

Machine Learning for Finance

Lecture 9

FNT633

Forecasting **Part 2**

Time Series Models

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- Forecasting
- Time Series Models

Week

10

Forecasting

FORECASTING



FORECASTING



What Kind of Problem do I Need to Solve? How do I Solve it?

The Problem to Solve	The Category of Techniques	Covered in this Course
I want to group items by similarity. I want to find structure (commonalities) in the data	Clustering	K-means clustering
I want to discover relationships between actions or items	Association Rules	Apriori
I want to determine the relationship between the outcome and the input variables	Regression	Linear Regression Logistic Regression
I want to assign (known) labels to objects	Classification	Naïve Bayes Decision Trees
I want to find the structure in a temporal process I want to forecast the behavior of a temporal process	Time Series Analysis	ACF, PACF, ARIMA
I want to analyze my text data	Text Analysis	Regular expressions, Document representation (Bag of Words).

Agenda

- Appreciate the importance of forecasting to every organization
- List different types of forecasting method
- Discuss the characteristics of judgmental forecasting
- Use a variety of approaches to judgmental forecasting
- Describe the characteristics of projective forecasting
- Understand the importance of time series
- Calculate errors and a tracking signal for forecasts
- Forecast using simple averages, moving averages, and exponential smoothing
- Forecast time series with seasonality and trend.

- **What is forecasting?**
 - Primary Function is to Predict the Future using (time series related or other) data we have in hand
- **Why are we interested?**
 - Affects the decisions we make today
- **Where is forecasting used ?**
 - forecast demand for products and services
 - forecast availability/need for manpower
 - forecast inventory and materiel needs daily

- **Generate forecasts for data with different patterns:
level, trend, seasonality, and cyclical**
- **Describe causal modeling using linear regression**
- **Compute forecast accuracy**
- **Explain how forecasting models should be selected.**

Forecasting steps

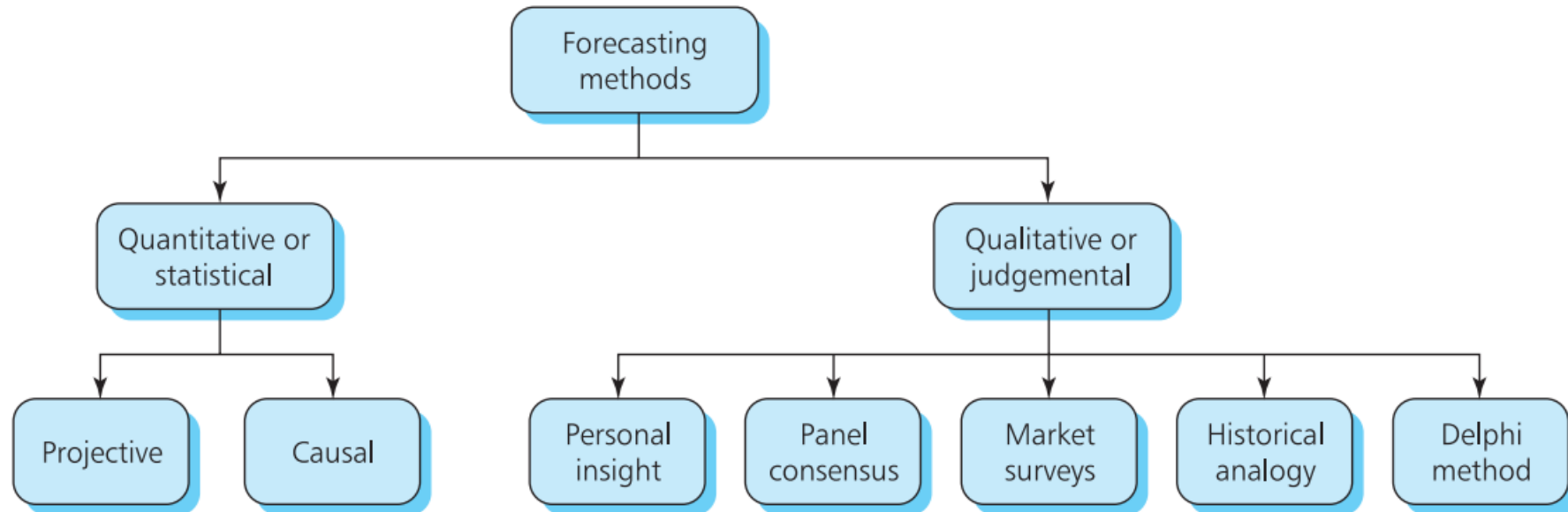
- Decide what needs to be forecast
 - Level of detail, units of analysis & time horizon required
- Evaluate and analyze appropriate data
 - Identify needed data & whether it's available
- Select and test the forecasting model
 - Cost, ease of use & accuracy
- Generate the forecast
- Monitor forecast accuracy over time

Methods of forecasting

- One classification of forecasting methods concerns the time in the future they cover
 - *Long-term forecasts*; look ahead **several years** – the time typically needed to **build a new factory**
 - *Medium-term forecasts* look ahead **between three months and two years** –the time typically needed to replace an old product by a new one.
 - *Short-term forecasts* cover **the next few weeks** – describing the continuing demand for a product.

Methods of forecasting

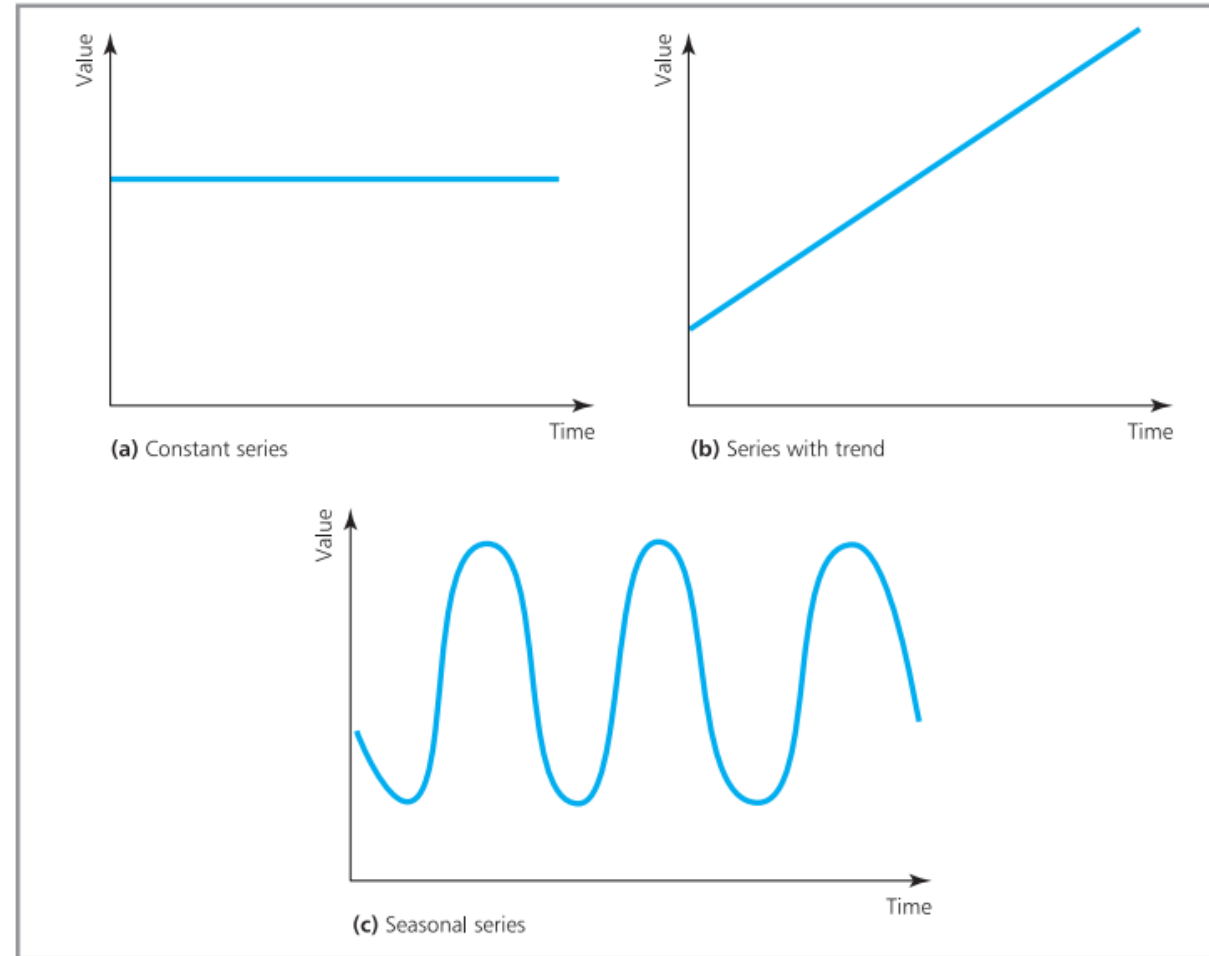
- Another classification of methods draws a distinction between qualitative and quantitative approaches



Time Series Pattern

- Projective forecasts often work with time series, such as monthly unemployment figures, daily rainfall, weekly sales, quarterly profit, and annual fuel consumption.
- When you have a time series it is always useful to draw a graph, and a simple scatter diagram shows any underlying patterns.
 - constant series (**Level**) ; with observations taking roughly the same value over time, such as annual rainfall
 - series with a **trend** ; with values either rising or falling steadily, such as the gross national product per capita
 - **seasonal** series; which have a **cyclical** component, such as the weekly sales of soft drinks.

