

Probability and Statistics

Prepared by Dr. Alfaisal Hasan

STA 201
Probability and Statistics

Lecture NO 2



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Chapter

Topics

- Chapter 1: Introduction To Statistics
- **Chapter 2: Descriptive Statistics**
- Chapter 3: Probability
- Chapter 4: Discrete Probability Distributions
- Chapter 5: Normal Probability Distributions
- Chapter 6: Confidence Intervals
- Chapter 7: Hypothesis Testing with One Sample
- Chapter 8: Hypothesis Testing with Two Samples
- Chapter 9: Correlation And Regression
- Chapter 10: Chi-Square Tests and The F -Distribution

الفصل الأول: مقدمة في الإحصاء

الفصل الثاني: الإحصاء الوصفي

الفصل الثالث: الاحتمالات

الفصل الرابع: التوزيعات الاحتمالية المنفصلة

الفصل الخامس: التوزيع الاحتمالي الطبيعي

الفصل السادس: فترات الثقة

الفصل السابع: اختبار الفرضيات لعينة واحدة

الفصل الثامن: اختبار الفرضيات لعينتين

الفصل التاسع: الارتباط والانحدار

الفصل العاشر: اختبارات كاي تربيع وتوزيع F

Descriptive Statistics

- | | |
|---|-------------------------------------|
| 1- Frequency Distributions and Their Graphs | التوزيعات التكرارية وتمثيلها بيانيا |
| 2- More Graphs and Displays | بعض نماذج التمثيل البياني |
| 3- Measures of Central Tendency | مقاييس النزعة المركزية |
| 4- Measures of Variation | مقاييس التشتت |
| 5- Measures of Position | مقاييس الموقع |

Chap **Frequency Distributions and their Graphs**

1- Frequency Distributions

التوزيعات التكرارية

2- Graphs of Frequency Distributions

تمثيل التوزيعات التكرارية بيانيا

Chap Frequency Distributions and their Graphs

A **frequency distribution** is a table that shows **classes** or **intervals** of data entries with a count of the number of entries in each class. The **frequency** f of a class is the number of data entries in the class.

To Draw the frequency table, determine the three items:

- 1- number of classes
- 2- Range
- 3- Class Width

التوزيع التكراري هو جدول يوضح فئات أو فترات إدخلات للفئة f البيانات مع حساب عدد الإدخالات في كل فئة. التردد هو عدد إدخلات البيانات في الفئة.

لرسم الجدول التكراري حدد العناصر الثلاثة:

- 1- عدد الفئات
- 2- المدى
- 3- عرض الفئة

Example

The following sample data set lists the prices (in dollars) of 30 portable global positioning system (GPS) navigators. Construct a frequency distribution that has seven classes.

**Construct a Frequency
Distribution from a Data Set**

90	130	200	400	350	70	325	250	150	250
275	270	150	130	59	200	160	450	300	130
220	100	200	400	200	250	95	180	170	150

تسرد مجموعة البيانات النموذجية
التالية الأسعار (بالدولار) لـ 30
ملاحًا محمولًا يعمل بنظام تحديد
أنشئ (GPS). المواقع العالمي)
توزيعًا تكراريًا يحتوي على سبع
فئات.

بناء التوزيع التكراري من
مجموعة البيانات

Solution of Example

Number of classes = 7

Range = Max – Min = 450 - 59 = 391

$$\text{Class Width} = \frac{\text{Range}}{\text{Number of classes}} = \frac{391}{7} = 55.86 \cong 56$$

90	130	200	400	350	70	325	250	150	250
275	270	150	130	59	200	160	450	300	130
220	100	200	400	200	250	95	180	170	150

