

IT111 - Introduction to Information Technology Bachelor of Science in Information Technology

LECTURE 2 NUMBER SYSTEMS

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After studying this chapter, the student should be able to:

- ☐ Understand the concept of number systems.
- ☐ Distinguish between non-positional and positional number systems.
- ☐ Describe the decimal, binary, hexadecimal and octal system.
- ☐ Convert a number in binary, octal or hexadecimal to a number in the decimal system.
- ☐ Convert a number in the decimal system to a number in binary, octal and hexadecimal.
- ☐ Convert a number in binary to octal and vice versa.
- ☐ Convert a number in binary to hexadecimal and vice versa.
- ☐ Find the number of digits needed in each system to represent a particular value.

2- INTRODUCTION

A **number system** defines how a number can be represented using distinct symbols. A number can be represented differently in different systems. For example, the two numbers $(2A)_{16}$ and $(52)_8$ both refer to the same quantity, $(42)_{10}$, but their representations are different.

Several number systems have been used in the past and can be categorized into two groups: **positional** and **non-positional** systems. Our main goal is to discuss the positional number systems, but we also give examples of non-positional systems.

2-2 POSITIONAL NUMBER SYSTEMS

In a **positional number system**, the position a symbol occupies in the number determines the value it represents. In this system, a number represented as:

$$\pm (S_{k-1} \dots S_2 S_1 S_0 . S_{-1} S_{-2} \dots S_{-l})_b$$

has the value of:

$$n = \pm S_{k-1} \times b^{k-1} + \dots + S_1 \times b^1 + S_0 \times b^0 + S_{-1} \times b^{-1} + S_{-2} \times b^{-2} + \dots + S_{-l} \times b^{-l}$$

in which S is the set of symbols, b is the **base** (or **radix**).

The decimal system (base 10)

The word decimal is derived from the Latin root **decemal** (ten). In this system the **base $b = 10$** and we use ten symbols

$$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

The symbols in this system are often referred to as **decimal digits** or just **digits**.

Integers

$$N = \pm S_{k-1} \times 10^{k-1} + S_{k-2} \times 10^{k-2} + \dots + S_2 \times 10^2 + S_1 \times 10^1 + S_0 \times 10^0$$

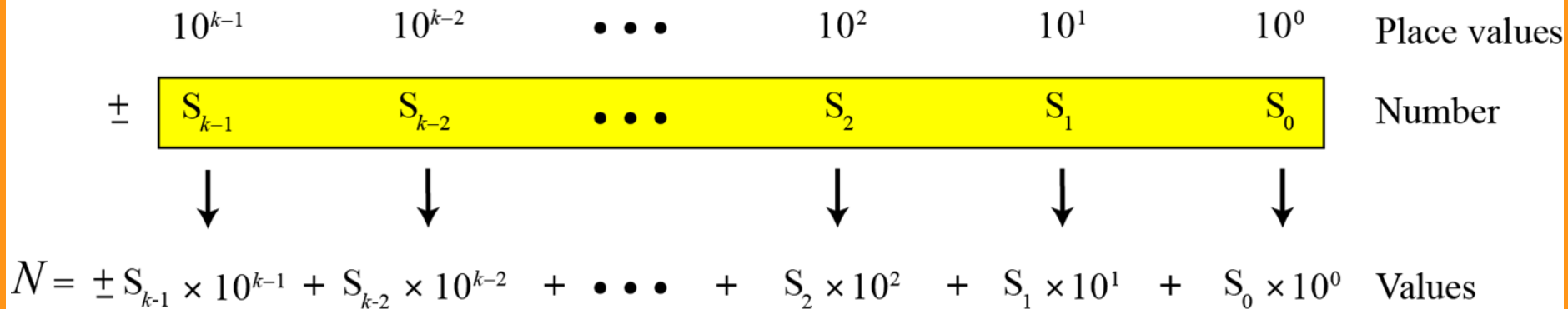


Figure 2.1 Place values for an integer in the decimal system

Example 2.1

The following shows the place values for the integer +224 in the decimal system.

	10^2	10^1	10^0	Place values
	2	2	4	Number
$N = +$	2×10^2	2×10^1	4×10^0	Values

Note that the digit 2 in position 1 has the value 20, but the same digit in position 2 has the value 200. Also note that we normally drop the plus sign, but it is implicit.

Example 2.2

The following shows the place values for the decimal number -7508 . We have used 1, 10, 100, and 1000 instead of powers of 10.

	1000	100	10	1	Place values
	7	5	0	8	Number
$N = -$	7×1000	$+$	5×100	$+$	0×10
				$+$	8×1
					) Values

Reals

Integral part

Fractional part

$$R = \pm \left(S_{k-1} \times 10^{k-1} + \dots + S_1 \times 10^1 + S_0 \times 10^0 \right) + \left(S_{-1} \times 10^{-1} + \dots + S_{-l} \times 10^{-l} \right)$$

Example 2.3

The following shows the place values for the real number +24.13.

	10^1	10^0		10^{-1}	10^{-2}	Place values
	2	4	•	1	3	Number
$R = +$	2×10	$+ 4 \times 1$	$+$	1×0.1	$+ 3 \times 0.01$	Values

The binary system (base 2)

The word binary is derived from the Latin root **bini** (or two by two). In this system the **base b = 2** and we use only two symbols,

$$S = \{0, 1\}$$

The symbols in this system are often referred to as **binary digits** or **bits** (binary digit).

