

Elasticity of Demand and Supply

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Lecture 04



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MTH 112
Business Mathematics

At the end of this lecture, you should be able to:

- Calculate the coefficient of price elasticity of demand at a point on the demand function
- Calculate the coefficient of price elasticity of demand averaged along an arc on the demand function
- Calculate the coefficient of price elasticity of supply

Elasticity:

- The concept of elasticity, denoted by the symbol ϵ , as used in economics, is the ratio of the percentage change in the quantity demanded (or supplied) to a percentage change in an economic variable. e.g.: price, income, etc.
- Elasticity may be used to predict the responsiveness of demand (and supply) to changes in such economic variables.

Price elasticity of demand:

Price elasticity of demand measures the responsiveness (sensitivity) of quantity demanded to changes in the good's own price. It is also referred to as own-price elasticity. It is represented by the general elasticity formula.

$$\epsilon_d = \frac{\text{Percentage change in quantity demanded}}{\text{Percentage change in price}} = \frac{\% \Delta Q_d}{\% \Delta P} = \frac{\frac{\Delta Q}{Q} 100}{\frac{\Delta P}{P} 100}$$

which simplifies to:

$$\epsilon_d = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

Price elasticity of demand:

There are two approaches to measuring price elasticity: **Point elasticity** and **arc elasticity**.

- **Point elasticity** measures elasticity at a point on the demand function.
- **Arc elasticity** measures elasticity over an interval on the demand function

Point elasticity of demand

Given the linear demand function, $P = a - bQ$,
the formula for point elasticity of demand at any point (P_0, Q_0) is

$$\epsilon_d = \frac{\Delta Q}{\Delta P} \frac{P}{Q}$$

or

$$\epsilon_d = -\frac{1}{b} \frac{P_0}{Q_0} \quad \text{since} \quad \text{slope} = \frac{\Delta Q}{\Delta P} = -\frac{1}{b}$$

Example:

Given the demand function for computers as $P = 2400 - 0.5Q$.

A. Determine the coefficient of point elasticity of demand when

- (i) $P = 1800$,
- (ii) $P = 1200$
- (iii) $P = 600$.

Give a verbal description of each result.

Solution A (i)

At $P = 1800$ the quantity of computers demanded, Q , is calculated by substituting $P = 1800$ into the demand function:

$$P = 2400 - 0.5Q \rightarrow 1800 = 2400 - 0.5Q \rightarrow 0.5Q = 600 \rightarrow Q = 1200$$

The value of point elasticity of demand at $(P = 1800, Q = 1200)$ is calculated by substituting these values along with $b = 0.5$ into formula

$$\epsilon_d = -\frac{1}{b} \frac{P_0}{Q_0}$$

Solution A (i):

$$\begin{aligned}\epsilon_d &= -\frac{1}{b} \frac{P_0}{Q_0} \\ &= -\frac{1}{0.5} \frac{1800}{1200} = -\frac{1800}{600} = -\frac{3}{1} = -3\end{aligned}$$

The coefficient of point elasticity of demand is $\epsilon_d = -3$, which indicates that at the price $P = 1800$, a 1% increase (decrease) in price will cause a 3% decrease (increase) in the quantity of computers demanded.

Demand is elastic, $|\epsilon_d| > 1$.

Solution A (ii) (iii):

The calculations for price elasticity of demand for $P = 1200$ and $P = 600$ are carried out in the same way as (i) and are summarized below:

P_0	1800	1200	600
Calculate Q_0	$P_0 = 2400 - 0.5Q_0$ $1800 = 2400 - 0.5Q_0$ $0.5Q_0 = 600$ $Q_0 = 1200$	$1200 = 2400 - 0.5Q_0$ $Q_0 = 2400$	$600 = 2400 - 0.5Q_0$ $Q_0 = 3600$

Solution A (ii) (iii):

P_0	1800	1200	600
Calculate ϵ_d	$\epsilon_d = -\frac{1}{b} \frac{P_0}{Q_0}$ $= -\frac{1}{0.5} \frac{1800}{1200}$ $= -3$	$= -\frac{1}{0.5} \frac{1200}{2400}$ $= -\frac{1200}{1200} = -1$	$= -\frac{1}{0.5} \frac{600}{3600}$ $= -\frac{1}{3} \approx -0.33$
Demand	Demand is elastic $ \epsilon_d > 1$	Demand is unit elastic $ \epsilon_d = 1$	Demand is inelastic $ \epsilon_d < 1$

Notes:

$\epsilon_d = -1$ indicates that at $P = 1200$, a 1% increase (decrease) in price will cause a 1% decrease (increase) in the quantity of computers demanded.