Frequency Distributions and their Graphs

Frequency Distribution for
Prices (in dollars) of GPS Navigators

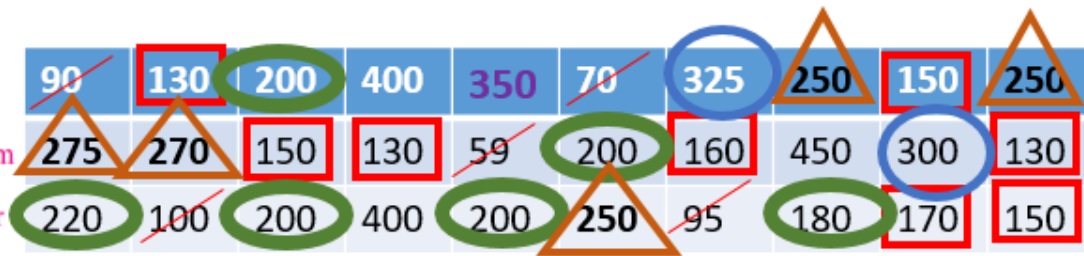
Prices

Class	Tally	Frequency, f
59–114		5
115–170		8
171–226		6
227–282		5
283–338		2
339–394		1
395–450		3
		$\Sigma f = 30$

Number of
GPS navigators

Check that the sum
of the frequencies
equals the number
in the sample.

Class Width = 56



Cumulative Frequency Distributions

1- The **midpoint** of a class = $\frac{\text{Lower class limit} + \text{Upper class Limit}}{2}$

نقطة المنتصف

2- The **relative frequency** of a class = $\frac{\text{Class Frequency}}{\text{Sample Size}} = \frac{f}{n}$

التكرار النسبي

3- The **cumulative frequency** of a class is the sum of the frequencies of that class and all previous classes. The cumulative frequency of the last class is equal to the sample size n .

التكرار التراكمي لفئة ما هو مجموع تكرارات تلك الفئة وجميع الفئات السابقة. التكرار التراكمي للفئة الأخيرة يساوي حجم العينة

Example

Chapter

The following sample data set lists the prices (in dollars) of 30 portable global positioning system (GPS) navigators. Construct a frequency distribution that has seven classes.

Find the mid point, Relative frequency and a Cumulative Frequency Distribution from a Data Set below

تسرد مجموعة البيانات النموذجية التالية الأسعار (بالدولار) لـ 30 ملاحًا محمولًا يعمل بنظام تحديد المواقع أنشئ توزيعًا تكراريًا يحتوي على سبع فئات. (GPS العالمي)

ابحث عن نقطة المنتصف والتكرار النسبي والتوزيع التكراري التراكمي من مجموعة البيانات أدناه

90	130	200	400	350	70	325	250	150	250
275	270	150	130	59	200	160	450	300	130
220	100	200	400	200	250	95	180	170	150

Solution of Example

Chapter

Class	Frequency f	Midpoint	Relative Frequency	Cumulative frequency
59-114	5	$\frac{59 + 114}{2} = 86.5$	$\frac{5}{30} = 0.17$	5
115-170	8	$\frac{115 + 170}{2} = 142.5$	$\frac{8}{30} = 0.27$	5+8=13
171-226	6	198.5	0.2	5+8+6=19
227-282	5	254.5	0.17	5+8+6+5=24
283-338	2	310.5	0.07	5+8+6+5+2=26
339-394	1	366.5	0.03	5+8+6+5+2+1=27
395-450	3	422.5	0.1	5+8+6+5+2+1+3=30
Total	30		1	

Graphs of Frequency Distributions

1- Frequency Histogram

الأعمدة التكرارية

2- Frequency Polygon

المضلع التكراري

3- Relative Frequency Histogram

الأعمدة التكرارية النسبية

Frequency Histogram

Frequency histogram is a bar graph that represents the frequency distribution of a data set. A histogram has the following properties.

1. The horizontal scale is quantitative and measures the data values.
2. The vertical scale measures the frequencies of the classes.
3. Consecutive bars must touch.

الأعمدة التكرارية هو رسم بياني شريطي يمثل التوزيع التكراري لمجموعة البيانات. يحتوي الرسم البياني على الخصائص التالية.

1. المقياس الأفقي كمي ويقاس قيم البيانات.
2. يقاس المقياس العمودي ترددات الفئات.
3. يجب أن تتلامس الأشرطة المتتالية.