Integers

$$N = \pm S_{k-1} \times 2^{k-1} + S_{k-2} \times 2^{k-2} + \dots + S_2 \times 2^2 + S_1 \times 2^1 + S_0 \times 2^0$$

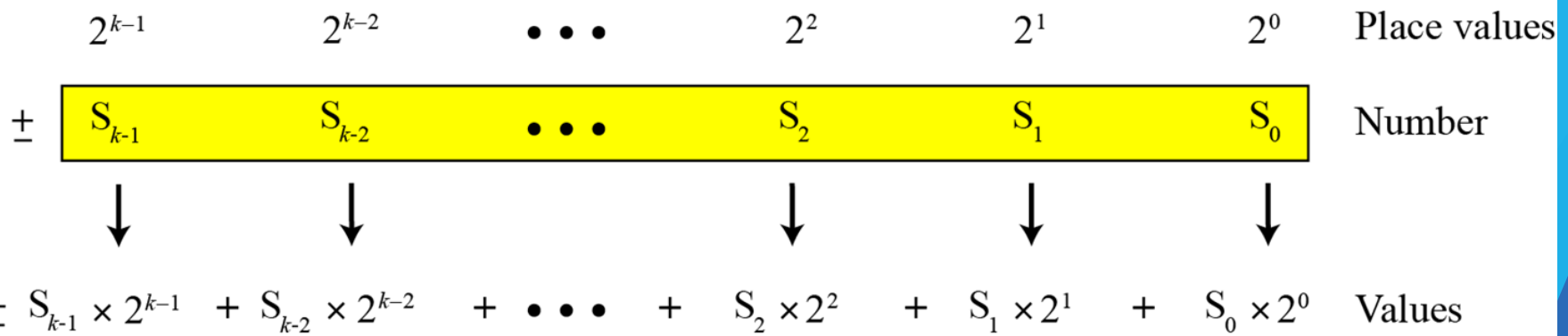


Figure 2.2 Place values for an integer in the binary system

Example 2.4

The following shows that the number $(11001)_2$ in binary is the same as 25 in decimal. The subscript 2 shows that the base is 2.

	2^4	2^3	2^2	2^1	2^0	Place values
	1	1	0	0	1	Number
$N =$	1×2^4	$+ 1 \times 2^3$	$+ 0 \times 2^2$	$+ 0 \times 2^1$	$+ 1 \times 2^0$	Decimal

The equivalent decimal number is $N = 16 + 8 + 0 + 0 + 1 = 25$.

Reals

$$R = \pm \left(S_{k-1} \times 2^{k-1} + \dots + S_1 \times 2^1 + S_0 \times 2^0 \right) + \left(S_{-1} \times 2^{-1} + \dots + S_{-l} \times 2^{-l} \right)$$

Integral part • Fractional part

Example 2.5

The following shows that the number $(101.11)_2$ in binary is equal to the number 5.75 in decimal.

	2^2	2^1	2^0	•	2^{-1}	2^{-2}	Place values
	1	0	1		1	1	Number
R =	1×2^2	$+ 0 \times 2^1$	$+ 1 \times 2^0$	$+ 1 \times 2^{-1}$	$+ 1 \times 2^{-2}$		Values

The hexadecimal system (base 16)

The word **hexadecimal** is derived from the Greek root **hex** (six) and the Latin root **decem** (ten). In this system the **base b = 16** and we use sixteen symbols to represent a number. The set of symbols is

$$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$$

Note that the symbols A, B, C, D, E, F are equivalent to 10, 11, 12, 13, 14, and 15 respectively. The symbols in this system are often referred to as **hexadecimal digits**.

Integers

$$N = \pm S_{k-1} \times 16^{k-1} + S_{k-2} \times 16^{k-2} + \dots + S_2 \times 16^2 + S_1 \times 16^1 + S_0 \times 16^0$$

	16^{k-1}	16^{k-2}	$\dots$	16^2	16^1	16^0	Place values
$\pm$	S_{k-1}	S_{k-2}	$\dots$	S_2	S_1	S_0	Number
	↓	↓		↓	↓	↓	
$N =$	$\pm S_{k-1} \times 16^{k-1}$	$+ S_{k-2} \times 16^{k-2}$	$+ \dots$	$+ S_2 \times 16^2$	$+ S_1 \times 16^1$	$+ S_0 \times 16^0$	Values

Figure 2.3 Place values for an integer in the hexadecimal system

Example 2.6

The following shows that the number $(2AE)_{16}$ in hexadecimal is equivalent to 686 in decimal.

	16^2	16^1	16^0	Place values
	2	A	E	Number
N =	2×16^2	$+ 10 \times 16^1$	$+ 14 \times 16^0$	Values

The equivalent decimal number is $N = 512 + 160 + 14 = 686$.

The octal system (base 8)

The word octal is derived from the Latin root **octo** (eight). In this system the **base b = 8** and we use eight symbols to represent a number. The set of symbols is

$$S = \{0, 1, 2, 3, 4, 5, 6, 7\}$$

Integers

$$N = \pm S_{k-1} \times 8^{k-1} + S_{k-2} \times 8^{k-2} + \dots + S_2 \times 8^2 + S_1 \times 8^1 + S_0 \times 8^0$$

	8^{k-1}	8^{k-2}	$\dots$	8^2	8^1	8^0	Place values
$\pm$	S_{k-1}	S_{k-2}	$\dots$	S_2	S_1	S_0	Number
	↓	↓		↓	↓	↓	
$N =$	$\pm S_{k-1} \times 8^{k-1}$	$+ S_{k-2} \times 8^{k-2}$	$+ \dots$	$+ S_2 \times 8^2$	$+ S_1 \times 8^1$	$+ S_0 \times 8^0$	Values

Figure 2.3 Place values for an integer in the octal system

Example 2.7

The following shows that the number $(1256)_8$ in octal is the same as 686 in decimal.

	8^3	8^2	8^1	8^0	Place values
	1	2	5	6	Number
$N =$	1×8^3	$+$	2×8^2	$+$	5×8^1
		$+$		$+$	6×8^0
					Values

Note that the decimal number is $N = 512 + 128 + 40 + 6 = 686$.

Summary of the four positional systems

Table 2.1 shows a summary of the four positional number systems discussed in this chapter.

Table 2.1 Summary of the four positional number systems

<i>System</i>	<i>Base</i>	<i>Symbols</i>	<i>Examples</i>
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	2345.56
Binary	2	0, 1	$(1001.11)_2$
Octal	8	0, 1, 2, 3, 4, 5, 6, 7	$(156.23)_8$
Hexadecimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F	$(A2C.A1)_{16}$

Table 2.2 shows how the number 0 to 15 is represented in different systems.

