

Mathematics 3 – MAT132

Dr. Israa Lewaa

Lecture 8 – Calculus Applications in real life



مجموعة جامعة
الجميع الذكية
للتعليم القابضة

Lecture Objective

- 1- Show several uses of Calculus in Real Life
- 2- Explain Different Applications using Differential
- 3- Explain Different Applications using Integration
- 4- Obtain Marginal cost
- 5- Obtain Marginal Revenue
- 6- Obtain Demand Function

Lecture Outline

- 1- Uses of Calculus in Real Life
- 2- Different Applications using Differential
- 3- Different Applications using Integration
- 4- Marginal cost
- 5- Marginal Revenue
- 6- Demand Function

Uses of Calculus in Real Life

Process of differentiation and Integration

Calculus focuses on the processes of differentiation and integration

However, many are uncertain what calculus is used for in real life. Fortunately for those toiling away with their textbooks, calculus has a variety of important practical uses in fields.

Various Fields of using Calculus

- 1. Uses of Calculus in Economics**
- 2. Uses of Calculus in Biology**
- 3. Uses of Calculus in Engineering**
- 4. Uses of Calculus in Architect**
- 5. Uses of Calculus in Finance**

Uses of Calculus in Economics

- # Economists use calculus to determine the right time to buy or sell something
- # The marginal value of a particular action
- # How much the price of an item affects how many people buy it
Or any other instance that requires measuring change over time in two or more related *variables*

Uses of Calculus in Biology

Uses of calculus in Biology

- # Some applications include modeling enzyme reactions as a function of nutrient concentration and the growth rate for bacteria
- # Modeling population growth of a certain species
- # And modeling the relationship between related physical traits, such as backbone length and skull length

Uses of Calculus in Engineering

Uses of calculus in Engineering

- # Engineering is the field in which calculus is used most often in a real-world context
- # Calculus is necessary to determine distances with precision, such as the length of a cable supported by two poles
- # Calculus is also required by engineers and architects to calculate the size of curved or dome-style surfaces

Uses of Calculus in Architect

Uses of calculus in Architect

An architect will use integration to determine the amount of materials necessary to construct a curved dome over a new sports arena, as well as calculate the weight of that dome and determine the type of support structure required.

Uses of Calculus in Finance

Uses of calculus in Finance

The minimum monthly payment on your credit card statement may vary from month to month depending on how much you owe, but it is not a number the credit card company arbitrarily decided. They use calculus to determine a minimum payment at the exact time your bill is due based on fluctuation of interest rates and changes in your available balance.

Applications for Calculus

Applications for Calculus

Applications for Calculus

- ☐ Marginal cost
- ☐ Marginal Revenue
- ☐ Demand Function

Extra Practice – Example 1

1) If $c = 500 + 10q + q^2$

- Find the marginal cost when $q = 100$

Extra Practice – Example 1 Solution

1) If $c = 500 + 10q + q^2$

- Find the marginal cost when $q = 100$

- **Solution**

First find $\frac{dc}{dq} = 10 + 2q$

$$\therefore \frac{dc}{dq} \text{ (at } q = 100) = 10 + 2(100) = 210$$

Extra Practice – Example 2

2) If $c = 700q + 100$

- Find the marginal cost at $q = 70$?

Extra Practice – Example 2 Solution

2) If $c = 700q + 100$

- Find the marginal cost at $q = 70$?
- **Solution**

$$\frac{dc}{dq} = 700$$

$$\therefore \frac{dc}{dq} = (\text{at } q = 70) = 700$$

Extra Practice – Example 3

3) If $R = 240q + 40q^2 - 2q^3$

- Find the marginal Revenue at $q = 20$

Extra Practice – Example 3 solution

3) If $R = 240q + 40q^2 - 2q^3$

- Find the marginal Revenue at $q = 20$
- **Solution**

$$\frac{dR}{dq} = 240 + 80q - 6q^2$$

$$\therefore \frac{dR}{dq} \text{ (at } q = 20) = 240 + 80(20) - 6(20)^2 = -560$$

Extra Practice – Example 4

What is the marginal cost at the given value(s) of q ?

$$c = 5000 + 6q$$

$$\text{at } q = 36$$

Extra Practice – Example 4 solution

$$c = 5000 + 6q$$

$$\text{at } q = 36$$

$$\frac{dc}{dq} = 6$$

$$\therefore \frac{dc}{dq} = 6 \text{ (constant) at } q = 36$$

Extra Practice – Example 5

What is the marginal cost at the given value(s) of q ?

$$c = 0.1 q^2 + 3q + 2 \quad \text{at } q = 3$$

Extra Practice – Example 5 Solution

$$c = 0.1 q^2 + 3q + 2 \quad \text{at } q = 3$$

$$\frac{dc}{dq} = 0.2q + 3$$

$$\therefore \frac{dc}{dq} = 0.2(3) + 3 = 3.6 \text{ at } q = 3$$

Extra Practice – Example 6

What is the marginal cost at the given value(s) of q ?