Time Series Pattern



Time Series Pattern

- If observations followed such simple patterns, there would be no problems with forecasting.
- Unfortunately, there are nearly always differences between actual observations and the underlying pattern.
- Random noise is applied to the underlying pattern

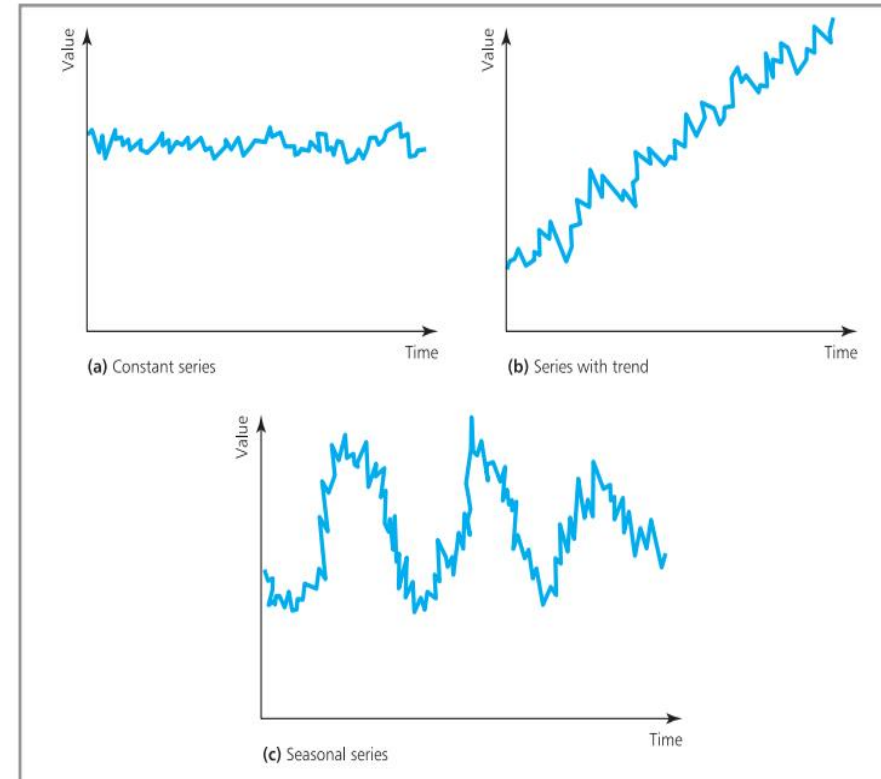
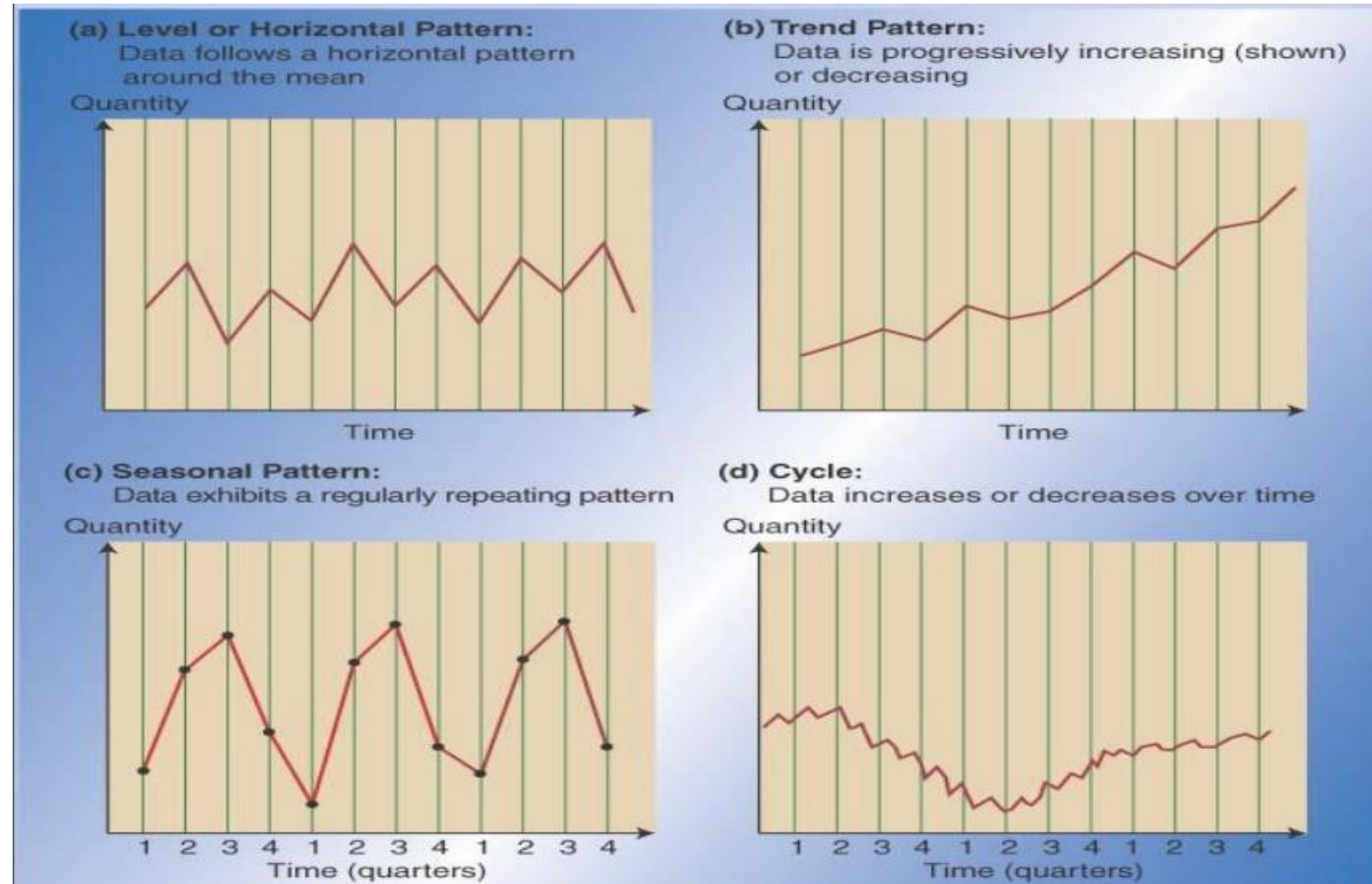


Figure 10.3 Patterns in time series including noise

Constant series, for example, does not always take exactly the same value, but is somewhere close. Then:
200 205 194 195 208 203 200 193 201 198
is a constant series of 200 with superimposed noise.

Time Series Pattern



Common Measures of Error

If we define y_t as the actual observation in period t , and F_t as the forecast value, the error is:

$$E_t = y_t - F_t$$

$$\text{mean error} = \frac{\sum E_t}{n} = \frac{\sum (y_t - F_t)}{n}$$

COMMON MEASURES OF ERROR

Mean Absolute Deviation (MAD)

$$\text{MAD} = \frac{\sum |\text{Actual} - \text{Forecast}|}{n}$$

Mean Squared Error (MSE)

$$\text{MSE} = \frac{\sum (\text{Forecast Errors})^2}{n}$$

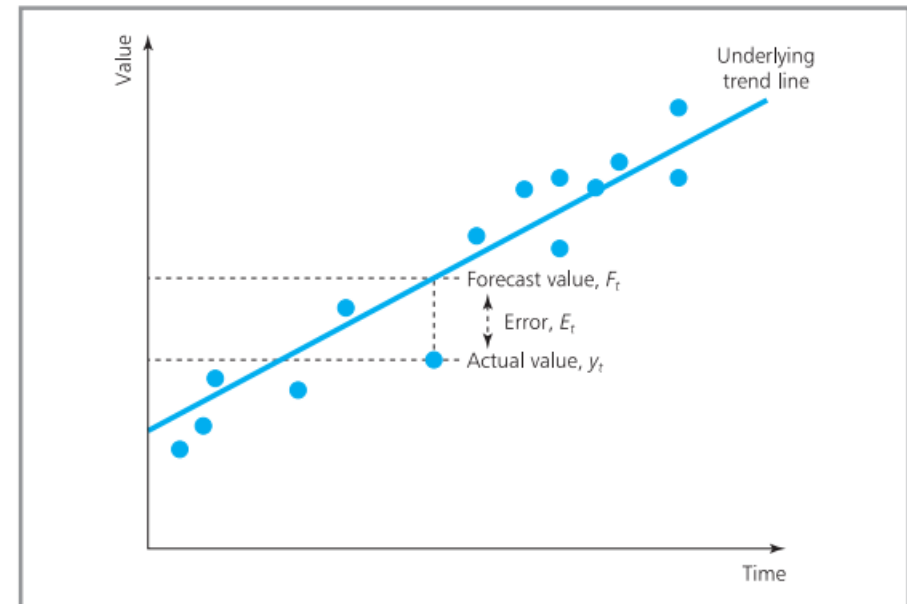


Figure 10.4 Errors in forecasts

E_t = error in the forecast in period t
= actual observation in period t – forecast value

◎ Naive:

$$F_{t+1} = A_t$$

- The forecast is equal to the actual value observed during the last period - good for level patterns

◎ Simple Mean:

$$F_{t+1} = \sum A_t / n$$

- The average of all available data - good for level patterns

◎ Moving Average:

$$F_{t+1} = \sum A_t / n$$

- The average value over a set time period (e.g.: the last four weeks)
- Each new forecast drops the oldest data point & adds a new observation
- More responsive to a trend but still lags behind actual data

Time Series Model

John Butler runs two dental surgeries, with the following numbers of patients visiting each over the past five weeks. Use simple averages to forecast the numbers of patients visiting in week six. How accurate are the forecasts? What are the forecasts for week 24?