Table 2.2 Comparison of numbers in the four systems

<i>Decimal</i>	<i>Binary</i>	<i>Octal</i>	<i>Hexadecimal</i>
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Conversion

We need to know how to convert a number in one system to the equivalent number in another system. Since the decimal system is more familiar than the other systems, we first show how to convert from any base to decimal. Then we show how to convert from decimal to any base. Finally, we show how we can easily convert from binary to hexadecimal or octal and vice versa.

Any base to decimal conversion

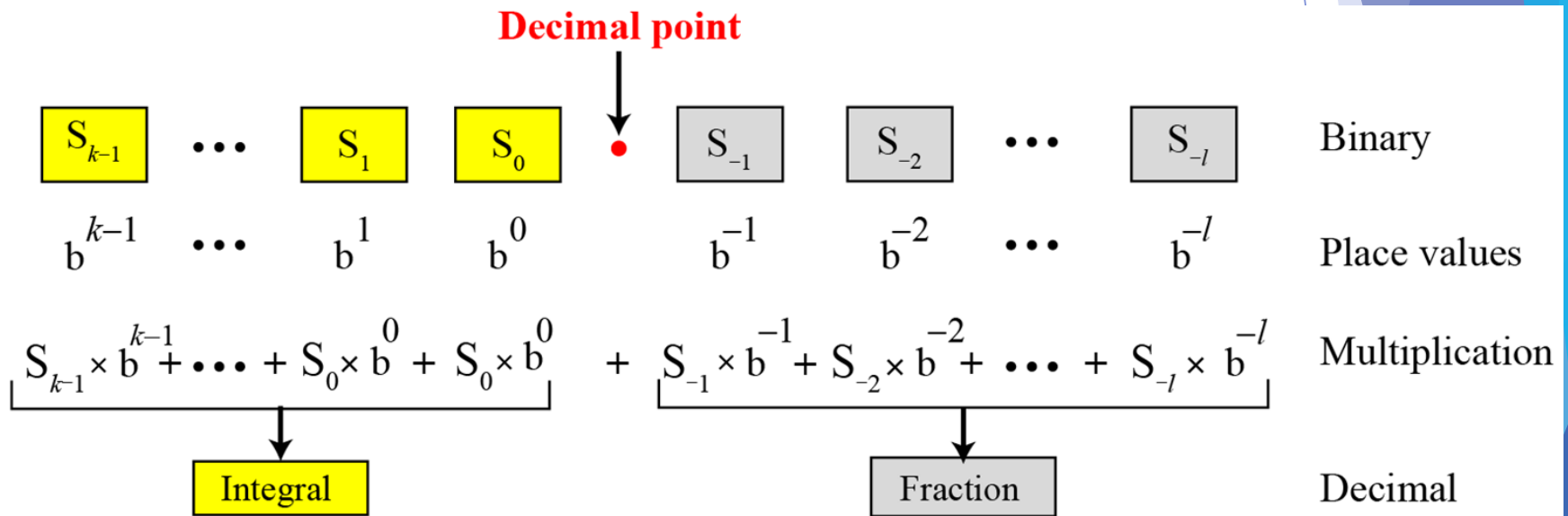


Figure 2.5 Converting other bases to decimal

Example 2.8

The following shows how to convert the binary number $(110.11)_2$ to decimal: $(110.11)_2 = 6.75$.

Binary	1	1	0	•	1	1
Place values	2^2	2^1	2^0		2^{-1}	2^{-2}
Partial results	4	2	0	+	0.5	0.25
Decimal: 6.75						

Example 2.9

The following shows how to convert the hexadecimal number $(1A.23)_{16}$ to decimal.

Hexadecimal	1	A	•	2	3
Place values	16^1	16^0		16^{-1}	16^{-2}
Partial result	16	+ 10	+ 0.125	+ 0.012	
Decimal:	26.137				

Note that the result in the decimal notation is not exact, because $3 \times 16^{-2} = 0.01171875$. We have rounded this value to three digits (0.012).

Example 2.10

The following shows how to convert $(23.17)_8$ to decimal.

Octal	2	3	•	1	7
Place values	8^1	8^0		8^{-1}	8^{-2}
Partial result	16	3	+	0.125	0.109
Decimal:	19.234				

This means that $(23.17)_8 \approx 19.234$ in decimal. Again, we have rounded up $7 \times 8^{-2} = 0.109375$.