$\epsilon_d = -0.33$ (correct to 2 decimal places) indicates that at $P = 600$, a 1 % increase (decrease) in price will cause a 0.33% decrease (increase) in the quantity of computers demanded.

Example:(Continued)

Given the demand function for computers as $P = 2400 - 0.5Q$.

B. If the price of computers increases by 12%, calculate the percentage change in the quantity demanded at $P = 1800$, $P = 1200$ and $P = 600$.

(i) **First use the definition of elasticity**

$$\epsilon_d = \frac{\text{Percentage change in quantity demanded}}{\text{Percentage change in price}} = \frac{\Delta Q_d(\%)}{\Delta P(\%)}$$

(ii) **Then calculate the exact percentage changes and compare them with the answers in (i).**

Solution B (i):

The definition

$$\epsilon_d = \frac{\text{Percentage change in quantity demanded}}{\text{Percentage change in price}} = \frac{\Delta Q_d(\%) }{\Delta P(\%)} \quad (2.13)$$

may be re arranged as

$$\Delta Q(\%) = \Delta P(\%) \times \epsilon_d$$

Solution B (i):

- At $P = 1800$ we have calculated that $\epsilon_d = -3$. To calculate the percentage change in Q , substitute $\epsilon_d = -3$ and $\Delta P = 12\%$ into equation (2.13):

$$\Delta Q = (12\%) (-3) = -36\%$$

The quantity demanded decreases by 36%, a larger percentage decrease than the 12% price increase. Demand is strongly responsive to price change and this is described as **elastic demand**.

Solution B (i):

- At $P = 1200$ we have calculated that $\epsilon_d = -1$. To calculate the percentage change in Q , substitute $\epsilon_d = -1$ and $\Delta P = 12\%$ into equation (2.13):

$$\Delta Q = (12\%) (-1) = -12\%$$

The quantity demanded decreases by 12%, a larger percentage decrease than the 12% price increase. This is described as **unit elastic demand**.

Solution B (i):

- At $P = 600$ we have calculated that $\epsilon_d = -0.33$. To calculate the percentage change in Q , substitute $\epsilon_d = -0.33$ and $\Delta P = 12\%$ into equation (2.13):

$$\Delta Q = (12\%) (-0.33) = -3.96\%$$

The quantity demanded decreases by 3.96%, a smaller percentage decrease than the 12% price increase. So, demand is weakly responsive to price, hence it is described as **inelastic demand**.

Solution B (ii):

The calculation of the exact percentage change in Q requires basic arithmetic. Start by calculating the price which results from an increase of 12% on the initial price

$$\text{Increase } P = 1800 \text{ by } 12 \% \rightarrow P_{\text{new}} = \frac{112}{100}P = 1.12(1800) = 2016$$

Solution B (ii):

Then calculate the corresponding values of Q from the equation of the demand function.

When $P = 1800$, $Q = 1200$ and when $P = 2016$, $Q = 768$.

Initial P, Q	New P, Q	Percentage change in Q by direct calculation	ϵ_d	$\Delta Q(\%) = \Delta P(\%) \epsilon_d$ By formula (2.13)
1800, 1200	2016, 768	$\frac{768 - 1200}{1200} \times 100 = -36\%$	-3	$\Delta Q = -3 \times 12 = -36\%$
1200, 2400	1344, 2122	$\frac{2112 - 2400}{2400} \times 100 = -12\%$	-1	$\Delta Q = -1 \times 12 = -12\%$
600, 3600	672, 3456	$\frac{3456 - 3600}{3600} \times 100 = -4\%$	$-\frac{1}{3}$	$\Delta Q = -\frac{1}{3} \times 12 = -4\%$

Comment: The exact value of elasticity, $-1/3$, not the rounded value, -0.33 , was used for accurate results when $P = 600$. The rounded value of elasticity, -0.33 , would give approximately the same answer, -3.96% , as obtained by direct calculation. At prices $P = 1800$ and $P = 1200$, no rounding of ed was required; the percentage change in demand was exactly the same, whether calculated directly or calculated by equation (2.13).

