

Business Mathematics

Prepared by Dr. Alfaisal Hasan

MTH 112
Business Mathematics

Lecture NO 3





مجموعة جامعة
الجميع الذكية
للتعليم القابضة

MIDOCEAN
UNIVERSITY
BY EVERYONE'S SMART UNIVERSITY

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Topics

- Chapter 1: Mathematical preliminaries
- Chapter 2: The straight line and applications
- **Chapter 3: Functions and Graphs**
- Chapter 4: Limits and Continuity
- Chapter 5: Financial Mathematics
- Chapter 6: Differentiation
- Chapter 7: Methods of Differentiation
- Chapter 8: Function of several variables
- Chapter 9: Integration and applications
- Chapter 10: Linear algebra and applications

Exponential Functions

Objectives

- To use the properties of exponents to:
 - Simplify exponential expressions.
 - Solve exponential equations.
- To sketch graphs of exponential functions.

Exponential Functions

- A polynomial function has the basic form: $f(x) = x^3$
- An exponential function has the basic form: $f(x) = 3^x$
- An exponential function has the variable in the exponent, not in the base.
- General Form of an Exponential Function:
$$f(x) = N^x, N > 0$$

Exponential Functions

Properties of Exponents

$$A^X \cdot A^Y = A^{X+Y}$$

$$\frac{A^X}{A^Y} = A^{X-Y}$$

$$(A^X)^Y = A^{XY}$$

$$A^{-X} = \frac{1}{A^X}$$

$$(AB)^X = A^X B^X$$

$$\frac{1}{A^{-X}} = A^X$$

$$\left(\frac{A}{B}\right)^X = \frac{A^X}{B^X}$$

$$A^{x/y} = \sqrt[y]{A^X} = \left(\sqrt[y]{A}\right)^X$$

Exponential Functions

■ Simplify: $2^2 \cdot 2^3 = 2^5 = 32$

$$2^2 \cdot 2^{-6} = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

$$(2^3)^2 = 2^6 = 64$$

Exponential Functions

■ Simplify: $\left(\frac{2}{3}\right)^{-3} = \frac{2^{-3}}{3^{-3}} = \frac{3^3}{2^3} = \frac{27}{8}$

$$\frac{3^7}{3^9} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$(2^{1/2})(8^{1/2}) = (2 \cdot 8)^{1/2} = 16^{1/2} = \sqrt{16} = 4$$

Exponential Functions

■ Solve: $5^x = 125$

$$5^x = 5^3$$

$$x = 3$$

Exponential Functions

■ Solve: $7^{(x-1)} = 7^{-1/2}$

$$x - 1 = -\frac{1}{2}$$
$$x = \frac{1}{2}$$

Exponential Functions

■ Solve: $8^x = 2$

$$(2^3)^x = 2^1$$

$$2^{3x} = 2^1$$

$$3x = 1$$

$$x = \frac{1}{3}$$

Exponential Functions

■ Solve: $8^x = 4$

$$(2^3)^x = 2^2$$

$$2^{3x} = 2^2$$

$$3x = 2$$

$$x = \frac{2}{3}$$

Exponential Functions

■ Solve: $\left(\frac{1}{3}\right)^x = 27$

$$\left(3^{-1}\right)^x = 27$$

$$3^{-x} = 3^3$$

$$-x = 3$$

$$x = -3$$

Exponential Functions

■ Solve: $x^{1/3} = 27$

$$\left(x^{1/3}\right)^3 = 27^3$$

$$x = 19,683$$

Not considered an
exponential equation,
because the variable
is now in the base.

Exponential Functions

■ Solve:

Not considered an exponential equation, because the variable is in the base.

$$x^{3/4} = 8$$

$$\left(x^{3/4}\right)^{4/3} = 8^{4/3}$$

$$x = \left(\sqrt[3]{8}\right)^4$$

$$x = (2)^4$$

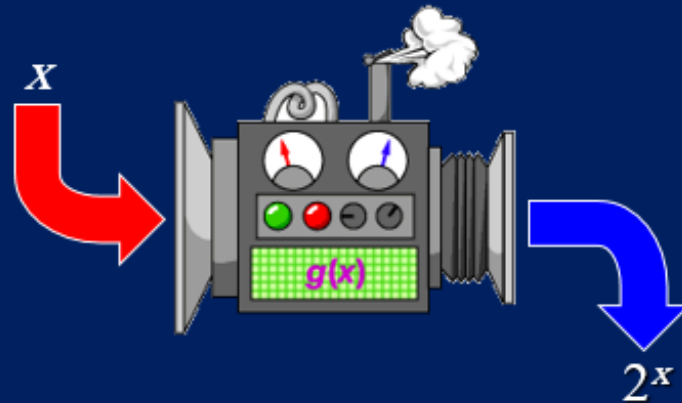
$$x = 16$$

Exponential Functions

- General Form of an Exponential Function:

$$f(x) = N^x, N > 0$$

$$g(x) = 2^x$$



$$g(3) = 8$$

$$g(2) = 4$$

$$g(1) = 2$$

$$g(0) = 1$$

$$g(-1) = 2^{-1} = \frac{1}{2}$$

$$g(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

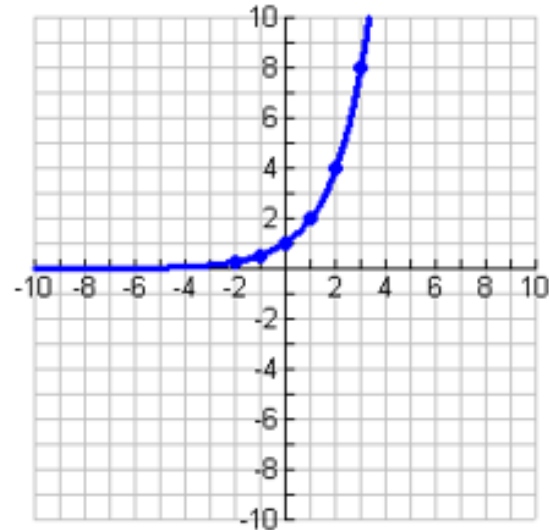
Exponential Functions

$$g(x) = 2^x$$

x	$g(x)$
2	4
1	2
0	1
-1	$\frac{1}{2}$
-2	$\frac{1}{4}$

Exponential Functions

$$g(x) = 2^x$$



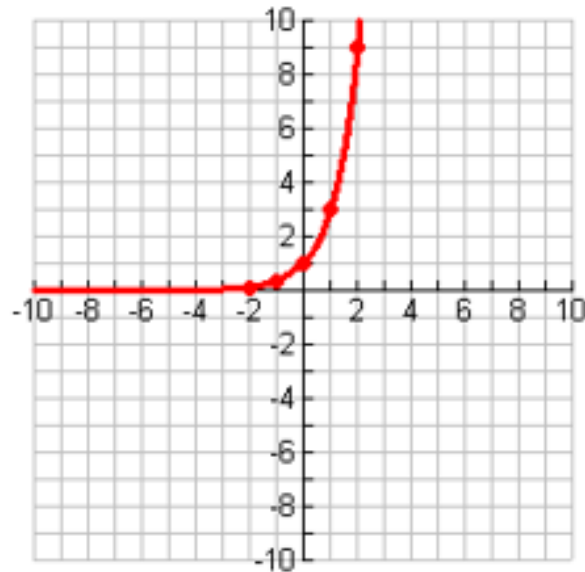
Exponential Functions

$$h(x) = 3^x$$

x	$h(x)$
2	9
1	3
0	1
-1	$\frac{1}{3}$
-2	$\frac{1}{9}$

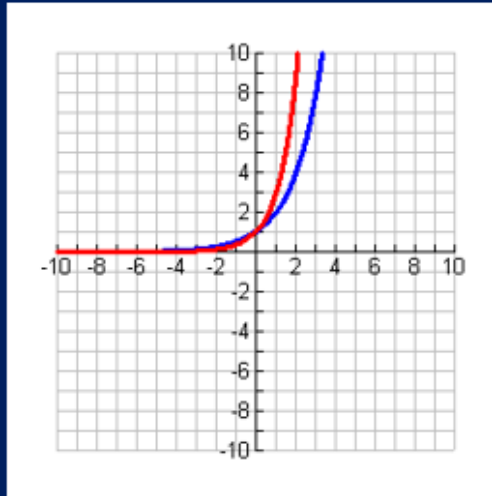
Exponential Functions

$$h(x) = 3^x$$



Exponential Functions

$$g(x) = 2^x \text{ (blue)}$$

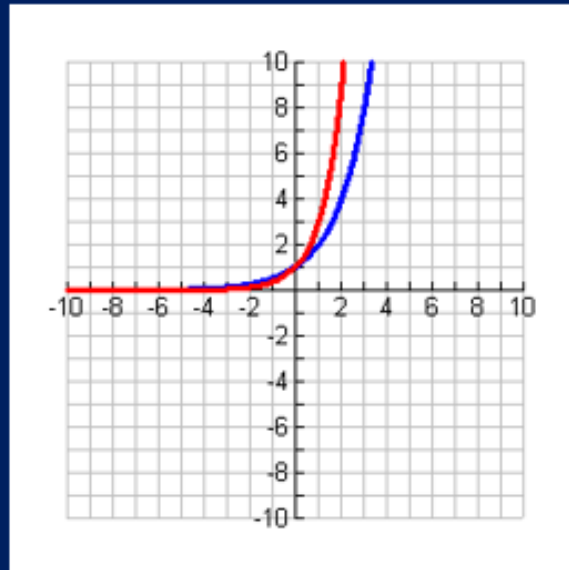


$$h(x) = 3^x \text{ (red)}$$