$$c = 0.04q^3 - 0.5q^2 + 4.4q + 7500 \text{ at } q=5, q=25, q=1000$$

Extra Practice – Example 6 Solution

$$c = 0.04q^3 - 0.5q^2 + 4.4q + 7500 \text{ at } q=5, q=25, q=1000$$

$$\frac{dc}{dq} = 0.12q^2 - q + 4.4$$

$$\therefore \frac{dc}{dq} = 2.4 \text{ at } q = 5$$

$$\therefore \frac{dc}{dq} = 54.4 \text{ at } q = 25$$

$$\therefore \frac{dc}{dq} = 119004.4 \text{ at } q = 1000$$

Extra Practice – Example 7

What is the marginal Revenue at the given value(s) of q ?

$$R = 0.8 q$$

$$\text{at } q = 9$$

Extra Practice – Example 7 Solution

$$R = 0.8 q$$

$$\text{at } q = 9$$

$$\frac{dR}{dq} = 0.8 \text{ at } q = 9$$

Extra Practice – Example 8

What is the marginal Revenue at the given value(s) of q ?

$$R = q \left(15 - \frac{1}{30} q \right) \quad \text{at } q = 5, q = 15$$

Extra Practice – Example 8 Solution

$$R = q \left(15 - \frac{1}{30}q \right)$$

at $q = 5, q = 15$

$$R = 15q - \frac{1}{30}q^2$$

$$\frac{dr}{dq} = 15 - \frac{1}{15}q$$

$$\therefore \frac{dr}{dq} = 14.667 \text{ at } q = 5$$

$$\therefore \frac{dr}{dq} = 14 \text{ at } q = 15$$

Extra Practice – Example 9

What is the marginal Revenue at the given value(s) of q ?

$$R = 2q(30 - 0.1q) \quad \text{at } q=10, q=20$$

Extra Practice – Example 9 Solution

$$R = 2q(30 - 0.1q)$$

$$\text{at } q = 10, q = 20$$

$$R = 60q - 0.2q^2$$

$$\frac{dr}{dq} = 60 - 0.4q$$

$$\therefore \frac{dr}{dq} = 60 - 0.4(10) = 56 \text{ at } q = 10$$

$$\therefore \frac{dr}{dq} = 52 \text{ at } q = 20$$

Extra Practice – Example 10

Find the demand function:

$$\frac{dR}{dq} = 0.7$$

Extra Practice – Example 10 Solution

$$\frac{dR}{dq} = 0.7$$

$$r = \int 0.7 dq = 0.7q + C$$

$$r = 0.7q.$$

$$p = \frac{r}{q} = \frac{0.7q}{q} = 0.7$$

The demand function is $p = 0.7$.

Extra Practice – Example 11

Find the demand function:

$$\frac{dR}{dq} = 10 - \frac{1}{16}q$$

Extra Practice – Example 11 Solution

$$\frac{dR}{dq} = 10 - \frac{1}{16}q$$

$$r = \int \left[10 - \frac{1}{16}q \right] dq = 10q - \frac{1}{32}q^2 + C$$

$$r = 10q - \frac{1}{32}q^2$$

$$p = \frac{r}{q} = 10 - \frac{1}{32}q .$$

The demand function is $p = 10 - \frac{1}{32}q$

Extra Practice – Example 12

Find the demand function:

$$\frac{dR}{dq} = 275 - q - 0.3q^2$$

Extra Practice – Example 12 Solution

$$\frac{dR}{dq} = 275 - q - 0.3q^2$$

$$\text{Thus } r = \int (275 - q - 0.3q^2) dq$$

$$r = 275q - 0.5q^2 - 0.1q^3 + C$$

$$r = 275q - 0.5q^2 - 0.1q^3$$

$$p = \frac{r}{q} = 275 - 0.5q - 0.1q^2$$

Thus the demand function is

$$p = 275 - 0.5q - 0.1q^2.$$

Extra Practice – Example 13

Find the demand function:

$$\frac{dR}{dq} = 5000 - 3(2q + 2q^3)$$

Extra Practice – Example 13 Solution

$$\frac{dR}{dq} = 5000 - 3(2q + 2q^3)$$

$$r = \int (5000 - 6q - 6q^3) dq$$

$$r = 5000q - 3q^2 - \frac{3q^4}{2} + C$$

$$r = 5000q - 3q^2 - \frac{3q^4}{2}$$

$$p = \frac{r}{q} = 5000 - 3q - \frac{3q^3}{2}.$$

Therefore the demand function is

$$p = 5000 - 3q - \frac{3q^3}{2}.$$

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