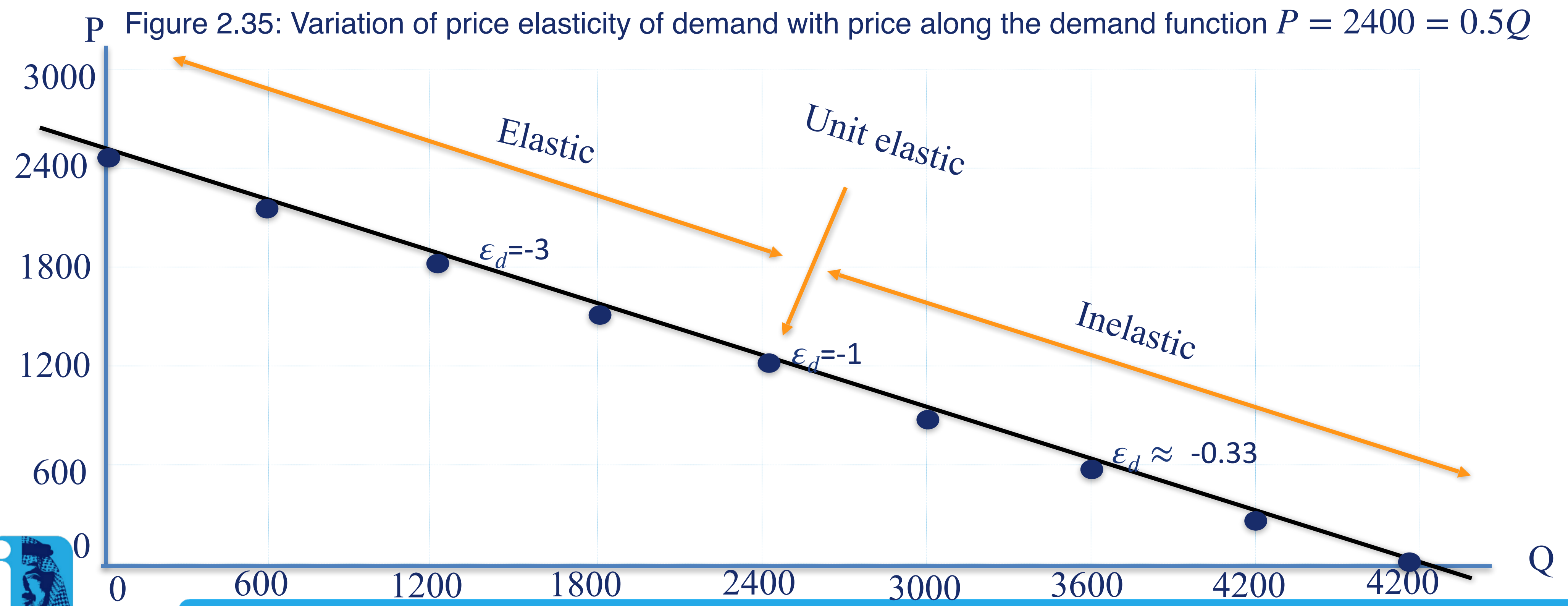
In general, the computer firm is not only interested in the percentage change in the price of computers, but also in the percentage change in demand which results from the price changes.

Example:(Continued)

Given the demand function for computers as $P = 2400 - 0.5Q$.

C. Graph the demand function, indicating where demand is elastic, unit elastic and inelastic.

(c) The graph of the demand function is given in Figure 2.35.



Coefficient of price elasticity of demand

There are three categories of price elasticity: **elastic**, **inelastic** and **unit elastic**.

- **Elastic demand** $-\infty < \epsilon_d < -1$: demand is strongly responsive to changes in price. that is, the percentage change in demand is greater than the percentage change in price. When $\epsilon_d \rightarrow -\infty$, demand for the good is perfectly elastic. The demand function is horizontal with slope = 0; that is,

$$\frac{\Delta P}{\Delta Q} = \frac{0}{\Delta Q} \rightarrow \frac{\Delta Q}{\Delta P} = \frac{\Delta Q}{0} = \infty \quad (P \neq 0)$$

Note: when $\epsilon_d < -1$, $|\epsilon_d| > 1$

Coefficient of price elasticity of demand:

- **Inelastic demand** $-1 < \epsilon_d < 0$: demand is weakly responsive to changes in price. that is, the percentage change in demand is less than the percentage change in price. When $\epsilon_d = 0$, demand for the good is perfectly inelastic. The demand function is vertical with slope $=\infty$; that is,

$$\frac{\Delta P}{\Delta Q} = \frac{\infty}{1} \rightarrow \frac{\Delta Q}{\Delta P} = \frac{1}{\infty} = 0 \quad (Q \neq 0)$$

Note: when $|\epsilon_d| < 1$

Coefficient of price elasticity of demand

- **Unit elastic demand** $\epsilon_d = -1$: the percentage change in demand is equal to -1. The numerical scale for the coefficient of price elasticity of demand is shown in Figure 2.36.