- Exponential functions with positive bases greater than 1 have graphs that are increasing.
- The function never crosses the x -axis because there is nothing we can plug in for x that will yield a zero answer.
- The x -axis is a left horizontal asymptote.

Exponential Functions

$$g(x) = 2^x \text{ (blue)}$$



$$h(x) = 3^x \text{ (red)}$$

- A smaller base means the graph rises more gradually.
- A larger base means the graph rises more quickly.
- Exponential functions will not have negative bases.

Exponential Functions

The Number e

A base often associated with exponential functions is:

$$e \approx 2.71828169$$

Exponential Functions

The Number e

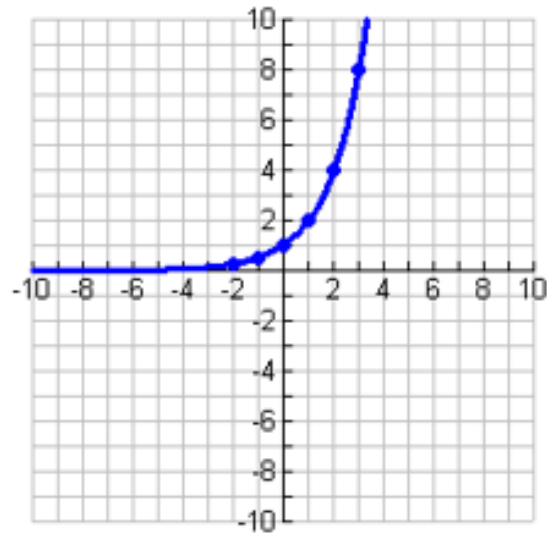
- Euler's number
- Leonhard Euler
(pronounced "oiler")
- Swiss mathematician
and physicist



