

# Operations Research

**Prepared by Dr. Mohamed Eassa**

**IT 262**  
**Operations Research**

## Lecture 5



# Linear Programming

## Simplex Method Part 2

### Lecture 5

IT 262

Operations Research

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Chapter

3

## *Lecture 5 Topics*

- ❖ **Simplex Method**
- ❖ **Minimization Problem**
- ❖ **Optimal Solution**

## Simplex Method

**Simplex Method** Can solve for more than two decision variables.

## Simplex Method

### *The advantage of Simplex Method*

**1- No graph**

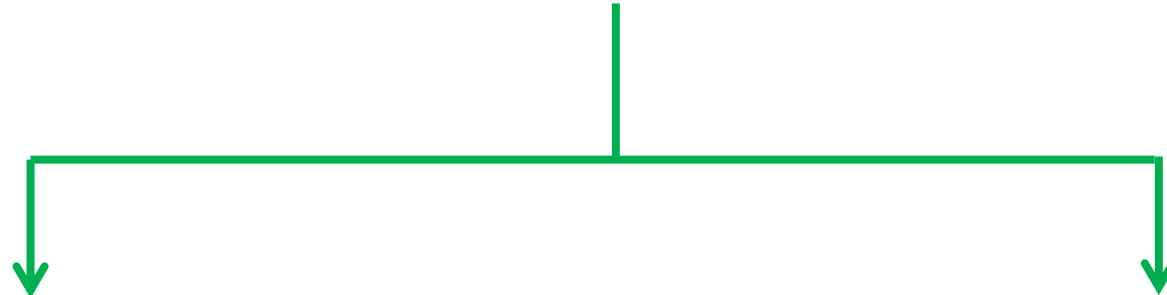
**2- can solve for more than 2 variables**

# Simplex Method

## *Simplex Method*

Maximization Problem

Minimization Problem



## Simplex Method

### *The First : Minimization Problem*

#### *Problem 1:*

$$\text{Minimize } Z = 16X_1 + 45X_2$$

*Subject to:*

$$2X_1 + 5X_2 \geq 50$$

$$X_1 + 3X_2 \geq 27$$

$$X_1, X_2 \geq 0$$

## Simplex Method: *Problem 1*

Convert the problem from minimization problem to maximization problem

Minimize  $Z = 16X_1 + 45X_2$

Subject to:

$$\begin{aligned} 2X_1 + 5X_2 &\geq 50 \\ X_1 + 3X_2 &\geq 27 \\ X_1, X_2 &\geq 0 \end{aligned}$$

Transformation  
Matrix



$$A = \begin{array}{ccc|c} X_1 & X_2 & \text{RHS} & \\ \hline 2 & 5 & 50 & R_1 \\ 1 & 3 & 27 & R_2 \\ 16 & 45 & Z & \text{Obj.} \end{array}$$

Min  $\rightarrow$  Max

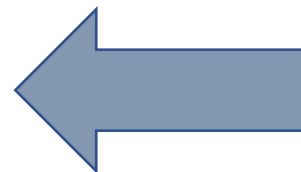


Maximize  $P = 50X_1' + 27X_2'$

Subject to.

$$\begin{aligned} 2X_1' + X_2' &\leq 16 \\ 5X_1' + 3X_2' &\leq 45 \\ X_1', X_2' &\geq 0 \end{aligned}$$

Max



$$A^T = \begin{array}{ccc|c} X_1' & X_2' & \text{RHS} & \\ \hline 2 & 1 & 16 & R_1 \\ 5 & 3 & 45 & R_2 \\ 50 & 27 & P & \text{Obj.} \end{array}$$

## Simplex Method: *Problem 1*

$$\text{Max } P = 50X_1' + 27X_2'$$

$$2X_1' + X_2' \leq 16$$

$$5X_1' + 3X_2' \leq 45$$

$$X_1', X_2' \geq 0$$

*Steps in the Simplex Method :*

1- **step 1:** Rewrite the problem in standard form.