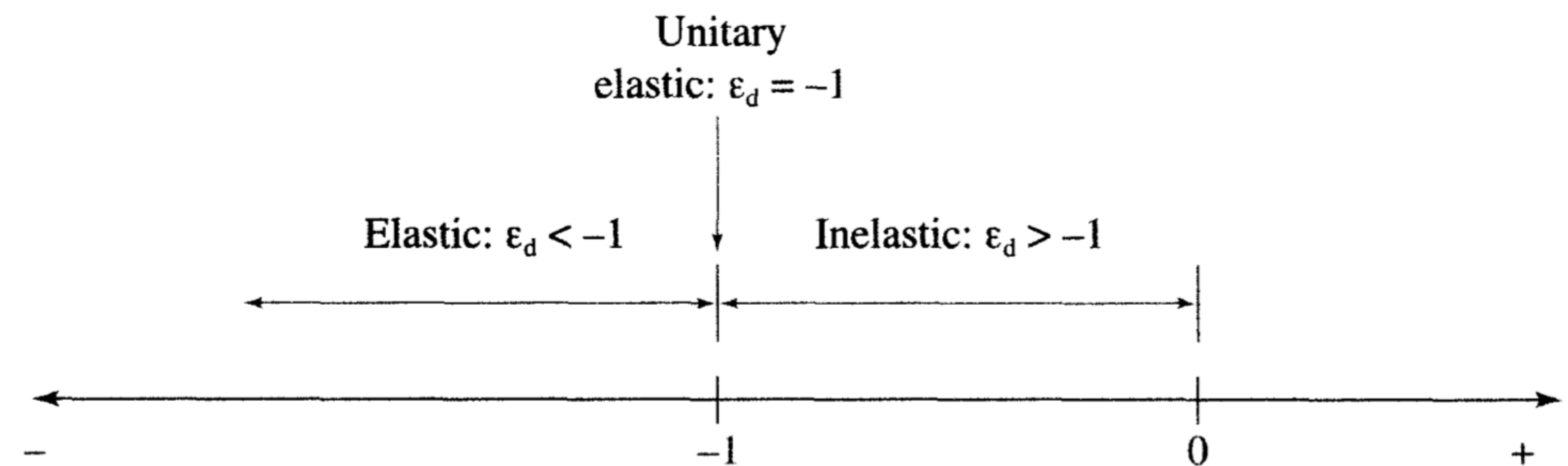


Figure 2.36 Numerical scale for the coefficient of price elasticity of demand

Arc price elasticity of demand measures the elasticity of demand over an interval on the demand function. Instead of using the price and quantity at a point as in point elasticity, arc elasticity uses the average of the prices and quantities at the beginning and end of the stated interval:

$$\epsilon_d = \frac{\Delta Q}{\Delta P} \cdot \frac{\frac{1}{2}[P_1 + P_2]}{\frac{1}{2}[Q_1 + Q_2]} = \frac{\Delta Q}{\Delta P} \cdot \frac{P_1 + P_2}{Q_1 + Q_2}$$

This formula is often referred to as the midpoints elasticity formula or arc-price elasticity formula.

Note: For linear functions, the arc-price elasticity of demand formula can also be written as

$$\epsilon_d = \frac{\Delta Q}{\Delta P} \cdot \frac{P_1 + P_2}{Q_1 + Q_2} = -\frac{1}{b} \cdot \frac{P_1 + P_2}{Q_1 + Q_2} \quad \text{since} \quad \frac{\Delta Q}{\Delta P} = -\frac{1}{b}$$

The same ideas as developed with respect to the price elasticity of demand can be applied to the analysis of the price elasticity of supply at a point on the supply function or averaged along an arc on the supply function. As the supply function is positively sloped, the coefficient of price elasticity of supply will be positive.

The coefficient of price elasticity of supply is given by the formula

$$\epsilon_s = \frac{\text{Percentage change in quantity supplied}}{\text{Percentage change in price}} = \frac{\% \Delta Q_s}{\% \Delta P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

Given the supply function, $P = c + dQ$, slope $= d = \Delta P / \Delta Q$. Then, $1/d = \Delta Q / \Delta P$; hence, the point elasticity of supply formula is

$$\epsilon_s = \frac{1}{d} \frac{P}{Q}$$

Note:

$\epsilon_s > 1$: supply is **elastic**;

$\epsilon_s < 1$: supply is **inelastic**;

$\epsilon_s = 1$: supply is **unit elastic**.