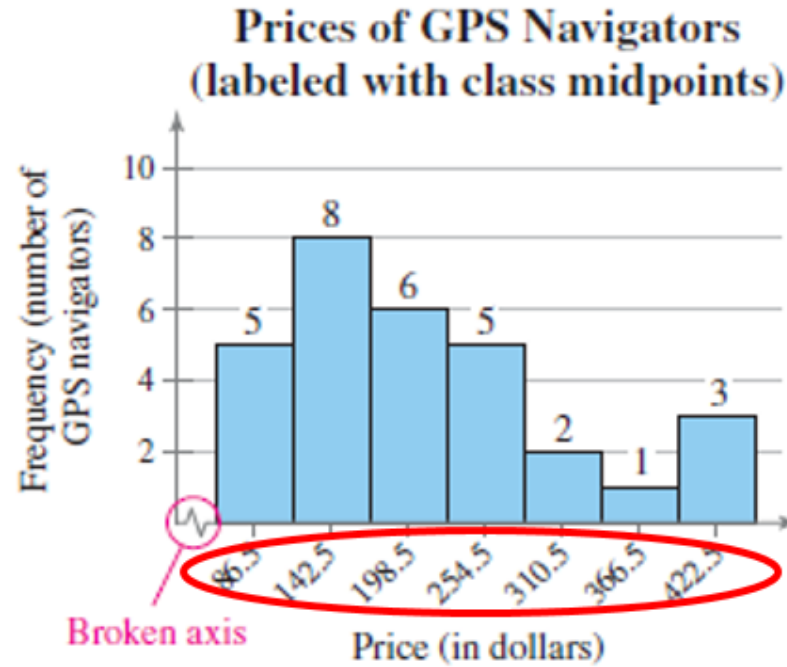
Example

Draw the frequency Histogram based on the data in the below table:

Class	Frequency <i>f</i>	Midpoint
59-114	5	86.5
115-170	8	142.5
171-226	6	198.5
227-282	5	254.5
283-338	2	310.5
339-394	1	366.5
395-450	3	422.5
Total	30	

Chapter

Frequency Histogram



Frequency Polygon

A **frequency polygon** is a line graph that emphasizes the continuous change in frequencies.

المضلع التكراري هو رسم بياني خطي يؤكد على التغيير المستمر في التكرارات.

