmetic Series

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$741 = \frac{n}{2}(9 + 105)$$

$$741 = \frac{n}{2}(114)$$

$$741 = 57n$$

$$n = 13$$

Arithmetic Series

Find the first 3 terms for an arithmetic series in which $a_1 = 9$, $a_n = 105$, $S_n = 741$.

9 $a_1 \rightarrow$ First term

105 $a_n \rightarrow$ nth term

13 ?? $n \rightarrow$ number of terms

741 $S_n \rightarrow$ sum of n terms

?? $d \rightarrow$ common difference

Arithmetic Series

$$a_n = a_1 + (n - 1)d$$

$$105 = 9 + (13 - 1)d$$

$$105 = 9 + (12)d$$

$$96 = 12d$$

$$8 = d$$

Arithmetic Series

Find the first 3 terms for an arithmetic series in which $a_1 = 9$, $a_n = 105$, $S_n = 741$.

9 $a_1 \rightarrow$ First term

105 $a_n \rightarrow$ nth term

13 ?? $n \rightarrow$ number of terms

741 $S_n \rightarrow$ sum of n terms

8 ?? $d \rightarrow$ common difference

Arithmetic Series

Find the first 3 terms for an arithmetic series in which $a_1 = 9$, $a_n = 105$, $S_n = 741$.

9 $a_1 \rightarrow$ First term

105 $a_n \rightarrow$ nth term

13 $n \rightarrow$ number of terms

741 $S_n \rightarrow$ sum of n terms

8 $d \rightarrow$ common difference

9, 17, 25

Un Solved Examples

Write the first five terms of the sequence $a_n = 3n + 1$.

Un Solved Examples

Given that $2, 6, 10, 14, \dots$ Is an arithmetic sequence?

Find the next 3 terms

Find the common difference d

Un Solved Examples

For an arithmetic sequence given that $a_1 = 9$, $a_n = 105$

Find the sum of first 15 terms

Un Solved Examples

For an arithmetic sequence given that $a_1 = -19$, $n=63$ and $d=6$

Find a_n



Thank You