Week	1	2	3	4	5
Surgery 1	98	100	98	104	100
Surgery 2	140	66	152	58	84

Simple Averages model

- Simple averages can give good forecasts for constant series
 - Simple averages do not work well when the underlying Pattern changes
 - The problem is that older data tends to swamp the latest figures and the forecast is very slow to follow the changing pattern.

Simple Average model limitations

- Suppose that demand for an item has been constant at 100 units a week for the past two years (104 weeks). Simple averages give a forecast demand for week 105 of 100 units.
- But if the actual demand in week 105 suddenly rises to 200 units, simple averages give a forecast for week 106

$$F_{106} = (104 \times 100 + 200)/105 = 100.95$$

- If demand continues at 200 units a week, following forecasts are:

$$F_{107} = 101.89 \quad F_{108} = 102.80, \quad F_{109} = 103.70, \quad F_{110} = 104.59, \dots \text{etc.}$$

- If demand continues at 200 units a week, following forecasts are:

Simple Average model limitations

- Simple averages only actually work for constant series.
- The forecasts are rising but the response is very slow.
- But very few time series are in fact stable over long periods, so this restriction makes them of limited value.
- The problem with simple averages is that old, out-of-date data tends to swamp newer, more relevant data.
- A way around this is to ignore old data and use only a number of the most recent observations.

- As patterns of demand tend to vary over time
 - **only a certain amount of historical data is relevant** to future forecasts.
 - The problem with simple averages is that **old, out-of-date data tends to swamp newer**, more relevant data
- the principle of moving averages is to **ignore old data and** use only a number of the most recent observations.

$$\begin{aligned} F_{t+1} &= \text{average of } n \text{ most recent observations} \\ &= \frac{\text{latest demand} + \text{next latest} + \dots + n\text{th latest}}{n} \\ &= \frac{y_t + y_{t-1} + \dots + y_{t-n+1}}{n} \end{aligned}$$

Moving Average method

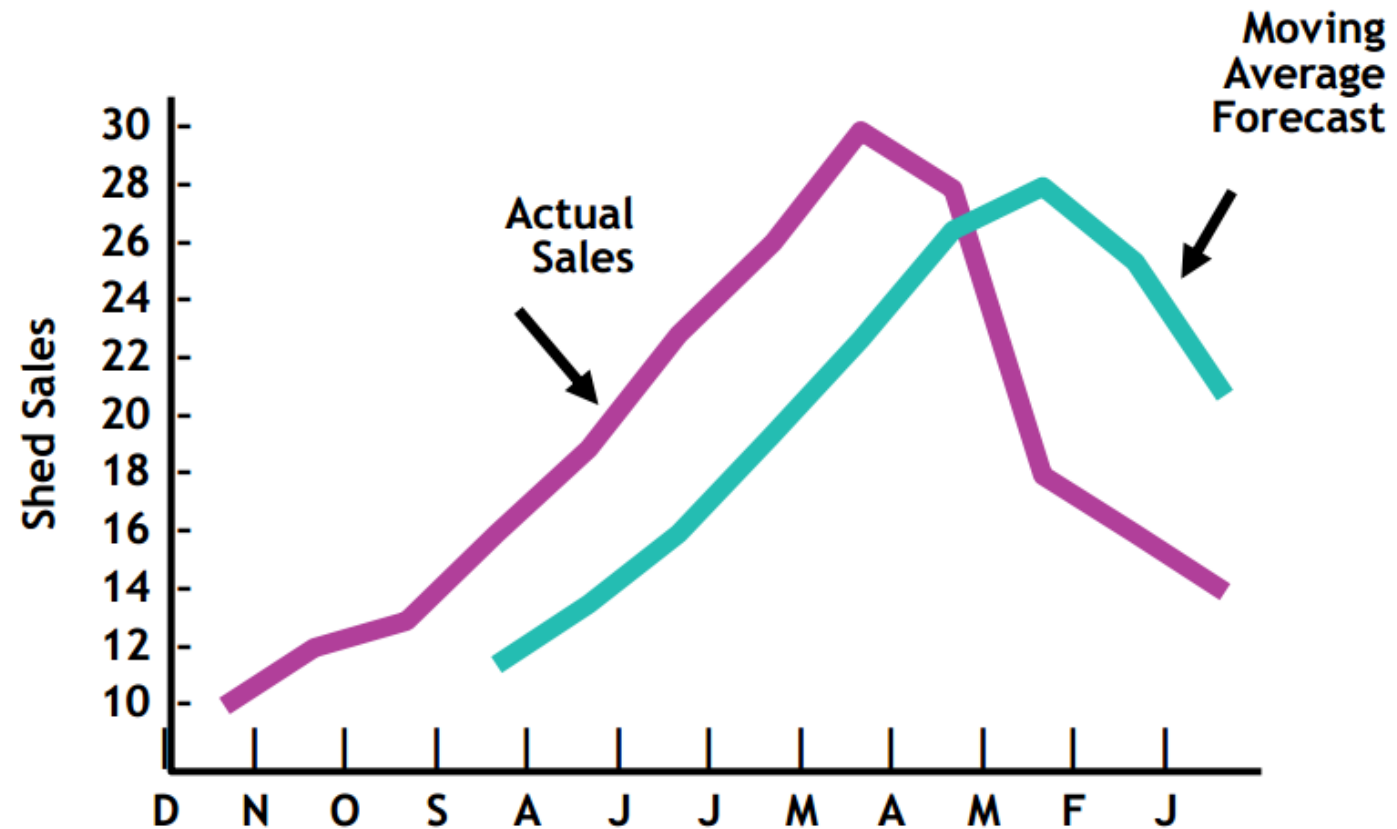
- ◆ MA is a series of arithmetic means
- ◆ Used if little or no trend
- ◆ Used often for smoothing
 - ◆ Provides overall impression of data over time

$$\text{Moving average} = \frac{\Sigma \text{ demand in previous } n \text{ periods}}{n}$$

Moving Average Example

3-Month Moving Average	Actual Shed Sales	Month
10	January	10
12	February	12
13	March	13
16	April	$(10 + 12 + 13)/3 = 11 \frac{2}{3}$
19	May	$(12 + 13 + 16)/3 = 13 \frac{2}{3}$
23	June	$(13 + 16 + 19)/3 = 16$
26	July	$(16 + 19 + 23)/3 = 19 \frac{1}{3}$

Graph of Moving Average



■ Weighted Moving Average model

Weighted Moving Average model

- ◆ Used when some trend might be present
 - ◆ Older data usually less important
- ◆ Weights based on experience and intuition

$$\text{Weighted moving average} = \frac{\sum (\text{weight for period } n) \times (\text{demand in period } n)}{\sum \text{weights}}$$

Weighted Moving Average model

WEIGHT

Weights Applied	Period
3	Last month
2	Two months ago
1	Three months ago
6	Sum of weights

3-Month Weighted Moving Average	Actual Shed Sales	Month
10	January	10
12	February	12
13	March	13
16	April	$[(3 \times 13) + (2 \times 12) + (10)]/6 = 12\frac{1}{6}$
19	May	$[(3 \times 16) + (2 \times 13) + (12)]/6 =$
23	June	$14\frac{1}{3}$
26	July	$[(3 \times 19) + (2 \times 16) + (13)]/6 = 17$
		$[(3 \times 23) + (2 \times 19) + (16)]/6 = 20\frac{1}{2}$

Weighted Moving Average model

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3-Month Weighted Moving Average	Actual Shed Sales	Month
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26	July	$[(3 \times 19) + (2 \times 16) + (13)]/6 = 17$
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POTENTIAL PROBLEMS WITH MOVING AVERAGE

- ◆ Increasing n smooths the forecast but makes it less sensitive to changes
- ◆ Do not forecast trends well
- ◆ Require extensive historical data