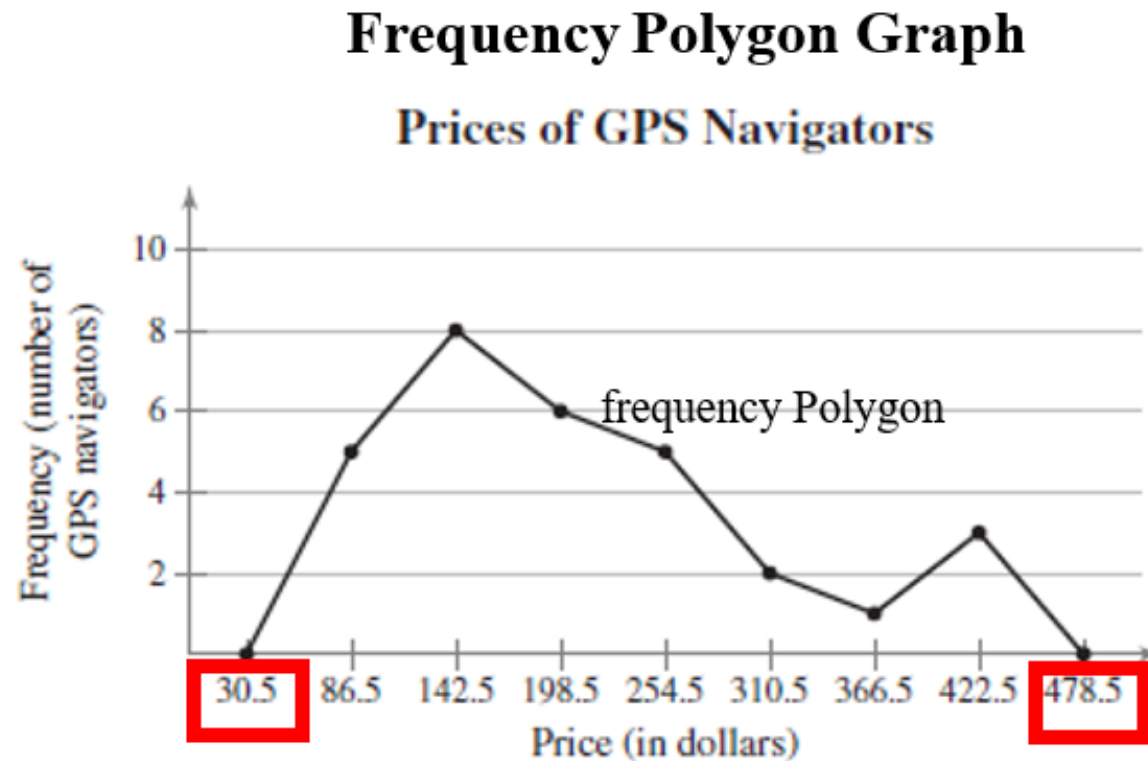
Example

Draw the frequency Polygon based on the data in this table:

Class	Frequency f	Midpoint
59-114	5	86.5
115-170	8	142.5
171-226	6	198.5
227-282	5	254.5
283-338	2	310.5
339-394	1	366.5
395-450	3	422.5
Total	30	

An increase of midpoints by 56 and the same for other points

Solution of Example



Midpoint

86.5

142.5

198.5

254.5

310.5

366.5

422.5

Relative Frequency Histogram

A **relative frequency histogram** has the same shape and the same horizontal scale as the corresponding frequency histogram. The difference is that the vertical scale measures the *relative* frequencies, not frequencies.

الأعمدة البيانية التكرارية النسبية له نفس الشكل ونفس المقياس الأفقي مثل الرسم البياني للتردد المقابل. والفرق هو أن المقياس الرأسي يقيس التكرارات النسبية وليس التكرارات.

Example

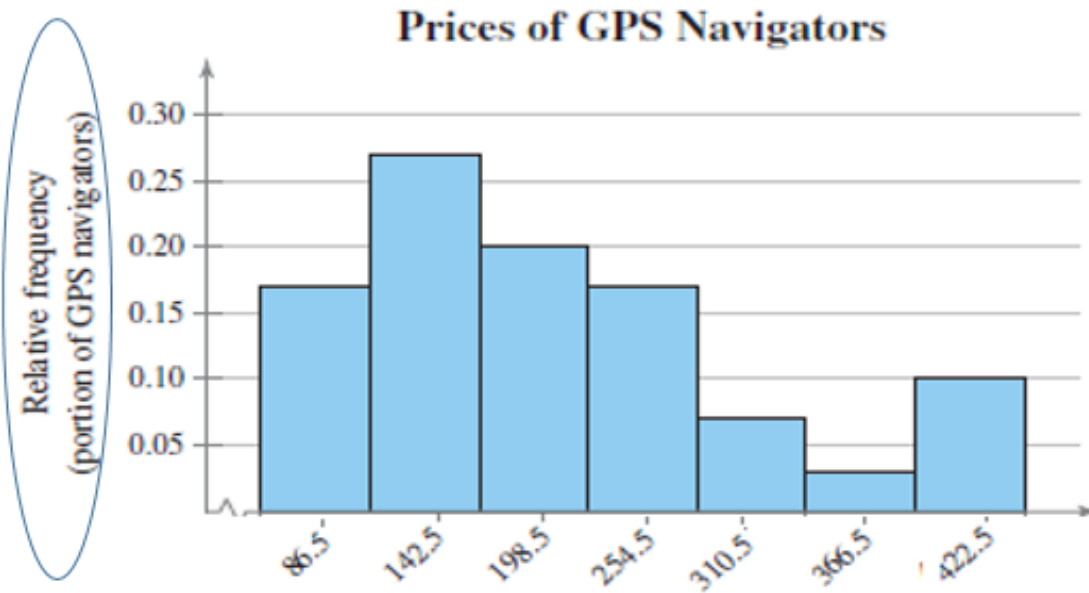
Chapter

Draw the relative frequency Histogram based on the data in this table:

Class	Frequency f	Midpoint	Relative Frequency
59-114	5	86.5	0.17
115-170	8	142.5	0.27
171-226	6	198.5	0.2
227-282	5	254.5	0.17
283-338	2	310.5	0.07
339-394	1	366.5	0.03
395-450	3	422.5	0.1
Total	30		1