Exponential Functions

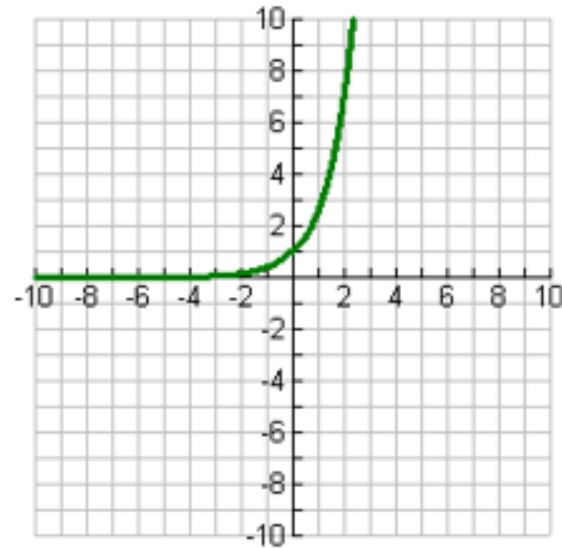
Remember

$$g(x) = 2^x$$



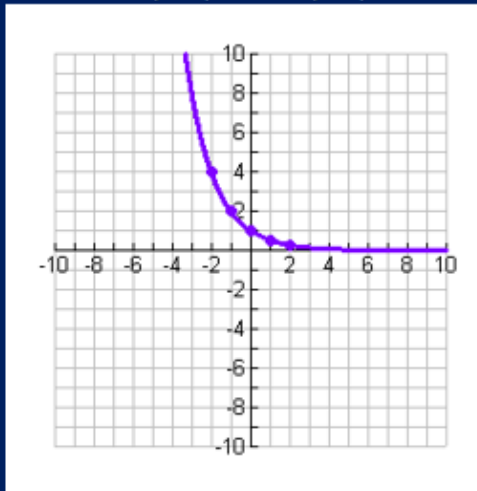
Exponential Functions

$$f(x) = e^x$$



Exponential Functions

$$j(x) = \left(\frac{1}{2}\right)^x$$



- Exponential functions with positive bases less than 1 have graphs that are decreasing.

Exponential Functions

Why study exponential functions?

- Exponential functions are used in our real world to measure growth, interest, and decay.
- Growth obeys exponential functions.
- Ex: rumors, human population, bacteria, computer technology, nuclear chain reactions, compound interest
- Decay obeys exponential functions.
- Ex: Carbon-14 dating, half-life, Newton's Law of Cooling

Exponential Functions

- You can use exponential functions to model the way certain populations increase or decrease over time.
- Increasing populations can be modeled by exponential growth function.

Exponential Functions

- Can you tell by looking at an equation whether the function models exponential growth or decay?

$$y = ab^x$$

- When $b > 1$, the function models exponential growth,
- and b is the **growth factor**.

Exponential Functions

When $0 < b < 1$,
the function models exponential **decay**,
and b is the **decay factor**.
It also called negative growth.

Exponential Functions

Now tell me just by looking at the equation if it represent exponential growth or decay.

$$y = 15(1.45)^x \quad \text{Exponential Growth}$$

$$y = 7.3(0.8)^x \quad \text{Exponential Decay}$$

Logarithmic Functions

For $x > 0$, $b > 0$ and b not equal to 1 the logarithm of x with base b is defined by the following:

$$\log_b x = y \Leftrightarrow x = b^y$$

Logarithmic Functions

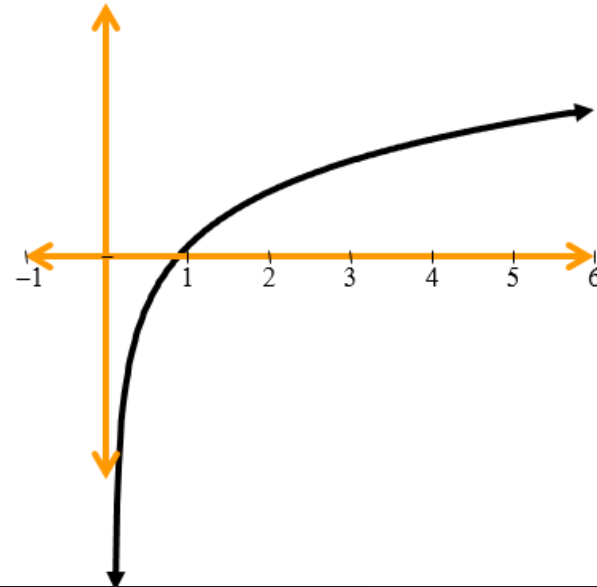
Properties of Logarithmic Function

- Domain: $\{x > 0\}$
- Range: all real numbers
- x intercept: $(1, 0)$
- No y intercept
- Approaches y axis as vertical asymptote
- Base determines shape.

Logarithmic Functions

Shape of logarithmic graphs

- For $b > 1$, the graph rises from left to right.



Logarithmic Functions

Common Logarithmic Function
The logarithmic function with
base 10

$$\log_{10} x = y \Leftrightarrow \log x = y$$

Logarithmic Functions

Natural logarithmic function
The logarithmic function with a
base of e

$$\log_e x = y \Leftrightarrow \ln x = y$$

Logarithmic Functions

Calculator Keys

- [LOG]
- [LN]

Logarithmic Functions

$$\log_b 1 = 0$$

$$\log_b b = 1$$

$$\log_b x = \log_b y \Leftrightarrow x = y$$

Logarithmic Functions

For $x > 0$, $y > 0$, $b > 0$ and $b \neq 1$
Product rule of Logarithms

$$\log_b (xy) = \log_b x + \log_b y$$

Logarithmic Functions

For $x > 0$, $y > 0$, $b > 0$ and b not 1
Quotient rule for Logarithms

$$\log_b \left(\frac{x}{y} \right) = \log_b x - \log_b y$$

Logarithmic Functions

For $x > 0$, $y > 0$, $b > 0$ and $b \neq 1$
Power rule for Logarithms

$$\log_b x^r = r (\log_b x)$$

Logarithmic Functions

- Apply the product, quotient, and power properties of logarithms.
- Combine and Expand logarithmic expressions