$$P - 50X_1' - 27X_2' = 0 \quad (\text{obj.})$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

$$X_1, X_2 \geq 0$$

*Maximize only*

*Objective function → with R.H.S = 0*

*Constraints:*

*(For  $\leq$ ) → equation by  $(+s_i)$*

*(For  $\geq$ ) → equation by  $(-s_i)$*

## Simplex Method: *Problem 1*

2- **step 2:** *Construct the table and search for the Pivot (Pivot column  $\cap$  Pivot Row)*

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | <i>Const.</i> |  |
|-------|--------|--------|-------|-------|---------------|--|
| $R_1$ | 2      | 1      | 1     | 0     | 16            |  |

$$P - 50X_1' - 27 X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

## Simplex Method: *Problem 1*

**2- *step 2*: Construct the table and search for the Pivot (Pivot column  $\cap$  Pivot Row)**

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. |  |
|-------|--------|--------|-------|-------|--------|--|
| $R_1$ | 2      | 1      | 1     | 0     | 16     |  |
| $R_2$ | 5      | 3      | 0     | 1     | 45     |  |

$$P - 50X_1' - 27 X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

## Simplex Method: *Problem 1*

2- **step 2:** *Construct the table and search for the Pivot (Pivot column  $\cap$  Pivot Row)*

|            | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. |  |
|------------|--------|--------|-------|-------|--------|--|
| $R_1$      | 2      | 1      | 1     | 0     | 16     |  |
| $R_2$      | 5      | 3      | 0     | 1     | 45     |  |
| <i>obj</i> | -50    | -27    | 0     | 0     | 0      |  |

$$P - 50X_1' - 27 X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

## Simplex Method: *Problem 1*

**2- *step 2*: Construct the table and search for the Pivot (Pivot column  $\cap$  Pivot Row)**

*For select the pivot column get the lower in objective function*

|            | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. |  |
|------------|--------|--------|-------|-------|--------|--|
| $R_1$      | 2      | 1      | 1     | 0     | 16     |  |
| $R_2$      | 5      | 3      | 0     | 1     | 45     |  |
| <i>obj</i> | -50    | -27    | 0     | 0     | 0      |  |

$$P - 50X_1' - 27 X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

## Simplex Method: *Problem 1*

**2- step 2:** *Construct the table and search for the Pivot (Pivot column  $\cap$  Pivot Row)*

*For select the pivot column get the lower in objective function*

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. |  |
|-------|--------|--------|-------|-------|--------|--|
| $R_1$ | 2      | 1      | 1     | 0     | 16     |  |
| $R_2$ | 5      | 3      | 0     | 1     | 45     |  |
| obj   | -50    | -27    | 0     | 0     | 0      |  |

$$P - 50X_1' - 27 X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

## Simplex Method: *Problem 1*

**2- step 2:** Construct the table and search for the Pivot (Pivot column  $\cap$  Pivot Row)

To select the Pivot row perform the ratio test and get the low value

*Ratio test = constant / value in pivot column for  $R_1$  &  $R_2$*

$$P - 50X_1' - 27 X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test   |
|-------|--------|--------|-------|-------|--------|--------------|
| $R_1$ | 2      | 1      | 1     | 0     | 16     | $16/2 = 8$   |
| $R_2$ | 5      | 3      | 0     | 1     | 45     | $45 / 5 = 9$ |
| obj   | -50    | -27    | 0     | 0     | 0      |              |

## Simplex Method: *Problem 1*

2- **step 2:** Construct the table and search for the Pivot (Pivot column  $\cap$  Pivot Row)

To select the Pivot row perform the ratio test and get the low value

Ratio test = constant / value in pivot column for  $R_1$  &  $R_2$

$$P - 50X_1' - 27X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test |
|-------|--------|--------|-------|-------|--------|------------|
| $R_1$ | 2      | 1      | 1     | 0     | 16     | $16/2 = 8$ |
| $R_2$ | 5      | 3      | 0     | 1     | 45     | $45/5 = 9$ |
| obj   | -50    | -27    | 0     | 0     | 0      |            |