Example:

Given the supply function, $P = 20 + 0.5Q$,

- a) Calculate the arc price elasticity of supply when price increases from £40 to £60.
- b) Calculate the percentage change in quantity supplied in response to a price increase of 10% when $P=£40$.
 - (i) By using the definition of elasticity in equation (2.16).
 - (ii) Exactly by using ordinary arithmetic.

Comment and explain the difference between (i) and (ii)

Solution:

$$P = 20 + 0.5Q$$

$$\varepsilon_s = \frac{\Delta Q}{\Delta P} \cdot \frac{P_1 + P_2}{Q_1 + Q_2}$$

$$P_1 = £40, \quad P_2 = £60$$

$$40 = 20 + \frac{1}{2}Q$$

$$Q_1 = (40 - 20) \times 2$$

$$= \boxed{40}$$

$$\frac{\Delta Q}{\Delta P} = \frac{1}{\frac{1}{2}} = 2$$

$$60 = 20 + \frac{1}{2}Q$$

$$Q_2 = (60 - 20) \times 2$$

$$= \boxed{80}$$

Solution:

$$\begin{aligned}\epsilon_s &= \frac{\Delta Q}{\Delta P} \cdot \frac{P_1 + P_2}{Q_1 + Q_2} \\ &= (2) \cdot \frac{40 + 60}{40 + 80} \\ &= (2) \cdot \frac{100}{120} \approx 1.666\end{aligned}$$

$$\begin{aligned}\frac{\Delta Q}{\Delta P} &= 2 \\ P_1 &= 40 \\ P_2 &= 60 \\ Q_1 &= 40 \\ Q_2 &= 80\end{aligned}$$

Solution:

$$\varepsilon_s \approx 1.666$$

$\varepsilon_s > 1 \Rightarrow$ the price elasticity
of supply is
elastic.

Example:

Given the supply function, $P = 20 + 0.5Q$,

- a) Calculate the arc price elasticity of supply when price increases from £40 to £60.
- b) Calculate the percentage change in quantity supplied in response to a price increase of 10% when $P = £40$.

(i) By using the definition of elasticity in equation (2.16).

(ii) Exactly by using ordinary arithmetic.

Comment and explain the difference between (i) and (ii)

Solution:

$$P = £40$$

$$\Delta P\% = 10\%$$

$$P = 40$$

$$(i) \quad \frac{(\Delta Q)\%}{?} = (\Delta P)\% (\epsilon_s)$$

$$\begin{aligned}\epsilon_s &= \frac{\Delta Q}{\Delta P} \cdot \frac{P_0}{Q_0} \\ &= (2) \left(\frac{40}{40} \right) = 2\end{aligned}$$

$$\begin{aligned}P &= 20 + \frac{1}{2}Q \\ 40 &= 20 + \frac{1}{2}Q \\ Q &= 40\end{aligned}$$

Solution:

$$\begin{aligned}(\Delta Q)\% &= (\Delta P)\% (\varepsilon_s) \\ &= (10)\% (2) \\ &= 20\%\end{aligned}$$

The quantity supplied increases by 20%,
when the price increases by 10%.

Example:

Given the supply function, $P = 20 + 0.5Q$,

- a) Calculate the arc price elasticity of supply when price increases from £40 to £60.
- b) Calculate the percentage change in quantity supplied in response to a price increase of 10% when $P = £40$.
 - (i) By using the definition of elasticity in equation (2.16).
 - (ii) Exactly by using ordinary arithmetic.

Comment and explain the difference between (i) and (ii)

Solution:

(ii)

old

$$P_{old} = 40$$

$$Q_{old} = 40$$

new

$$P_{new} = 44$$

$$Q_{new} = 48$$



Solution:

$$P = 20 + \frac{1}{2}Q$$

$$44 = 20 + \frac{1}{2}Q$$

$$Q = (24) (2)$$

$$Q = 48$$

new



Solution:

$$\begin{aligned}\frac{Q_{\text{new}} - Q_{\text{old}}}{Q_{\text{old}}} \times 100 &= \frac{48 - 40}{40} \times 100 \\ &= \frac{8}{40} \times 100 \\ &= \frac{80}{4} = 20\%\end{aligned}$$

Thank you ..