Decimal to any base

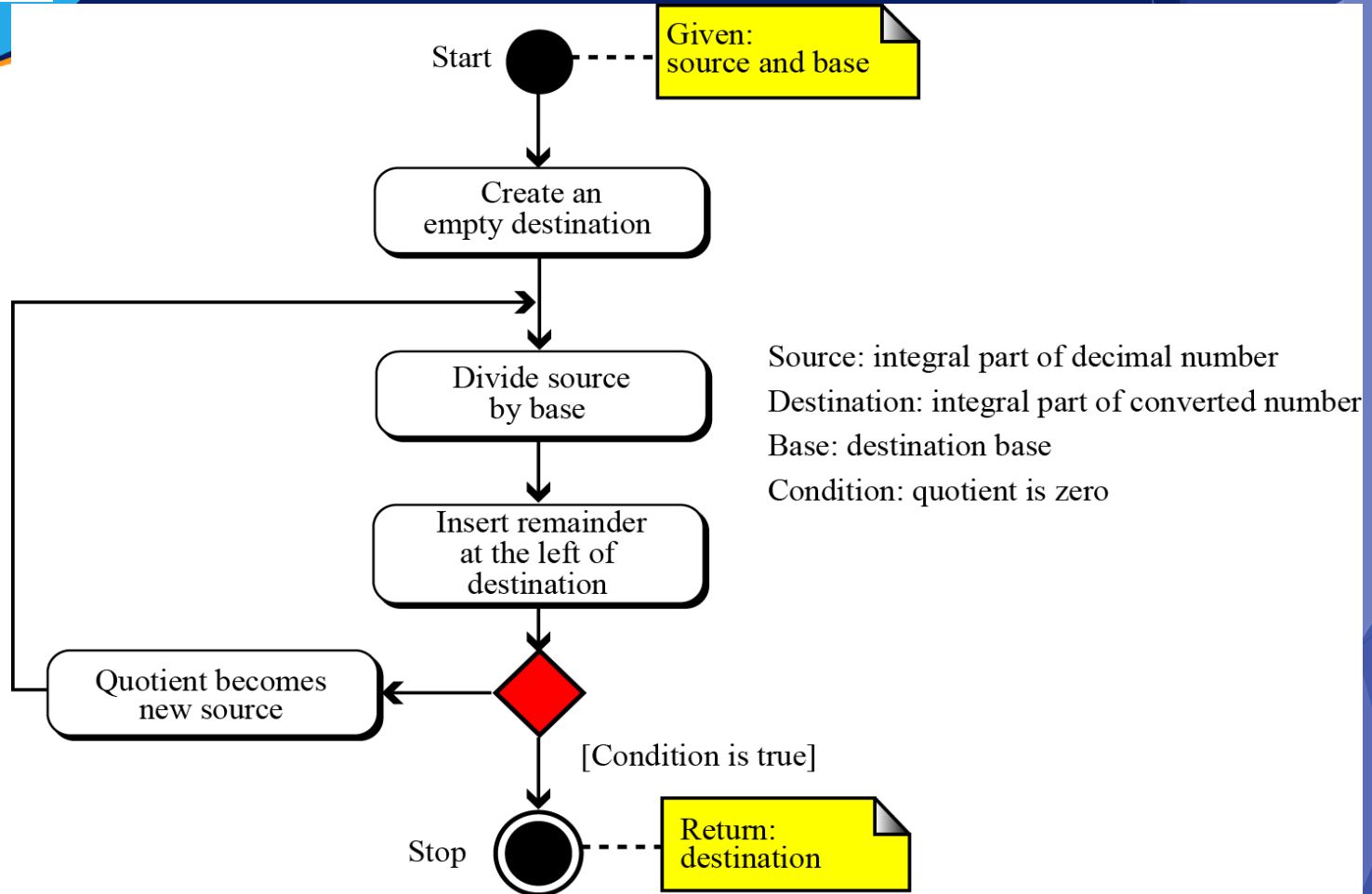
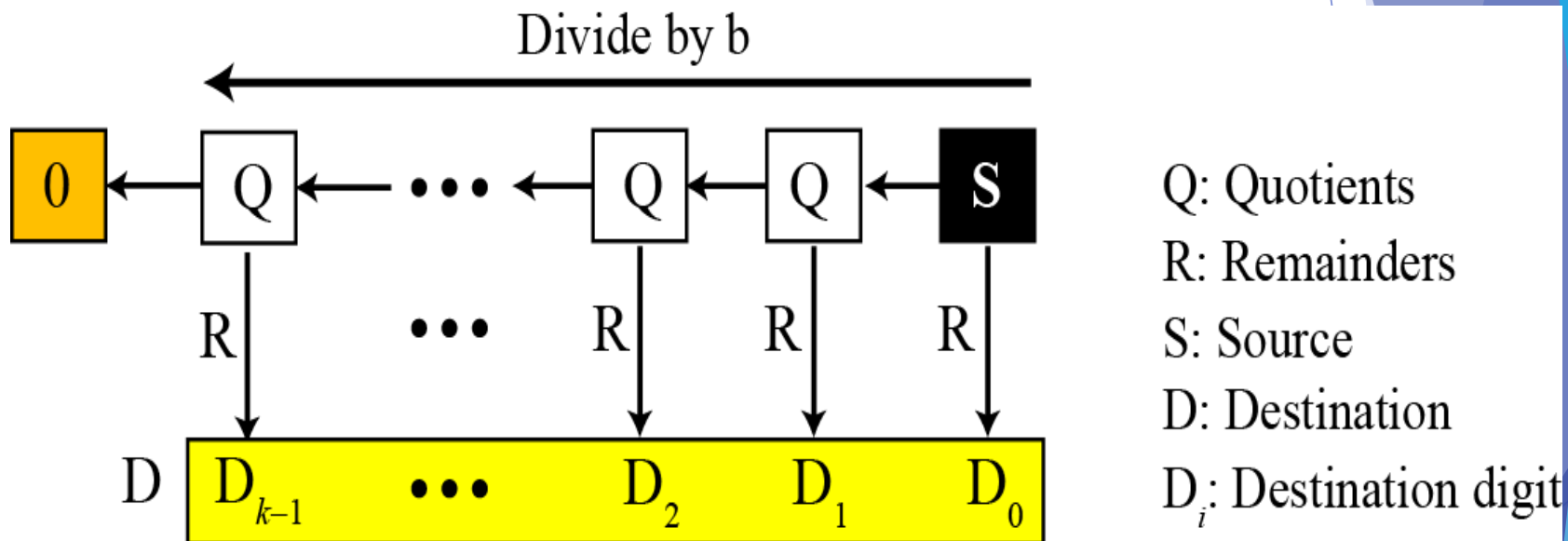
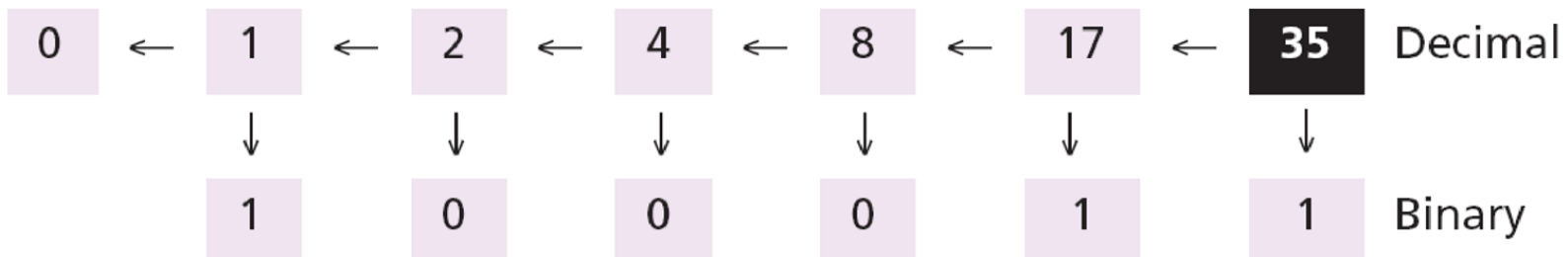