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MOVING AVERAGE AND WEIGHTED MOVING AVERAGE

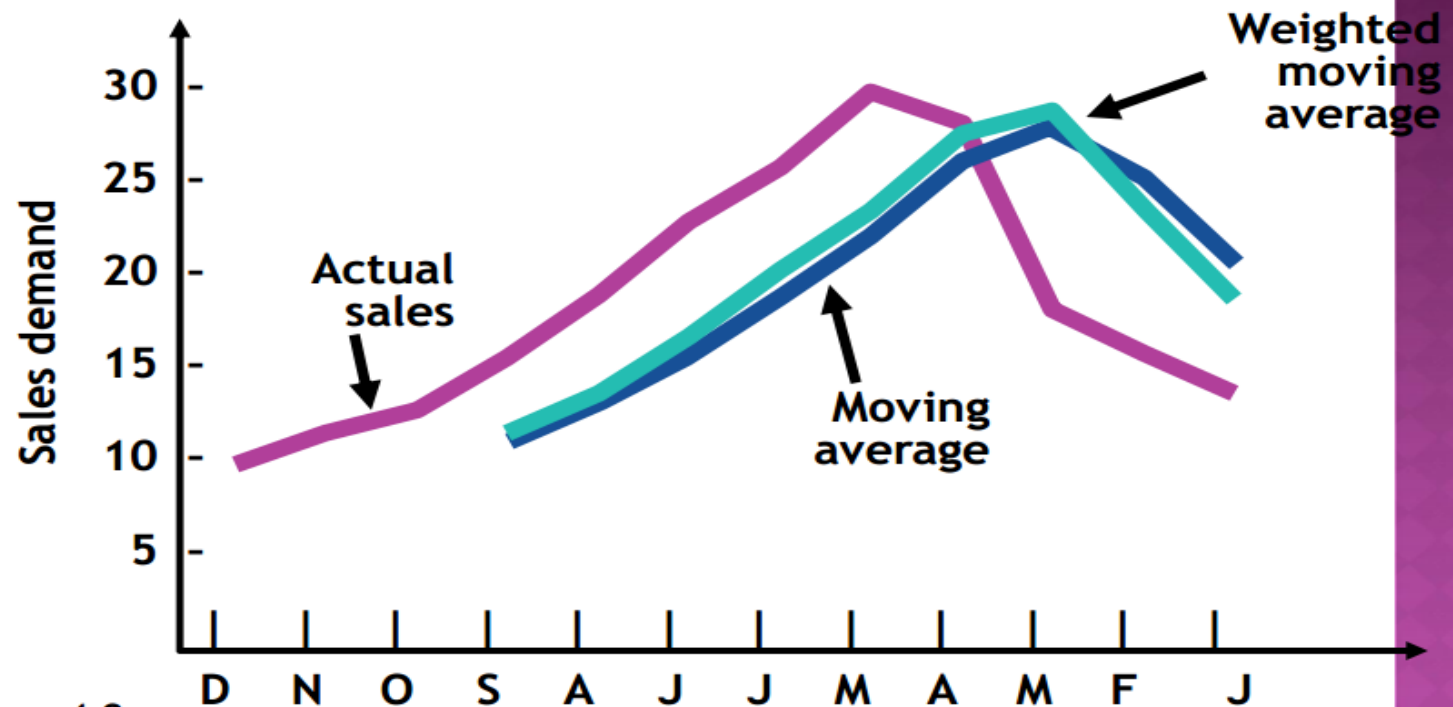


Figure 4.2

10.1 Use linear regression to forecast values for periods 11 to 13 for the following time series.

Period	1	2	3	4	5	6	7	8	9	10
Observation	121	133	142	150	159	167	185	187	192	208

10.2 Use simple averages to forecast values for the data in problem 10.1. Which method gives better results?

10.3 Use a four-period moving average to forecast values for the data in problem 10.1.

10.4 Find the two-, three- and four-period moving averages for the following time series, and use the errors to say which gives the best results.

<i>T</i>	1	2	3	4	5	6	7	8	9	10
<i>Y</i>	280	240	360	340	300	220	200	360	410	280

MOVING AVERAGE AND WEIGHTED MOVING AVERAGE

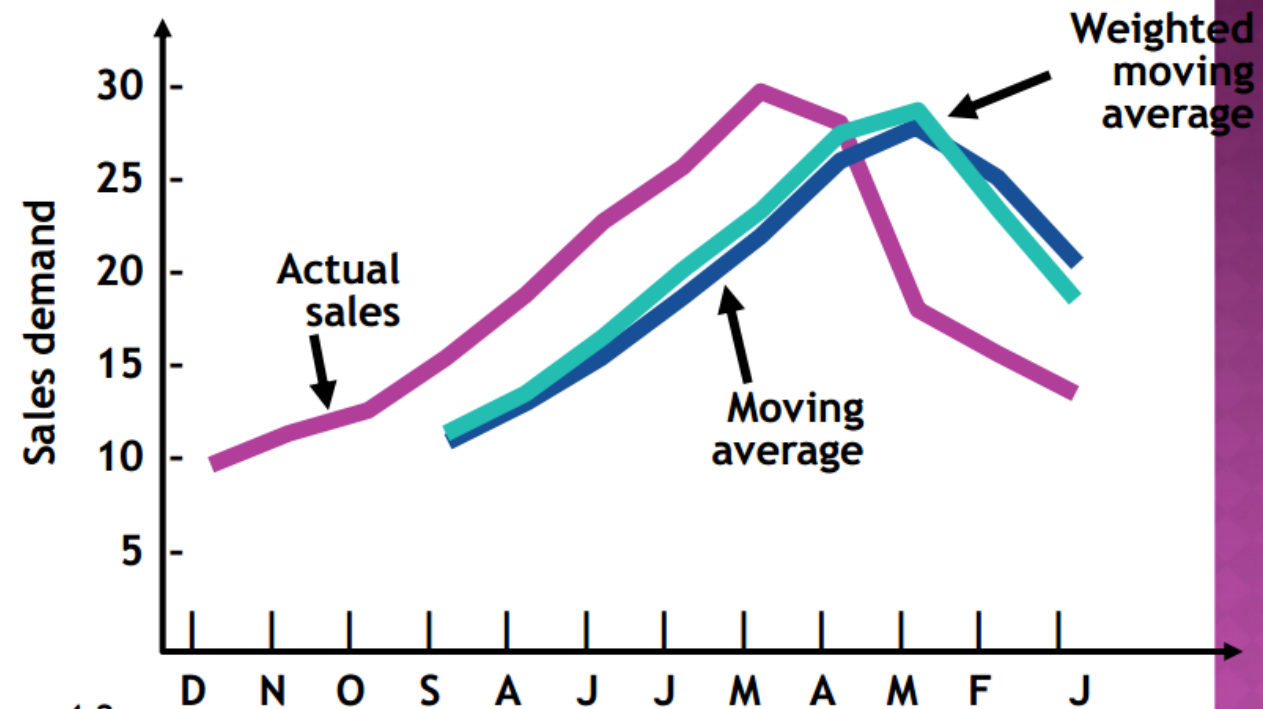
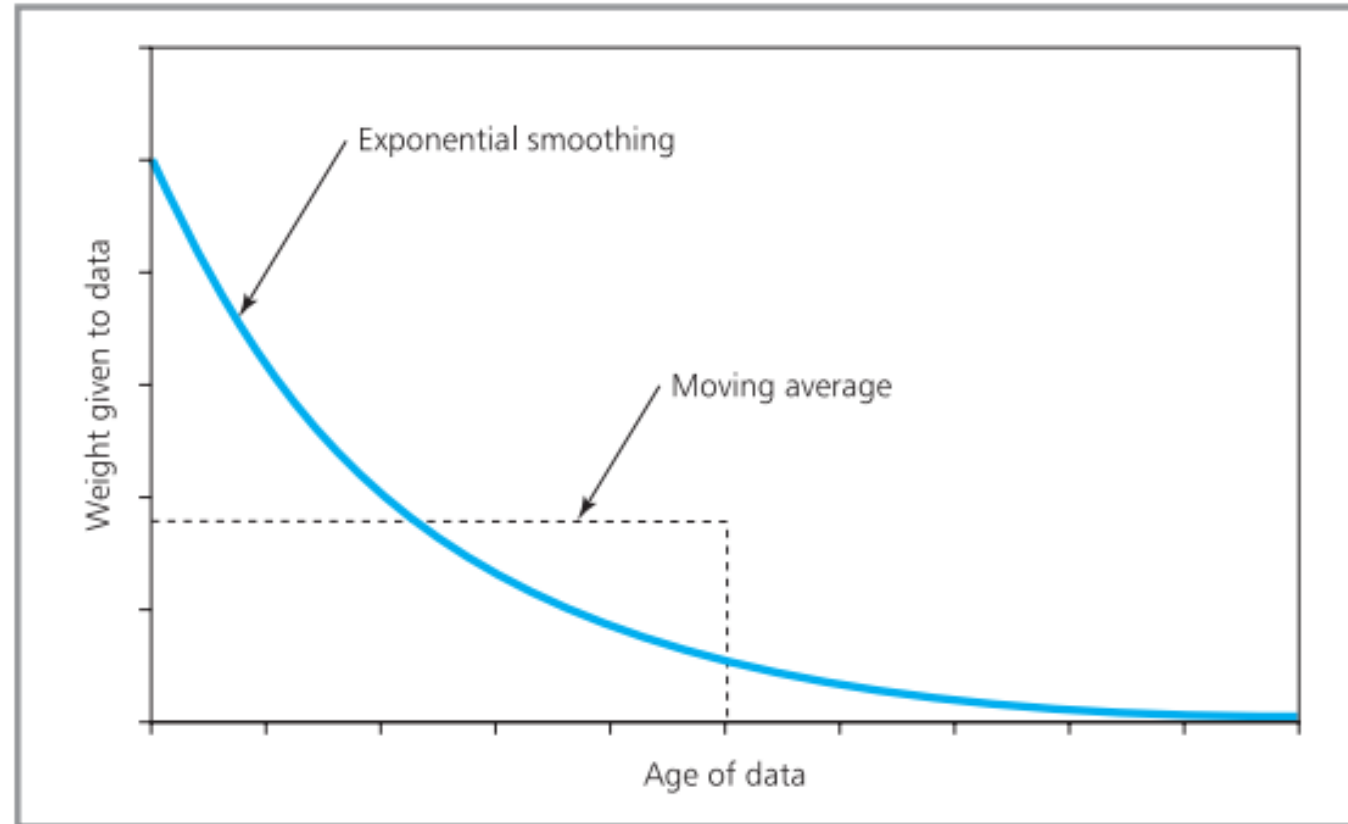