**Pivot**

## Simplex Method: *Problem 1*

### 3- *step 3*: The Pivot must be 1

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test   |
|-------|--------|--------|-------|-------|--------|--------------|
| $R_1$ | 2      | 1      | 1     | 0     | 16     | $16/2 = 8$   |
| $R_2$ | 5      | 3      | 0     | 1     | 45     | $45 / 5 = 9$ |
| $obj$ | -50    | -27    | 0     | 0     | 0      |              |

$$P - 50X_1' - 27 X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

|                                       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test |
|---------------------------------------|--------|--------|-------|-------|--------|------------|
| $(R_1)_{New} = \frac{(R_1)_{old}}{2}$ | 1      | $1/2$  | $1/2$ | 0     | 8      |            |

## Simplex Method: *Problem 1*

4- **step 4:** use row operation such that in the Pivot column the other element must be 0

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test |
|-------|--------|--------|-------|-------|--------|------------|
| $R_1$ | 2      | 1      | 1     | 0     | 16     | $16/2 = 8$ |
| $R_2$ | 5      | 3      | 0     | 1     | 45     | $45/5 = 9$ |
| obj   | -50    | -27    | 0     | 0     | 0      |            |

$$P - 50X_1' - 27X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

Step 4: must be 0 with the new Row

|                                             | $X_1'$ | $X_2'$ | $S_1$  | $S_2$ | Const. | Ratio Test |
|---------------------------------------------|--------|--------|--------|-------|--------|------------|
| $(R_1)_{New} = \frac{(R_1)_{old}}{2}$       | 1      | $1/2$  | $1/2$  | 0     | 8      |            |
| $(R_2)_{New} = -5(R_1)_{New} + (R_2)_{old}$ | 0      | $1/2$  | $-5/2$ | 1     | 5      |            |

## Simplex Method: *Problem 1*

**4- step 4:** use row operation such that in the Pivot column the other element must be 0

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test   |
|-------|--------|--------|-------|-------|--------|--------------|
| $R_1$ | 2      | 1      | 1     | 0     | 16     | $16/2 = 8$   |
| $R_2$ | 5      | 3      | 0     | 1     | 45     | $45 / 5 = 9$ |
| obj   | -50    | -27    | 0     | 0     | 0      |              |

$$P - 50X_1' - 27 X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

|                                             | $X_1'$ | $X_2'$ | $S_1$  | $S_2$ | Const. | Ratio Test |
|---------------------------------------------|--------|--------|--------|-------|--------|------------|
| $(R_1)_{New} = \frac{(R_1)_{old}}{2}$       | 1      | $1/2$  | $1/2$  | 0     | 8      |            |
| $(R_2)_{New} = -5(R_1)_{New} + (R_2)_{old}$ | 0      | $1/2$  | $-5/2$ | 1     | 5      |            |
| $(obj)_{New} = 50(R_1)_{New} + (obj)_{old}$ | 0      | -2     | 25     | 0     | 400    |            |

## Simplex Method: *Problem 1*

4- **step 4:** use row operation such that in the Pivot column the other element must be 0

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test   |
|-------|--------|--------|-------|-------|--------|--------------|
| $R_1$ | 2      | 1      | 1     | 0     | 16     | $16/2 = 8$   |
| $R_2$ | 5      | 3      | 0     | 1     | 45     | $45 / 5 = 9$ |
| $obj$ | -50    | -27    | 0     | 0     | 0      |              |

$$P - 50X_1' - 27 X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

|               | $X_1'$ | $X_2'$ | $S_1$  | $S_2$ | Const. | Ratio Test |
|---------------|--------|--------|--------|-------|--------|------------|
| $(R_1)_{New}$ | 1      | $1/2$  | $1/2$  | 0     | 8      |            |
| $(R_2)_{New}$ | 0      | $1/2$  | $-5/2$ | 1     | 5      |            |
| $(obj)_{New}$ | 0      | -2     | 25     | 0     | 400    |            |

## Simplex Method: *Problem 1*

5- **step 5:** Repeat step 2 & 3 & 4 until there is no negative numbers in the obj. row.