Figure 2.7 Converting the integral part of a number in decimal to other bases



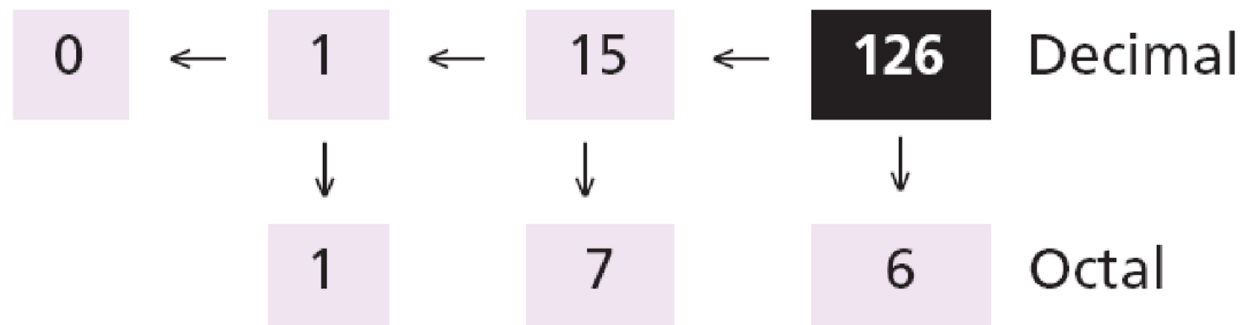
Example 2.11

The following shows how to convert 35 in decimal to binary. We start with the number in decimal, we move to the left while continuously finding the quotients and the remainder of division by 2. The result is $35 = (100011)_2$.



Example 2.12

The following shows how to convert 126 in decimal to its equivalent in the octal system. We move to the right while continuously finding the quotients and the remainder of division by 8. The result is $126 = (176)_8$.



Example 2.13

The following shows how we convert 126 in decimal to its equivalent in the hexadecimal system. We move to the right while continuously finding the quotients and the remainder of division by 16. The result is $126 = (7E)_{16}$