Figure 4.2

Weighted Moving Average model



- Weights given to data with exponential smoothing and moving average

Week

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Exponential smoothing

Exponential smoothing

- Exponential smoothing is based on the idea that as data gets older it becomes less relevant and should be given less weight.
- we can do weighting by using the latest observation to update a previous forecast.
- The calculation for the new forecast can be calculated by

new forecast = $\alpha \times$ latest observation + $(1 - \alpha) \times$ last forecast

$$F_{t+1} = \alpha x_t + (1 - \alpha)F_t$$

α is a smoothing constant

EXPONENTIAL SMOOTHING

New forecast = Last period's forecast
+ α (Last period's actual demand
- Last period's forecast)

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

where

F_t = new forecast

F_{t-1} = previous forecast

A_{t-1} = Actual demand

α = smoothing (or weighting)
constant ($0 \leq \alpha \leq 1$)

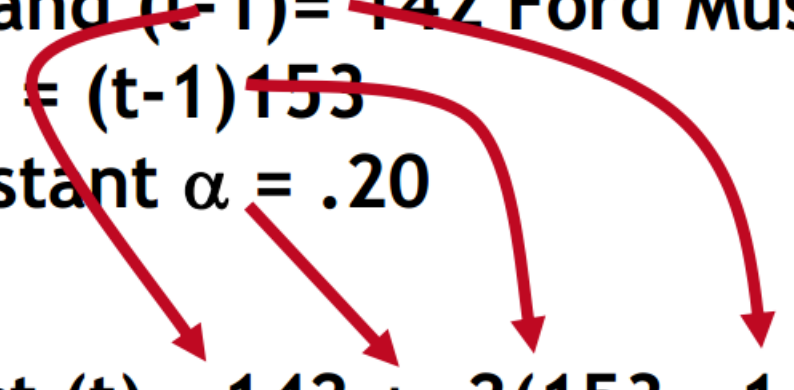
EXPONENTIAL SMOOTHING EXAMPLE

Predicted demand (t-1) = 142 Ford Mustangs

Actual demand = (t-1) 153

Smoothing constant $\alpha = .20$

New forecast (t) = $142 + .2(153 - 142)$



EXPONENTIAL SMOOTHING EXAMPLE

Predicted demand = 142 Ford Mustangs

Actual demand = 153

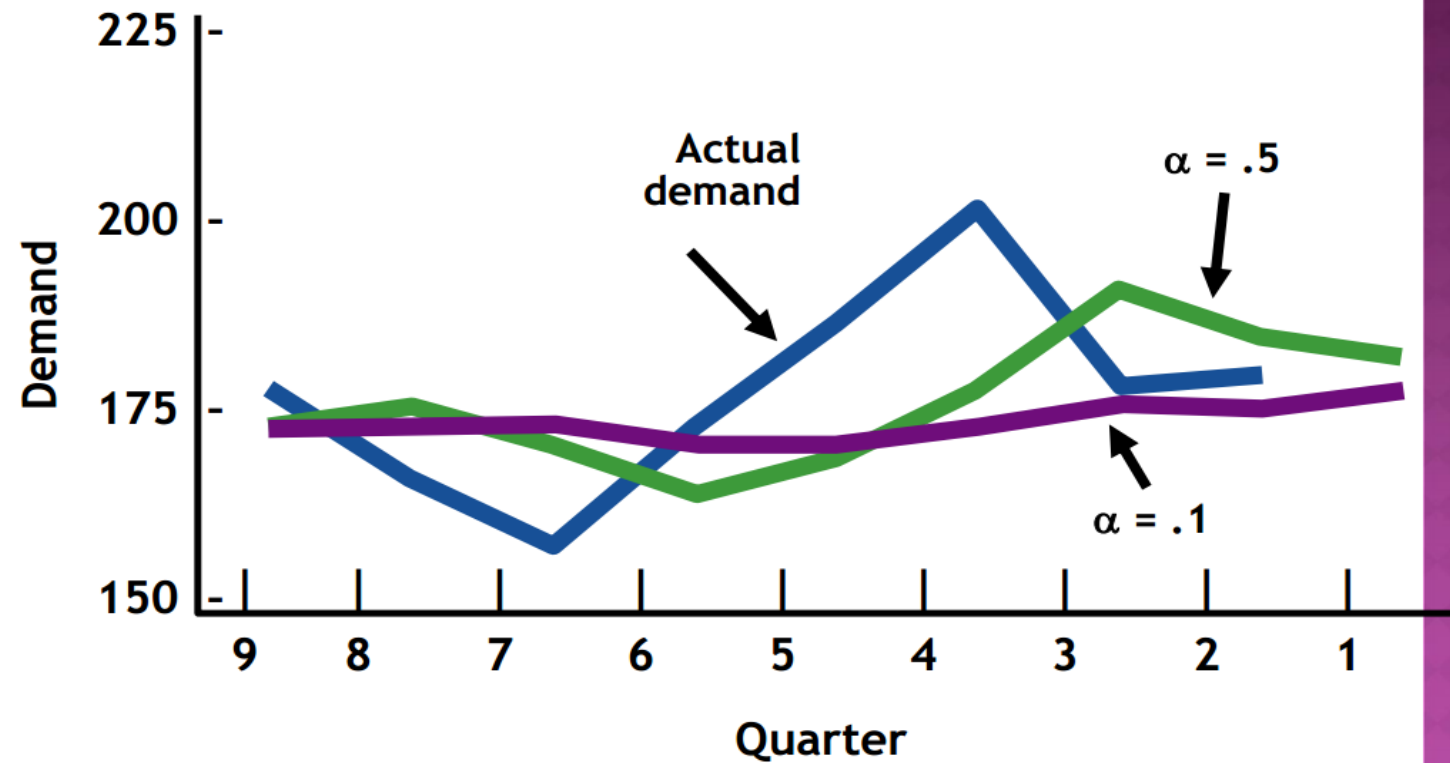
Smoothing constant $\alpha = .20$

$$= 142 + .2(153 - 142) \text{ New forecast}$$

$$= 142 + 2.2$$

$$= 144.2 \approx 144 \text{ cars}$$

IMPACT OF DIFFERENT α



WORKED EXAMPLE 10.6

Use exponential smoothing with $\alpha = 0.2$ and an initial value of $F_1 = 170$ to get forecasts one period ahead for the following time series.

Month	1	2	3	4	5	6	7	8
Demand	178	180	156	150	162	158	154	132

Exponential smoothing

WORKED EXAMPLE 10.6

Use exponential smoothing with $\alpha = 0.2$ and an initial value of $F_1 = 170$ to get forecasts one period ahead for the following time series.

Month	1	2	3	4	5	6	7	8
Demand	178	180	156	150	162	158	154	132

Solution

We know that $F_1 = 170$ and $\alpha = 0.2$. Substituting these values gives a forecast for the second period:

$$F_2 = \alpha y_1 + (1 - \alpha)F_1 = 0.2 \times 178 + 0.8 \times 170 = 171.6$$

Then substituting for F_2 gives:

$$F_3 = \alpha y_2 + (1 - \alpha)F_2 = 0.2 \times 180 + 0.8 \times 171.6 = 173.3$$

$$F_4 = \alpha y_3 + (1 - \alpha)F_3 = 0.2 \times 156 + 0.8 \times 173.3 = 169.8$$

and so on, giving the following results.

Month, t	1	2	3	4	5	6	7	8	9
Demand, Y_t	178	180	156	150	162	158	154	132	
Forecast, F_t	170	171.6	173.3	169.8	165.8	165	163.6	161.7	155.8

WORKED EXAMPLE 10.7

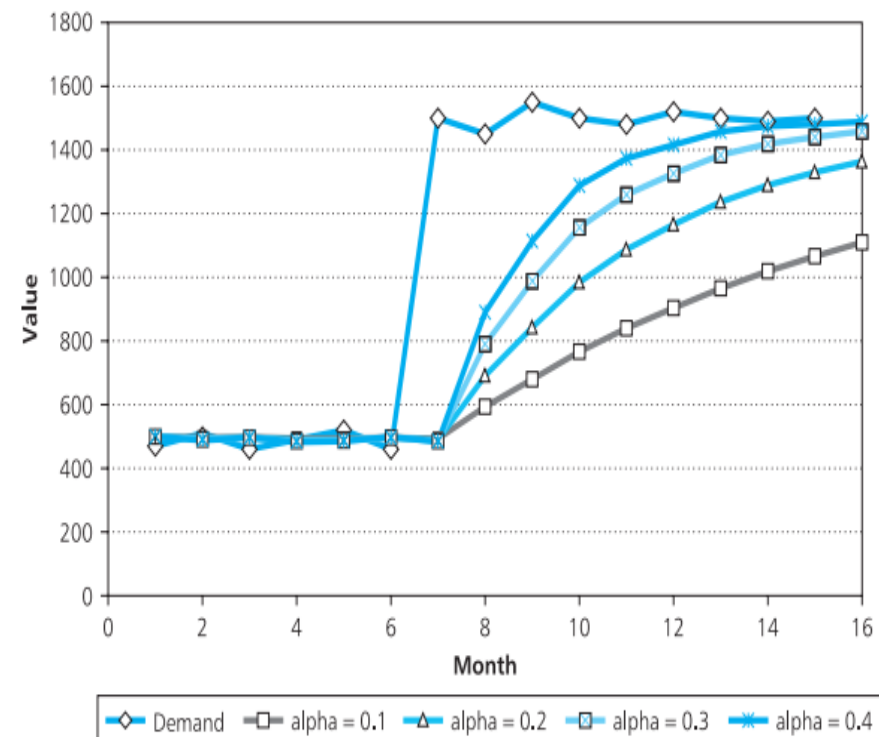
Remko van Rijn collected the demand figures shown in Figure 10.8. Use an initial forecast of 500 to compare exponential smoothing forecasts with different values of α .