Solution of Example

Relative frequency Histogram Graph



Frequency Distributions and their Graphs

- 1- Frequency Distributions and Their Graphs التوزيعات التكرارية وتمثيلها بيانيا
- 2- More Graphs and Displays بعض نماذج التمثيل البياني
- 3- Measures of Central Tendency مقاييس النزعة المركزية
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- 5- Measures of Position مقاييس الموقع

Measures of Central Tendency

We have three measures of Central Tendency: مقياس النزعة المركزية

1- Mean المتوسط الحسابي

2- Median الوسيط

3- Mode المنوال

Important Notes

- Mean is not preferred to use as a central measure for the data contained an outlier.
- An **outlier** is a data entry that is far removed from the other entries in the data set.
- **Sample Mean** = $\bar{X} = \frac{\sum X}{n}$
- **Population Mean** = $\mu = \frac{\sum X}{N}$

Example

Given the following Macroeconomics exam grades for a sample of 6 students:

81	75	66	16	90	90
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Calculate the central tendency measures and state the best central measure for this data.

Solution of Example

1- Mean:

$$\text{Sample Mean} = \bar{X} = \frac{\sum X}{n} = \frac{81+75+66+16+90+90}{6} = 69.67$$

The mean mark is equal to 69.67 which represents the value at the Centre of the data where the majority of observations are around.

المتوسط الحسابي يساوي 69.67 والتي تمثل القيمة في مركز البيانات حيث توجد غالبية الملاحظات.

Solution of Example

2- Median:

- put data in order: (smallest Largest)

16, 66, 75, 81, 90, 90

- **The value of the median is the value between 75 and 81. So,**

$$\text{value of median: } \frac{75+81}{2} = 78$$

Solution of Example

3- Mode:

The most repeated number in the data. So,

Mode = 90

Example

Given the following Macroeconomics exam grades for a sample of 7 students:

5 6 9 10 14 11 12

Calculate the central tendency measures and state the best central measure for this data.

Solution of Example

1- Mean:

$$\text{Sample Mean} = \bar{X} = \frac{\sum X}{n} = \frac{5+6+9+10+14+11+12}{7} = 9.57$$

The mean mark is equal to 9.57 which represents the value at the Centre of the data where the majority of observations are around.

Solution of Example

2- Median:

- put data in order: (smallest Largest)

5, 6, 9, 10, 11, 12, 14

- **The value of the median is the value 10**

Solution of Example

3- Mode: 5 6 9 10 14 11 12

The most repeated number in the data. So,

The data has **No mode عديمة المنوال**

Example

Find Mean – Median – Mode for the following data

أوجد المتوسط الحسابي – الوسيط – المنوال للبيانات التالية

18 29 43 24 31 9 12 19 16 14

Solution of Example

Mean المتوسط الحسابي

$$= \frac{18 + 29 + 43 + 24 + 31 + 9 + 12 + 19 + 16 + 14}{10} = \frac{215}{10} = 21.5$$

Solution of Example

Median الوسيط

9 12 14 16 18 19 24 29 31 43

$$\frac{18+19}{2} = 18.5$$

Solution of Example

Mode المنوال

The data has **No mode** عديمة المنوال

Example

Find Mean – Median – Mode for the following data

أوجد المتوسط الحسابي – الوسيط – المنوال للبيانات التالية

14 16 9 8 12 15 20 9 10 24 22 10 10

Solution of Example

Mean المتوسط الحسابي

14 16 9 8 12 15 20 9 10 24 22 10 10

$$= \frac{14+16+9+8+12+15+20+9+10+24+22+10+10}{13} = \frac{179}{13} = 13.769 \approx 13.77$$

Solution of Example

Median الوسيط

14 16 9 8 12 15 20 9 10 24 22 10 10

8 9 9 10 10 10 12 14 15 16 20 22 24

Median = 12

Solution of Example

Mode المنوال

8 9 9 10 10 10 12 14 15 16 20 22 24

Mode = 10

Example

In the following three groups, it is required to calculate the mean, median, and mode:

Group 1: 2 – 4 – 4 – 6

Group 2: -2 – 4 – 4 - 10

Group 3: -70 – 4 – 4 - 78

Solution of Example

Group	Mean	Median	Mode
1	4	4	4
2	4	4	4
3	4	4	4

Measures of Variation

We have three measures of Variations:

1- Range

المدى

2- Variance

التباين

3- Standard deviation

الانحراف المعياري

Chapter 10 Important Notes

- **Population Variance** = $\sigma^2 = \frac{\sum(X-\mu)^2}{N}$
- **Sample Variance** = $S^2 = \frac{\sum(X-\bar{X})^2}{n-1}$
- **Population Standard deviation** = $\sigma = \sqrt{\frac{\sum(X-\mu)^2}{N}}$
- **Sample Standard deviation** = $S = \sqrt{\frac{\sum(X-\bar{X})^2}{n-1}}$

Example

Given the following Mathematics exam grades for a sample of 5 students.

8 12 20 7 3

Calculate the dispersion measures for the above data

Solution of Example

1- Range = Max – Min = 20- 3 = 17

2- Sample Variance = $S^2 = \frac{\sum(X-\bar{X})^2}{n-1}$

X	$\bar{X}$	$(X - \bar{X})$	$(X - \bar{X})^2$
8	10	-2	4
12	10	2	4
20	10	10	100
7	10	-3	9
3	10	-7	49
Σ		zero	166

Solution of Example

1- Range = Max – Min = 20- 3 = 17

2- Sample Variance = $S^2 = \frac{\sum(X-\bar{X})^2}{n-1}$

X	$\bar{X}$	$(X - \bar{X})$	$(X - \bar{X})^2$
8	10	-2	4
12	10	2	4
20	10	10	100
7	10	-3	9
3	10	-7	49
Σ		zero	166

3- Sample Standard deviation = $S = \sqrt{\frac{\sum(X-\bar{X})^2}{n-1}} = \sqrt{\frac{166}{4}} = 6.44$

Example

Given the following Macroeconomics exam grades for a sample of 5 students.

81

75

66

84

96

Calculate the dispersion measures for the above data

Solution of Example

1- Range = Max – Min = 96- 66 = 30

2- Sample Variance = $S^2 = \frac{\sum(X-\bar{X})^2}{n-1}$

x	$\bar{x}$	$(x-\bar{x})$	$(x-\bar{x})^2$
81	80.4	0.6	0.36
75	80.4	-5.4	29.16
66	80.4	-14.4	207.36
84	80.4	3.6	12.96
96	80.4	15.6	243.36
Σ		Zero	493.2

Solution of Example

1- Range = Max – Min = 96- 66 = 30

2- Sample Variance = $S^2 = \frac{\sum(X-\bar{X})^2}{n-1} = \frac{493.2}{5-1} = 123.3$

x	$\bar{x}$	$(x-\bar{x})$	$(x-\bar{x})^2$
81	80.4	0.6	0.36
75	80.4	-5.4	29.16
66	80.4	-14.4	207.36
84	80.4	3.6	12.96
96	80.4	15.6	243.36
Σ		Zero	493.2

3- Sample Standard deviation = $S = \sqrt{\frac{\sum(X-\bar{X})^2}{n-1}} = \sqrt{123.3} = 11.1$

Example

Find Mean – Median – Mode and Range for the following data

أوجد المتوسط الحسابي – الوسيط – المنوال والمدى للبيانات التالية

14 16 9 8 12 15 20 9 10 24 22 10 10 9 13

Solution of Example

Mean المتوسط الحسابي

14 16 9 8 12 15 20 9 10 24 22 10 10 9 13

$$= \frac{14+16+9+8+12+15+20+9+10+24+22+10+10+9+13}{15} = \frac{201}{15} = 13.4$$