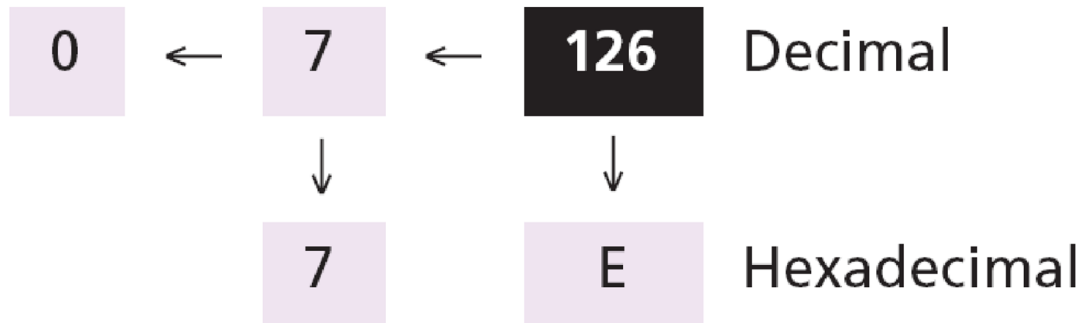


Figure 2.8 Converting the fractional part of a number in decimal to other bases

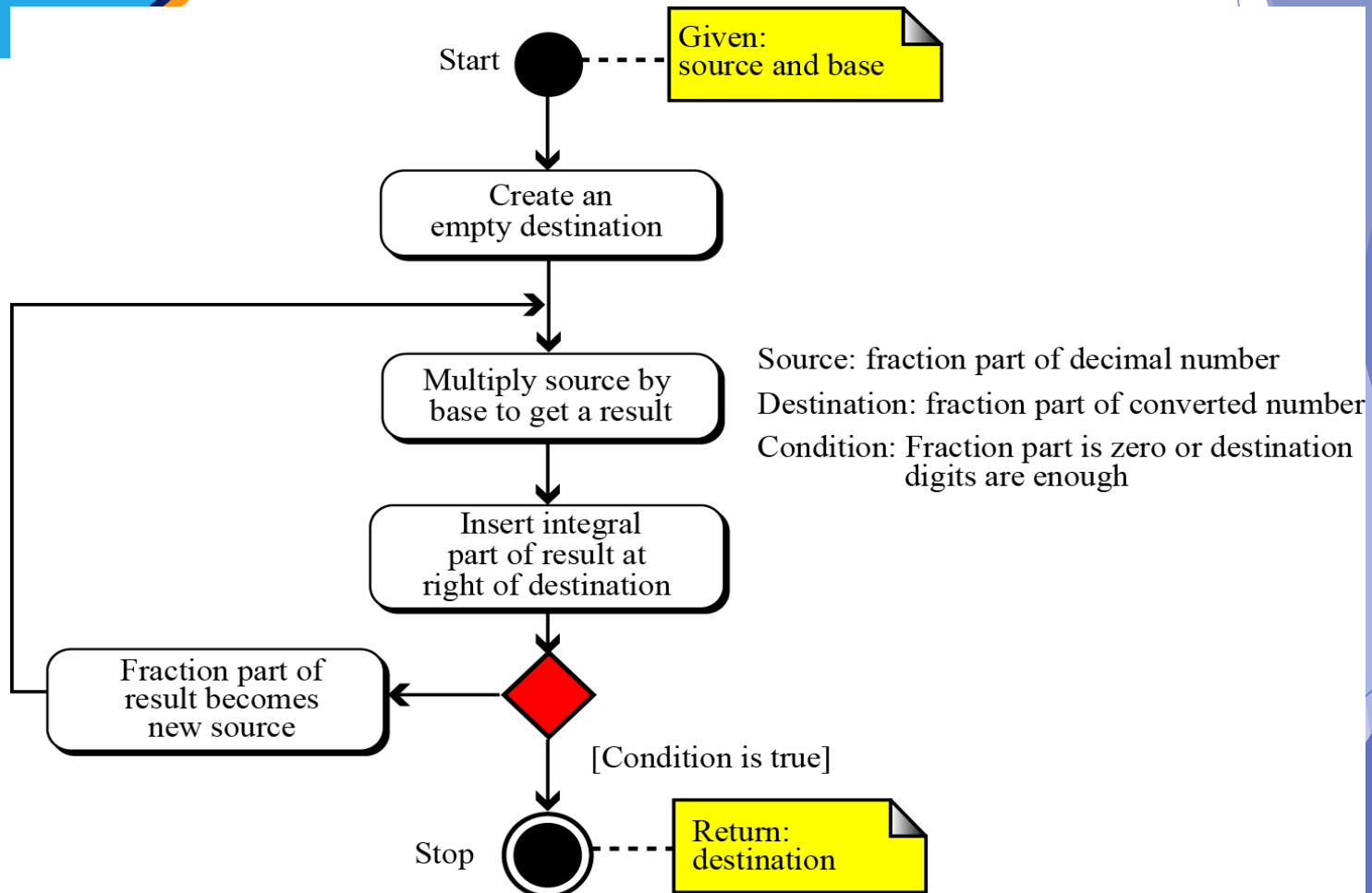
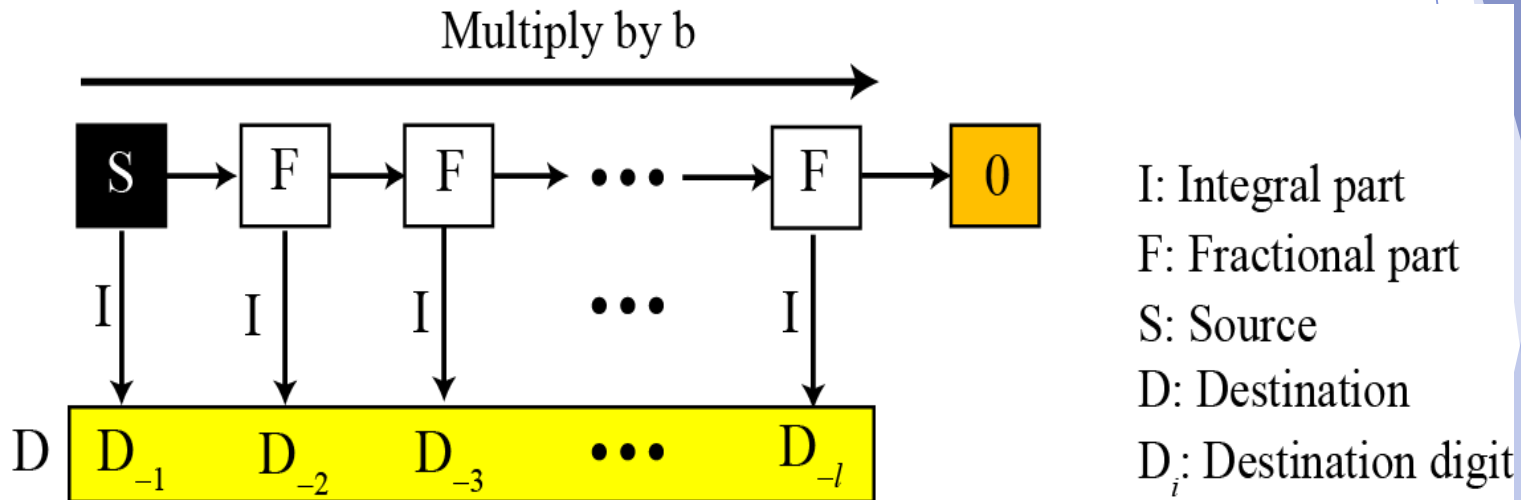


Figure 2.9 Converting the fractional part of a number in decimal to other bases



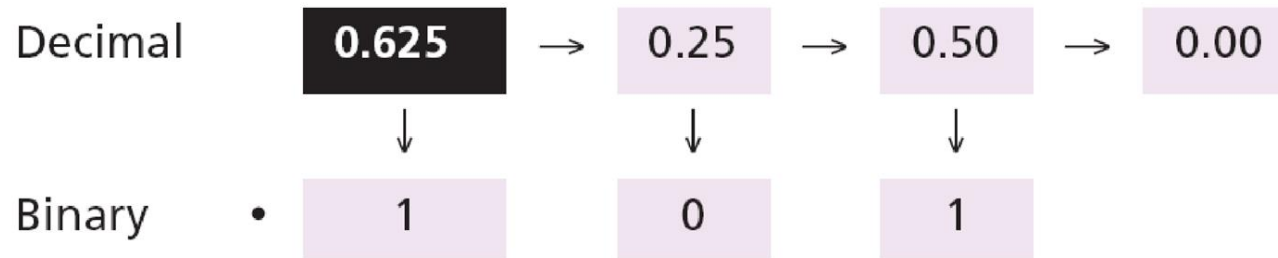
Note:

The fraction may never become zero.

Stop when enough digits have been created.

Example 2.14

Convert the decimal number 0.625 to binary.



Since the number $0.625 = (0.101)_2$ has no integral part, the example shows how the fractional part is calculated.



Example 2.15

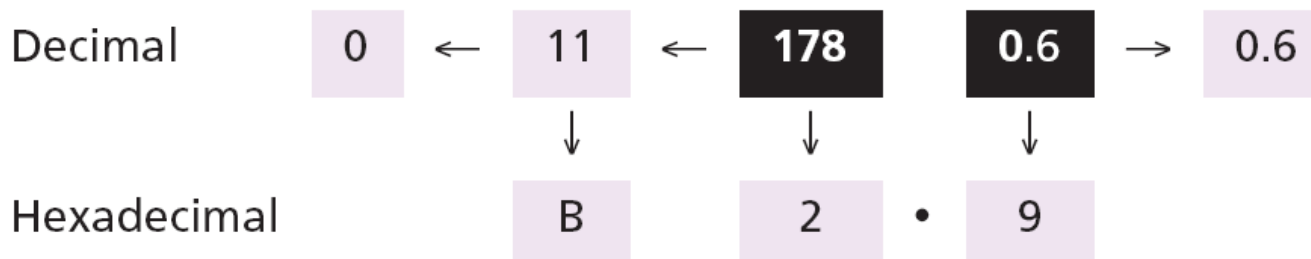
The following shows how to convert 0.634 to octal using a maximum of four digits. The result is $0.634 = (0.5044)_8$.