α is 0.1 , 0.2 , 0.3 , and 0.4

Exponential Smoothing	
Month	Demand
1	470
2	510
3	460
4	490
5	520
6	460
7	1500
8	1450
9	1550
10	1500
11	1480
12	1520
13	1500
14	1490
15	1500
16	

Exponential smoothing

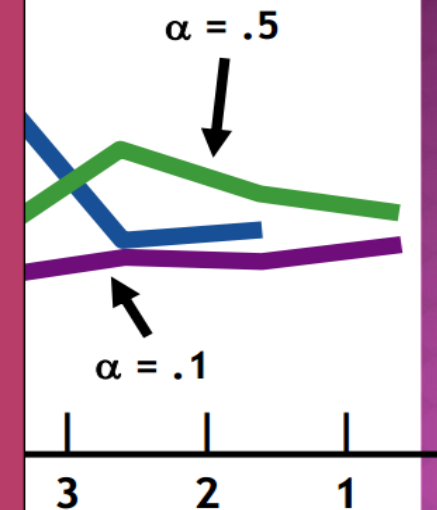
Exponential Smoothing					
Month	Demand	Forecast $\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$
1	470	500.0	500.0	500.0	500.0
2	510	497.0	494.0	491.0	488.0
3	460	498.3	497.2	496.7	496.8
4	490	494.5	489.8	485.7	482.1
5	520	494.0	489.8	487.0	485.2
6	460	496.6	495.8	496.9	499.1
7	1500	493.0	488.7	485.8	483.5
8	1450	593.7	690.9	790.1	890.1
9	1550	679.3	842.8	988.1	1114.1
10	1500	766.4	984.2	1156.6	1288.4
11	1480	839.7	1087.4	1259.6	1373.1
12	1520	903.8	1165.9	1325.8	1415.8
13	1500	965.4	1236.7	1384.0	1457.5
14	1490	1018.8	1289.4	1418.8	1474.5
15	1500	1066.0	1329.5	1440.2	1480.7
16		1109.4	1363.6	1458.1	1488.4



IMPACT OF DIFFERENT α

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- ◆ Chose **high values** of α when underlying average is likely to change
- ◆ Choose **low values** of α when underlying average is stable



Quarter

CHOOSING α

The objective is to obtain the most accurate forecast no matter the technique

We generally do this by selecting the model that gives us the lowest forecast error

= Actual demand - Forecast valueForecast error

$$= A_t - F_t$$

CHOOSING α

The objective is to obtain the most accurate forecast no matter the technique

We generally do this by selecting the model that gives us the lowest forecast error

= Actual demand - Forecast valueForecast error

= $A_t - F_t$

COMMON MEASURES OF ERROR

Mean Absolute Deviation (MAD)

$$MAD = \frac{\sum |\text{Actual} - \text{Forecast}|}{n}$$

Mean Squared Error (MSE)

$$MSE = \frac{\sum (\text{Forecast Errors})^2}{n}$$

COMMON MEASURES OF ERROR

Mean Absolute Percent Error (MAPE)

$$\text{MAPE} = \frac{\sum_{i=1}^n |\text{Actual}_i - \text{Forecast}_i| \% / \text{Actual}_i}{n}$$

EXAMPLE

- ◉ During the past 8 quarters, the Port of Baltimore has unloaded large quantities of grain from ships. The Port's **Operations Manager** wants to test the forecasting method *exponential smoothing* to see how well the this method works in predicting tonnage unloaded.
- ◉ He guesses that the forecast of grain unloaded in the first quarter was 175 tons.

COMPARISON OF FORECAST ERROR

Quarter	Actual Tonnage Unloaded	Rounded Forecast with $\alpha = .10$	Absolute Deviation for $\alpha = .10$	Rounded Forecast with $\alpha = .50$	Absolute Deviation for $\alpha = .50$
1	180	175	5.00	175	5.00
2	168	175.5	7.50	177.50	9.50
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.30
			<u>82.45</u>		<u>98.62</u>

COMPARISON OF FORECAST

$$MAD = \frac{\sum | \text{deviations} |}{n}$$

$$\text{For } \alpha = .10 \\ = 82.45/8 = 10.31$$

$$\text{For } \alpha = .50 \\ = 98.62/8 = 12.33$$

8	182	178.22	3.78
			<u>82.45</u>

Actual Tonnage Unloaded	Quarter
175	5.00
177.50	9.50
172.75	13.75
165.88	9.12
170.44	19.56
180.22	24.78
192.61	12.61
186.30	4.30
	<u>98.62</u>

COMPARISON OF FORECAST

$$MSE = \frac{\sum (\text{forecast errors})^2}{n}$$

For $\alpha = .10$

$$= 1,526.54/8 = 190.82$$

For $\alpha = .50$

$$= 1,561.91/8 = 195.24$$

8	182	178.22	3.78
			82.45
		MAD	10.31

Actual Tonnage Unloaded	Quarter
175	5.00
177.50	9.50
172.75	13.75
165.88	9.12
170.44	19.56
180.22	24.78
192.61	12.61
186.30	4.30
	98.62
	12.33

COMPARISON OF FORECAST

$$MAPE = \frac{\sum_{i=1}^n 100 | \text{deviation}_i | / \text{actual}_i}{n}$$

For $\alpha = .10$

$$= 44.75 / 8 = 5.59\%$$

For $\alpha = .50$

$$= 54.05 / 8 = 6.76\%$$

8	182	178.22	3.78	186.30
			82.45	98.62
		MAD	10.31	12.33
		MSE	190.82	195.24

Quarter
5.00
9.50
13.75
9.12
19.56
24.78
12.61
4.30

COMPARISON OF FORECAST ERROR

Quarter	Actual Tonnage Unloaded	Rounded Forecast with $\alpha = .10$	Absolute Deviation for $\alpha = .10$	Rounded Forecast with $\alpha = .50$	Absolute Deviation for $\alpha = .50$
1	180	175	5.00	175	5.00
2	168	175.5	7.50	177.50	9.50
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.30
			82.45		98.62
			MAD 10.31		12.33
			MSE 190.82		195.24
			MAPE 5.59%		6.76%

Week

10

Forecasts with seasonality and trend

Forecasts with seasonality and trend

- The earlier methods we have described so far give good results for constant time series
- but they need adjusting for other patterns
 - The easiest way of doing this is to divide the underlying pattern into separate components,
 - Then forecast each component separately
- Then we get the final forecast by recombining the separate components.

The observation is made up of three components

- Trend(T) is the long-term direction of a time series
- Seasonal factor(S) is the regular variation around the trend, which shows the variation in demand over a year
- Residual(R) is the random noise that we cannot properly explain.

Forecasts with seasonality and trend

- Adding these three components gives an 'additive model' which assumes that an observation, y

$$y = T + S + R$$

Forecasts with seasonality and trend

- the seasonal factor, S , is an amount we add to the trend to allow for the season.
 - If summer sales are 100 units higher than the trend, S has a value of 100;
 - if winter sales are 100 units lower than the trend, S has a value of -100.
- This additive model is easy to establish,
 - but it can underestimate variations, particularly when there is a significant trend.