Solution of Example

Median الوسيط

14 16 9 8 12 15 20 9 10 24 22 10 10 9 13

8 9 9 9 10 10 10 12 13 14 15 16 20 22 24

Median = 12

Solution of Example

Mode المنوال

8 9 9 9 10 10 10 12 13 14 15 16 20 22 24

Mode = 9 ,10

Solution of Example

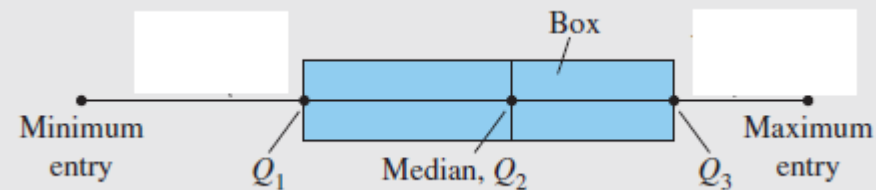
Range **المدى**

8 9 9 9 10 10 10 12 13 14 15 16 20 22 24

$$\text{Range} = 24 - 8 = 16$$

Measures of Position

- Q1 is the first Quartile الربع الأول
- Q2 is the Second Quartile (Median) الربع الثاني (الوسيط)
- Q3 is the third Quartile الربع الثالث
- Interquartile range $IQR = Q3 - Q1$ المدى الربيعي
- Box Plot

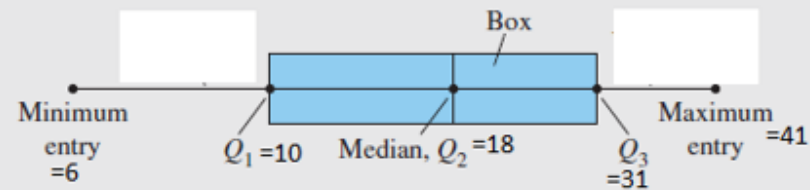


Example

The number of nuclear power plants in the top 15 nuclear power-producing countries in the world are listed.

7 18 11 6 59 17 18 54 104 20 31 8 10 15 19

- Find the first, second, and third quartiles of the data set.
- Calculate IQR
- Draw the Box plot



Example

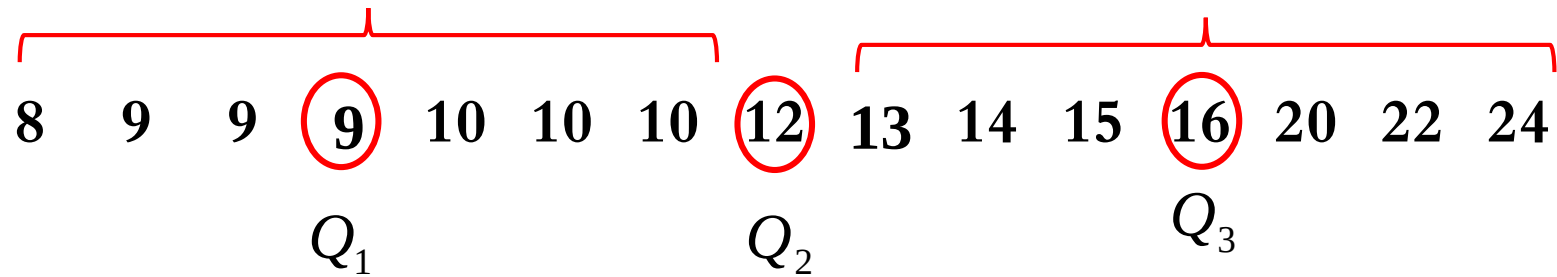
For the following data.

14 16 9 8 12 15 20 9 10 24 22 10 10 9 13

- Find the first, second, and third quartiles of the data set.
- Calculate IQR

Solution of Example

- Put data in order: (smallest Largest)



$$\text{IQR} = Q_3 - Q_1 = 16 - 9 = 7$$

Chapter Summary

Frequency Distributions

Measures of Central Tendency:

- 1- Mean
- 2- Median
- 3- Mode

Measures of Position:

- 1- First Quartile
- 2- Second Quartile
- 3- Third Quartile
- 4- IQR
- 5- Box Plot

Graphs and Displays:

- 1- Frequency Histogram
- 2- Frequency Polygon
- 3- Relative Frequency Histogram

Measures of Variation:

- 1- Range
- 2- Variance
- 3- Standard deviation

Un Solved Examples

Chapter

Given the following data

11 25 30 24 22 10 12

Find Mean – Median – Mode - Range

Un Solved Examples

Chapter

The value of Q1 for the values [12,10,33,49,18,90,100]

Un Solved Examples

Chapter

The value of Q3 for the values [12,10,33,49,18,90,100]

Un Solved Examples

Chapter

The value of Q2 for the values [12,10,33,49,18,90,100]

Un Solved Examples

Chapter

The value of IQR for the values [12,10,33,49,18,90,100]



Thank You