Note that we multiple by 8 (base octal).

Decimal	0.634	→	0.072	→	0.576	→	0.608	→	0.864
	↓		↓		↓		↓		
Octal	• 5		0		4		4		

Example 2.16

The following shows how to convert 178.6 in decimal to hexadecimal using only one digit to the right of the decimal point. The result is $178.6 = (B2.9)_{16}$. Note that we divide or multiply by 16 (base hexadecimal).



Example 2.17

An alternative method for converting a small decimal integer (usually less than 256) to binary is to break the number as the sum of numbers that are equivalent to the binary place values shown:

Place values	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Decimal equivalent	128	64	32	16	8	4	2	1

Decimal 165 =	128	+	0	+	32	+	0	+	0	+	4	+	0	+	1
Binary	1		0		1		0		0		1		0		1

Example 2.18

A similar method can be used to convert a decimal fraction to binary when the denominator is a power of two:

Place values	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}	2^{-7}				
Decimal equivalent	1/2	1/4	1/8	1/16	1/32	1/64	1/128				
Decimal = 27/64	16/64	+	8/64	+	2/64	+	1/64				
	1/4	+	1/8	+	1/32	+	1/64				
Decimal 27/64 =	0	+	1/4	+	1/8	+	0	+	1/32	+	1/64
Binary	0		1		1		0		1		1

The answer is then $(0.011011)_2$

Binary-hexadecimal conversion

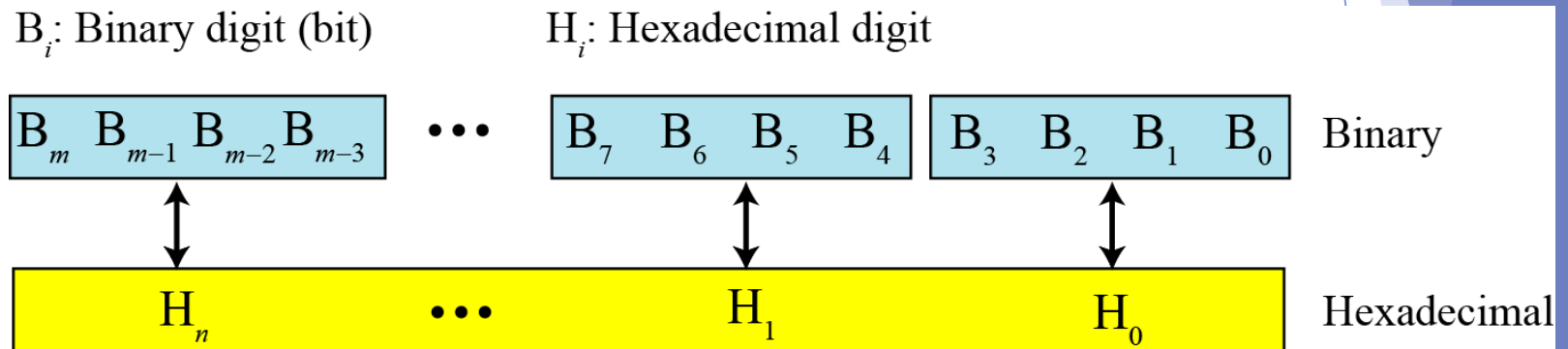


Figure 2.10 Binary to hexadecimal and hexadecimal to binary conversion

Example 2.19

Show the hexadecimal equivalent of the binary number $(110011100010)_2$.

Solution

We first arrange the binary number in 4-bit patterns:

1100 1110 0010

Note that the leftmost pattern can have one to four bits. We then use the equivalent of each pattern shown in Table 2.2 on page 25 to change the number to hexadecimal: $(CE2)_{16}$.

Example 2.20

What is the binary equivalent of $(24C)_{16}$?

Solution

Each hexadecimal digit is converted to 4-bit patterns:

$$2 \rightarrow 0010, 4 \rightarrow 0100, \text{ and } C \rightarrow 1100$$

The result is $(001001001100)_2$.

Binary-octal conversion

B_i : Binary digit (bit) O_i : Octal digit

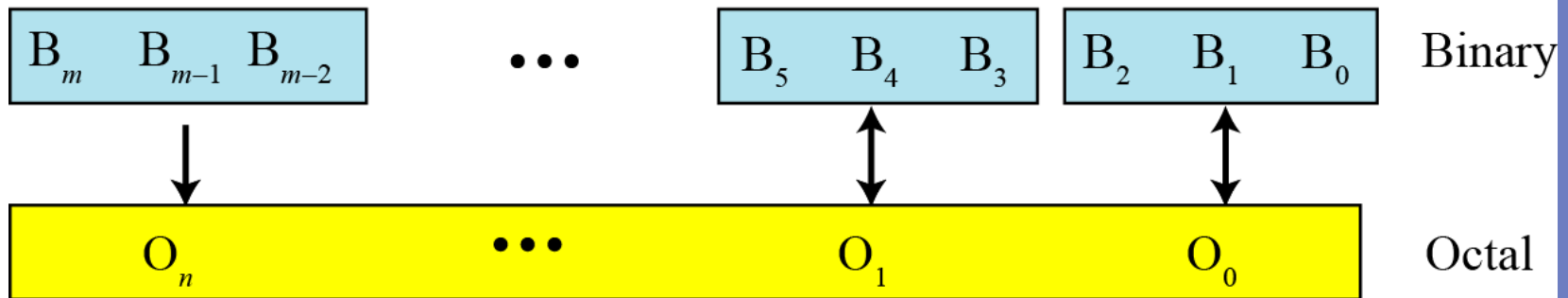


Figure 2.10 Binary to octal and octal to binary conversion

Example 2.21

Show the octal equivalent of the binary number $(101110010)_2$.

Solution

Each group of three bits is translated into one octal digit. The equivalent of each 3-bit group is shown in Table 2.2 on page 25.

101 110 010

The result is $(562)_8$.

Example 2.22

What is the binary equivalent of for $(24)_8$?