Forecasts using multiplicative model

- it is better to use indices for seasonal variations, and put these into a 'multiplicative model

$$y = T \times S \times R$$

- If summer sales are 50% higher than the trend, S has a value of 1.5
- if winter sales are 50% lower than the trend, S has a value of 0.5
- As we do not know the random elements R, we cannot include this in forecasts

- Additive model: $F = T + S$
- Multiplicative model: $F = T \times S$

Week

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Exponential smoothing with Trend adjustment

EXPONENTIAL SMOOTHING WITH TREND ADJUSTMENT

When a trend is present, exponential smoothing must be modified to respond to trend

$$\begin{array}{l} \text{Forecast} \\ \text{including} \\ \text{trend} \end{array} (FIT_t) = \begin{array}{l} \text{Exponentially} \\ \text{smoothed} \\ \text{forecast} \end{array} (F_t) + \begin{array}{l} \text{Exponentially} \\ \text{smoothed} \\ \text{trend} \end{array} (T_t)$$

EXPONENTIAL SMOOTHING WITH
TREND ADJUSTMENT

$$F_t = \alpha(A_{t-1}) + (1 - \alpha)(F_{t-1} + T_{t-1})$$

$$T_t = \beta(F_t - F_{t-1}) + (1 - \beta)T_{t-1}$$

Step 1: Compute F_t

Step 2: Compute T_t

Step 3: Calculate the forecast $FIT_t = F_t + T_t$

EXAMPLE

- ◉ A Portland manufacturer wants to forecast the demand for a pollution-control equipment.
- ◉ Past data shows that there is an increasing trend.
- ◉ The company assumes the initial forecast for month 1 was 11 units and the trend over that period was 2 units.
- ◉ $\alpha = 0.2$ $\beta = 0.4$

EXPONENTIAL SMOOTHING WITH
TREND ADJUSTMENT EXAMPLE

Month(t)	Actual Demand (A_t)	Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend, FIT_t
1	12	11	2	13.00
2	17			
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10				

Exponential smoothing with Trend adjustment

EXPONENTIAL SMOOTHING WITH TREND ADJUSTMENT EXAMPLE

Month(t)	Actual Demand (A_t)	Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend, FIT_t
1	12	11	2	13.00
2	17			
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10				

Step 1: Forecast for Month 2

$$\begin{aligned}
 &= \alpha A_1 + (1 - \alpha)(F_1 + T_1)F_2 \\
 &= (.2)(12) + (1 - .2)(11 + 2)F_2 \\
 &= 2.4 + 10.4 = 12.8 \text{ units}
 \end{aligned}$$

EXPONENTIAL SMOOTHING WITH TREND ADJUSTMENT EXAMPLE

Month(<i>t</i>)	Actual Demand (<i>A_t</i>)	Smoothed Forecast, <i>F_t</i>	Smoothed Trend, <i>T_t</i>	Forecast Including Trend, <i>FIT_t</i>
1	12	11	2	13.00
2	17	12.80		
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10				

Step 2: Trend for Month 2

$$\begin{aligned}
 &= \beta(F_2 - F_1) + (1 - \beta)T_1T_2 \\
 &= (.4)(12.8 - 11) + (1 - .4)(2)T_2 \\
 &= .72 + 1.2 = 1.92 \text{ units}
 \end{aligned}$$

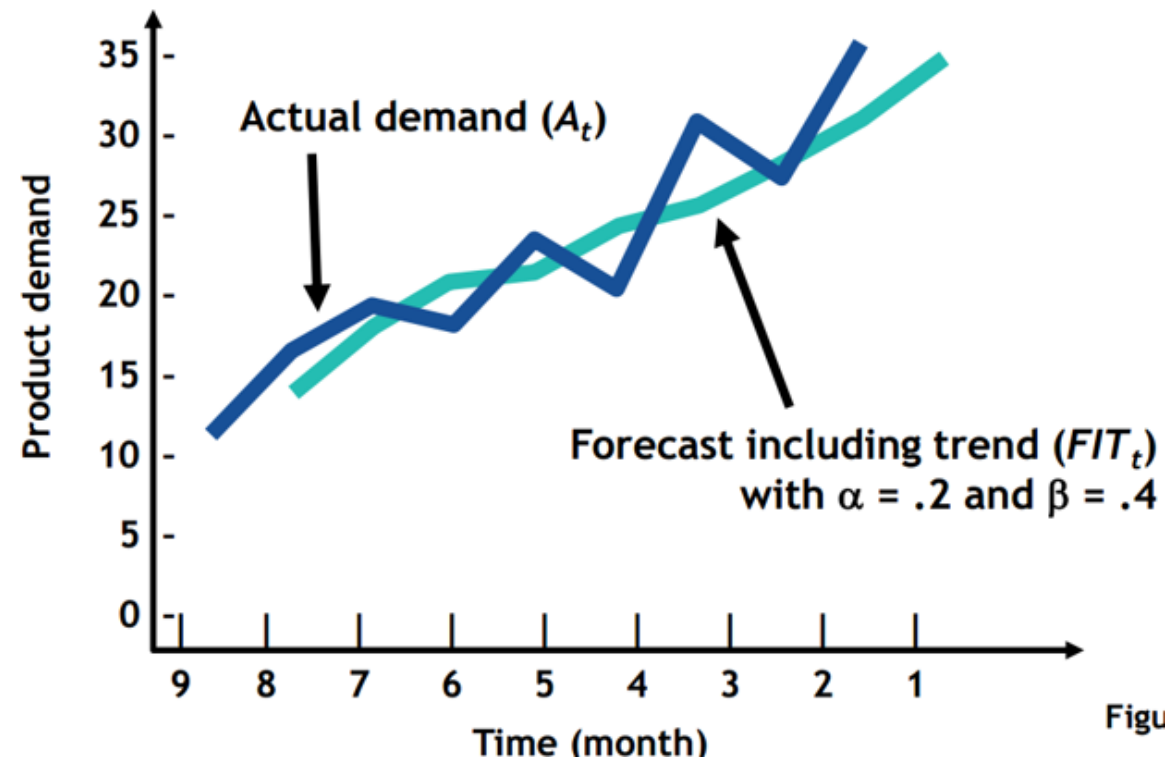
Table 4.1

EXPONENTIAL SMOOTHING WITH TREND ADJUSTMENT EXAMPLE

Month(t)	Actual Demand (A_t)	Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10				

Step 3: Calculate FIT for Month 2

$$\begin{aligned}
 &= F_2 + T_2 FIT_2 \\
 &= 12.8 + 1.92 FIT_2 \\
 &= 14.72 \text{ units}
 \end{aligned}$$

EXPONENTIAL SMOOTHING WITH
TREND ADJUSTMENT EXAMPLE

Week

10

Trend Projection

TREND PROJECTIONS

Fitting a trend line to historical data points to project into the medium to long-range

Linear trends can be found using the least squares technique

$$\hat{y} = a + bx$$

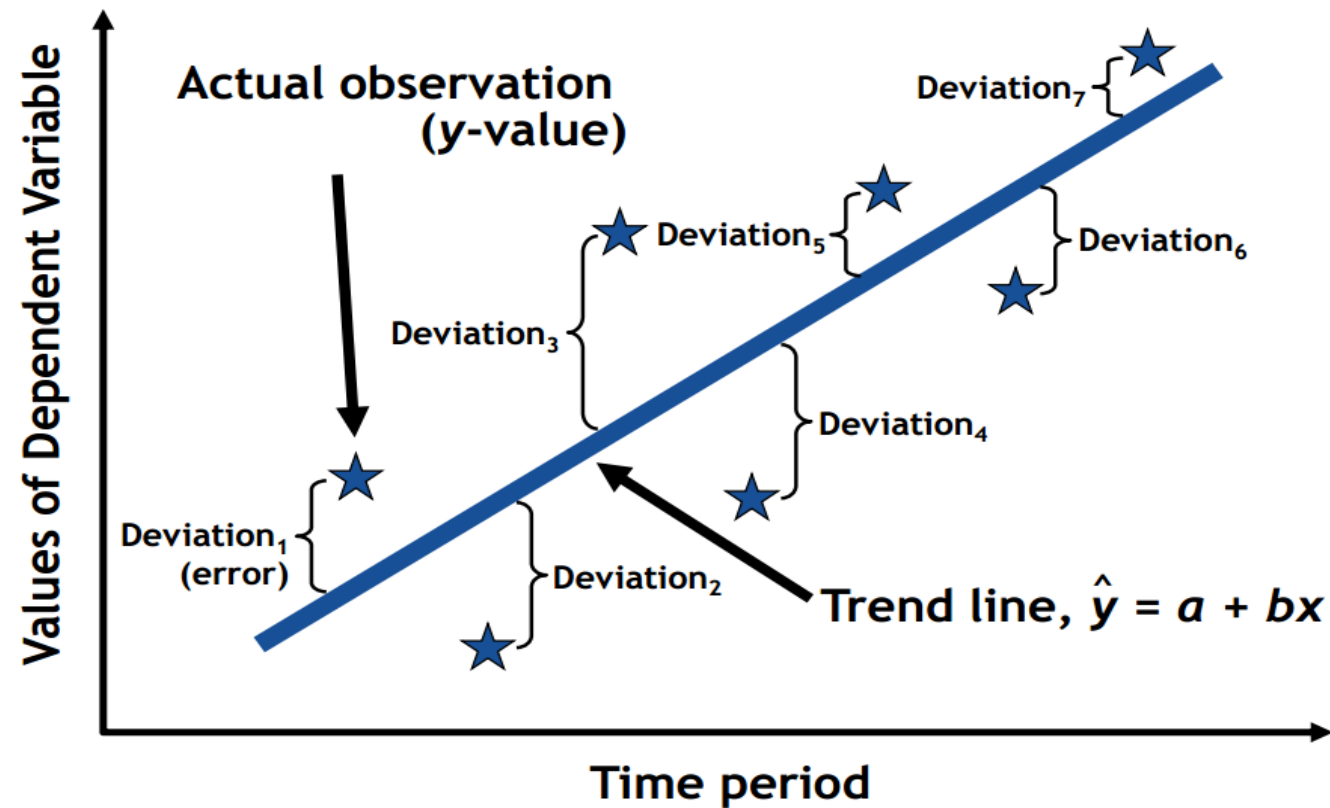
where $\hat{y}$ = computed value of the variable to be predicted
(dependent variable)