Logarithmic Functions

$$\log_b (xy) = \log_b x + \log_b y$$

$$\log_b \left(\frac{x}{y} \right) = \log_b x - \log_b y$$

$$\log_b x^r = r (\log_b x)$$

Logarithmic Functions

$$\ln(xy) = \ln x + \ln y$$

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

$$\ln x^r = r(\ln x)$$

Exponential and Logarithmic Functions

- A **logarithm**, or **log**, is defined in terms of an exponential.

$$\text{If } b^x = a, \text{ then } \log_b a = x$$

$$\text{If } 5^2 = 25, \text{ then } \log_5 25 = 2$$

Exponential and Logarithmic Functions

Solve each logarithmic equation.

$$\ln 3x = 2$$

Exponential and Logarithmic Functions

$$3x = e^2$$

Inverse Property

$$x = \frac{1}{3}e^2$$

Multiply each side by $\frac{1}{3}$.

$$x \approx 2.46$$

Use a calculator.

The solution is $x = \frac{1}{3}e^2$
 ≈ 2.46

Check this in the original equation.

Exponential and Logarithmic Functions

Solve the equation

$$\log_5 x - 2\log_5 7 = \log_5 125$$

Exponential and Logarithmic Functions

$$\log_5 x - 2\log_5 7 = \log_5 125$$

$$\log_5 x - \log_5 7^2 = \log_5 125$$

$$\log_5 x - \log_5 49 = \log_5 125$$

Exponential and Logarithmic Functions

$$\log_5 \frac{x}{49} = \log_5 125$$

$$\frac{x}{49} = 125$$

$$x = 125 \times 49 = 6125$$

Exponential and Logarithmic Functions

Solve the equation

$$\log_2 x + 2\log_2 5 = 2\log_2(x - 6)$$

Exponential and Logarithmic Functions

$$\log_2 x + 2\log_2 5 = 2\log_2 (x - 6)$$

$$\log_2 x + \log_2 5^2 = \log_2 (x - 6)^2$$

$$\log_2 x + \log_2 25 = \log_2 (x - 6)^2$$

Exponential and Logarithmic Functions

$$\log_2(25x) = \log_2(x - 6)^2$$

$$25x = (x - 6)^2$$

$$25x = x^2 - 12x + 36$$

$$0 = x^2 - 37x + 36$$

Exponential and Logarithmic Functions

$$(x - 36)(x - 1) = 0$$

$$x - 36 = 0 \quad x - 1 = 0$$

$$x = 36$$

$$\textcircled{x = 1}$$

Reject solutions
that would mean
taking the log of
a negative number

Un Solved Examples

Given that $f(x) = 5^x$

Find the value of $f(-2)$

Un Solved Examples

Given that $f(x) = 5^x$

Find the value of $f(0)$

Un Solved Examples

Given that $f(x) = 5^x$

Find the value of $f(1)$

Un Solved Examples

Given that $f(x) = 5^x$

Find the value of $f(-1)$

Un Solved Examples

Given that $f(x) = \left(\frac{1}{4}\right)^x$

Find the value of $f(-1)$

Un Solved Examples

Given that $f(x) = \left(\frac{1}{4}\right)^x$

Find the value of $f(0)$

Un Solved Examples

Given that exponential form $(16)^{\frac{1}{2}} = 4$

Find the logarithmic form

Un Solved Examples

Given that logarithmic form $\log_8 16 = \frac{3}{4}$

Find the exponential form

Un Solved Examples

Given that logarithmic form $\log 100 = 2$

Find the exponential form

Un Solved Examples

Given the function $y = 28(0.3)^x$

Is exponential decay or growth

Un Solved Examples

Given the function $y = 2(4.3)^x$

Is exponential decay or growth

Un Solved Examples

Given the expression $\ln \frac{y}{x^3}$
Simplify it

Un Solved Examples

Solve the equation $\ln\left(\frac{x}{10}\right) = \ln(15)$



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Thank You