Solution

Write each octal digit as its equivalent bit pattern to get

$$2 \rightarrow 010 \text{ and } 4 \rightarrow 100$$

The result is $(010100)_2$.

Octal-hexadecimal conversion

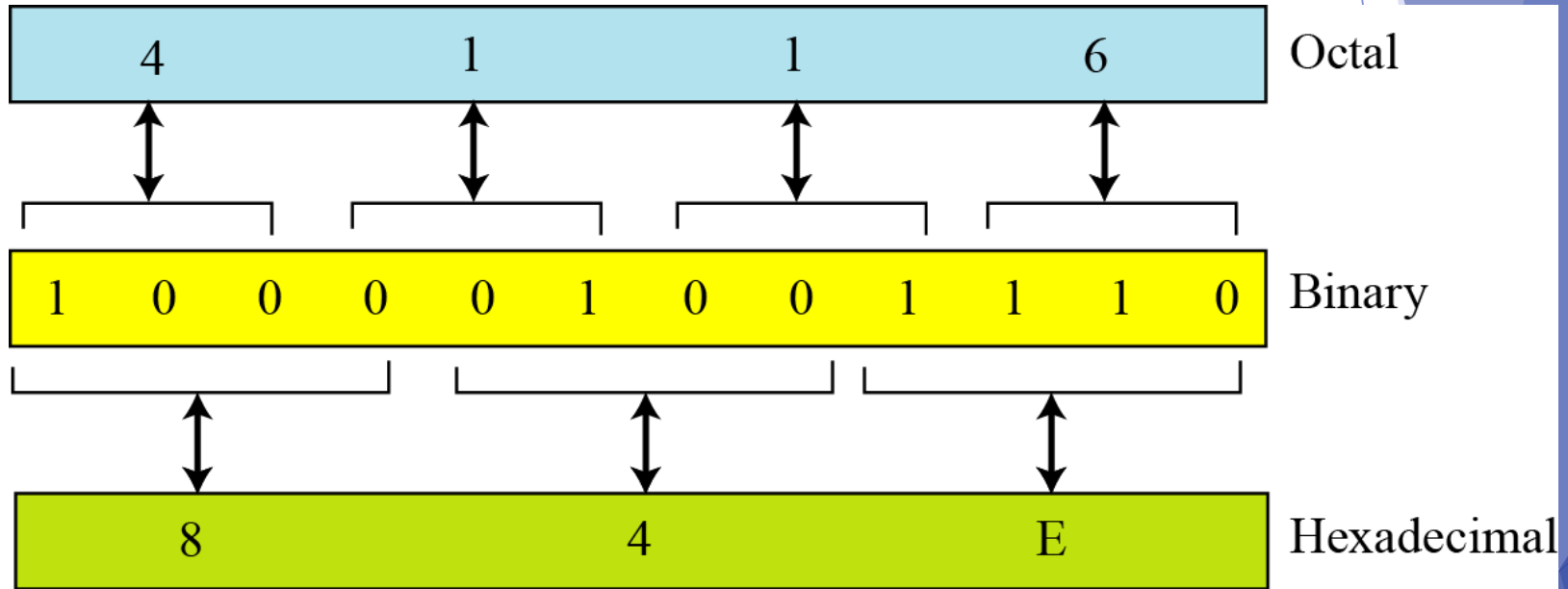


Figure 2.12 Octal to hexadecimal and hexadecimal to octal conversion

Number of digits

In conversion from one base to another, we often need to know the minimum number of digits we need in the destination system if we know the maximum number of digits in the source system.

$$x = \lceil k \times (\log b_1 / \log b_2) \rceil$$

Example 2.23

Find the minimum number of binary digits required to store decimal integers with a maximum of six digits.

Solution

$k = 6$, $b_1 = 10$, and $b_2 = 2$. Then

$$x = \lceil k \times (\log b_1 / \log b_2) \rceil = \lceil 6 \times (1 / 0.30103) \rceil = 20.$$

The largest six-digit decimal number is 999,999 and the largest 20-bit binary number is 1,048,575. Note that the largest number that can be represented by a 19-bit number is 524,287, which is smaller than 999,999. We definitely need twenty bits.

2-3 NONPOSITIONAL NUMBER SYSTEMS

Although **non-positional number systems** are not used in computers, we give a short review here for comparison with positional number systems. A non-positional number system still uses a limited number of symbols in which each symbol has a value. However, the position a symbol occupies in the number normally bears no relation to its value—the value of each symbol is fixed. To find the value of a number, we add the value of all symbols present in the representation.



In this system, a number is represented as:

$$S_{k-1} \dots S_2 S_1 S_0 \cdot S_{-1} S_{-2} \dots S_{-l}$$

and has the value of:

$$n = \pm \begin{array}{c} \text{Integral part} \\ S_{k-1} + \dots + S_1 + S_0 \end{array} + \begin{array}{c} \text{Fractional part} \\ S_{-1} + S_{-2} + \dots + S_{-l} \end{array}$$

There are some exceptions to the addition rule we just mentioned, as shown in Example 2.24.

Example 2.24

Roman numerals are a good example of a non-positional number system. This number system has a set of symbols $S = \{I, V, X, L, C, D, M\}$. The values of each symbol are shown in Table 2.3

Table 2.3 Values of symbols in the Roman number system

Symbol	<i>I</i>	<i>V</i>	<i>X</i>	<i>L</i>	<i>C</i>	<i>D</i>	<i>M</i>
Value	1	5	10	50	100	500	1000

To find the value of a number, we need to add the value of symbols subject to specific rules (See the textbook).

Example 2.24 (Continued)

The following shows some Roman numbers and their values.

III	→	$1 + 1 + 1$	=	3
IV	→	$5 - 1$	=	4
VIII	→	$5 + 1 + 1 + 1$	=	8
XVIII	→	$10 + 5 + 1 + 1 + 1$	=	18
XIX	→	$10 + (10 - 1)$	=	19
LXXII	→	$50 + 10 + 10 + 1 + 1$	=	72
CI	→	$100 + 1$	=	101
MMVII	→	$1000 + 1000 + 5 + 1 + 1$	=	2007
MDC	→	$1000 + 500 + 100$	=	1600

Thank You