a = y-axis intercept

b = slope of the regression line

x = the independent variable

LEAST SQUARES METHOD



LEAST SQUARES METHOD

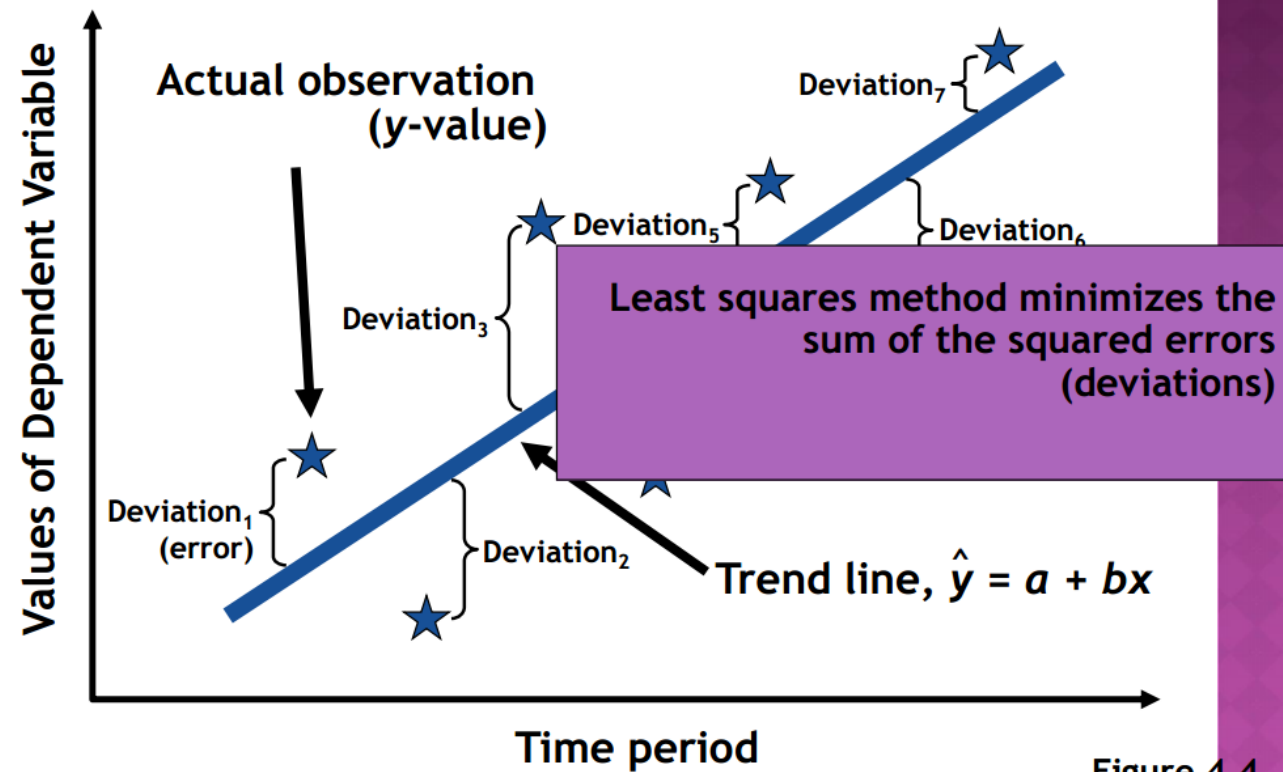


Figure 4.4

Trend Projection

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LEAST SQUARES METHOD

Equations to calculate the regression variables

$$\hat{y} = a + bx$$

$$b = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n\bar{x}^2}$$

$$a = \bar{y} - b\bar{x}$$

LEAST SQUARES METHOD

Equations to calculate the regression variables

$$\hat{y} = a + bx$$

$$b = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n\bar{x}^2}$$

$$a = \bar{y} - b\bar{x}$$

LEAST SQUARES EXAMPLE

Year	Time Period (x)	Electrical Power Demand (megawatt)	x^2	xy
2006	1	74	1	74
2007	2	79	4	158
2008	3	80	9	240
2009	4	90	16	360
2010	5	105	25	525
2011	6	142	36	852
2012	7	122	49	854
	<u> </u>	<u> </u>	<u> </u>	<u> </u>
	$\Sigma x = 28$	$\Sigma y = 692$	$\Sigma x^2 = 140$	$\Sigma xy = 3,063$
	$\bar{x} = 4$	$\bar{y} = 98.86$		

$$b = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n\bar{x}^2} = \frac{3,063 - (7)(4)(98.86)}{140 - (7)(4^2)} = 10.54$$

$$a = \bar{y} - b\bar{x} = 98.86 - 10.54(4) = 56.70$$

LEAST SQUARES EXAMPLE

Electrical Power xy	x ²	Time Demand	Period (x)	Year
2002	1	74	1	74
			4	158
			9	240
			16	360
			25	525
			36	852
			49	854
<u>Σxy = 3,063</u>	<u>Σx² = 140</u>	<u>Σy = 692</u>		
<u>Σx = 28</u>		<u>ȳ = 98.86</u>		
<u>x̄ = 4</u>				

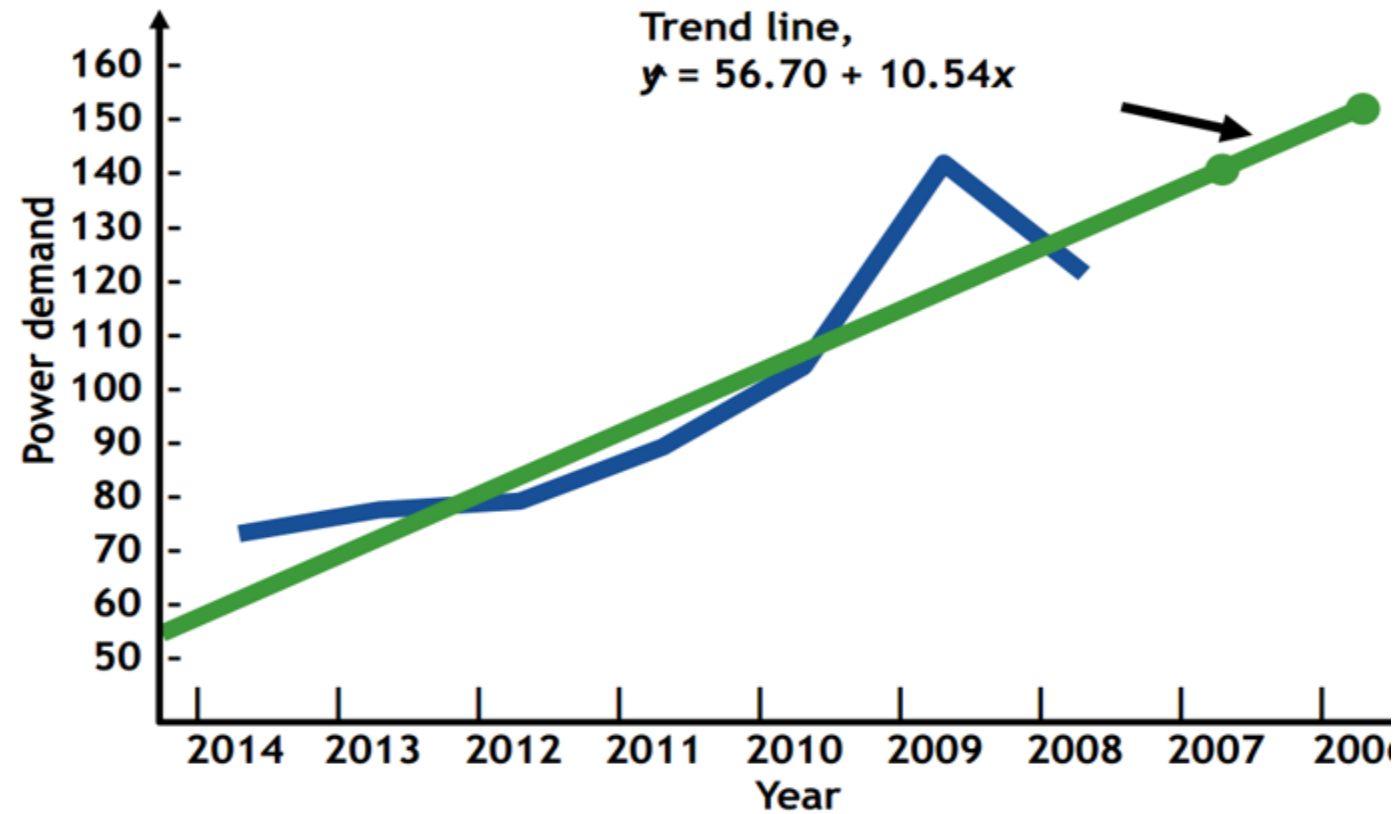
The trend line is

$$\hat{y} = 56.70 + 10.54x$$

$$b = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n\bar{x}^2} = \frac{3,063 - (7)(4)(98.86)}{140 - (7)(4^2)} = 10.54$$

$$a = \bar{y} - b\bar{x} = 98.86 - 10.54(4) = 56.70$$

LEAST SQUARES EXAMPLE





Thank You