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test   |
|-------|--------|--------|-------|-------|--------|--------------|
| $R_1$ | 2      | 1      | 1     | 0     | 16     | $16/2 = 8$   |
| $R_2$ | 5      | 3      | 0     | 1     | 45     | $45 / 5 = 9$ |
| obj   | -50    | -27    | 0     | 0     | 0      |              |

$$P - 50X_1' - 27 X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

|               | $X_1'$ | $X_2'$ | $S_1$  | $S_2$ | Const. | Ratio Test |
|---------------|--------|--------|--------|-------|--------|------------|
| $(R_1)_{New}$ | 1      | $1/2$  | $1/2$  | 0     | 8      |            |
| $(R_2)_{New}$ | 0      | $1/2$  | $-5/2$ | 1     | 5      |            |
| $(obj)_{New}$ | 0      | -2     | 25     | 0     | 400    |            |

**Repeat Step 2: Get the lower value in objective function to get the Pivot column**



## Simplex Method: *Problem 1*

5- **step 5:** Repeat step 2 & 3 & 4 until there is no negative numbers in the obj. row.

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test   |
|-------|--------|--------|-------|-------|--------|--------------|
| $R_1$ | 2      | 1      | 1     | 0     | 16     | $16/2 = 8$   |
| $R_2$ | 5      | 3      | 0     | 1     | 45     | $45 / 5 = 9$ |
| obj   | -50    | -27    | 0     | 0     | 0      |              |

$$P - 50X_1' - 27 X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

|               | $X_1'$ | $X_2'$ | $S_1$  | $S_2$ | Const. | Ratio Test |
|---------------|--------|--------|--------|-------|--------|------------|
| $(R_1)_{New}$ | 1      | $1/2$  | $1/2$  | 0     | 8      |            |
| $(R_2)_{New}$ | 0      | $1/2$  | $-5/2$ | 1     | 5      |            |
| $(obj)_{New}$ | 0      | -2     | 25     | 0     | 400    |            |

**Repeat Step 2: Get the lower value in objective function to get the Pivot column**

## Simplex Method: *Problem 1*

5- **step 5:** Repeat step 2 & 3 & 4 until there is no negative numbers in the obj. row.

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test   |
|-------|--------|--------|-------|-------|--------|--------------|
| $R_1$ | 2      | 1      | 1     | 0     | 16     | $16/2 = 8$   |
| $R_2$ | 5      | 3      | 0     | 1     | 45     | $45 / 5 = 9$ |
| obj   | -50    | -27    | 0     | 0     | 0      |              |

$$P - 50X_1' - 27 X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

|               | $X_1'$ | $X_2'$ | $S_1$  | $S_2$ | Const. | Ratio Test       |
|---------------|--------|--------|--------|-------|--------|------------------|
| $(R_1)_{New}$ | 1      | $1/2$  | $1/2$  | 0     | 8      | $8 / (1/2) = 16$ |
| $(R_2)_{New}$ | 0      | $1/2$  | $-5/2$ | 1     | 5      | $5 / (1/2) = 10$ |
| $(obj)_{New}$ | 0      | -2     | 25     | 0     | 400    |                  |

**Step 2: Calculate the low value in the ratio test to get the Pivot Row**

## Simplex Method: *Problem 1*

5- **step 5:** Repeat step 2 & 3 & 4 until there is no negative numbers in the obj. row.

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test   |
|-------|--------|--------|-------|-------|--------|--------------|
| $R_1$ | 2      | 1      | 1     | 0     | 16     | $16/2 = 8$   |
| $R_2$ | 5      | 3      | 0     | 1     | 45     | $45 / 5 = 9$ |
| obj   | -50    | -27    | 0     | 0     | 0      |              |

$$P - 50X_1' - 27 X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

|         | $X_1'$ | $X_2'$ | $S_1$  | $S_2$ | Const. | Ratio Test       |
|---------|--------|--------|--------|-------|--------|------------------|
| $(R_1)$ | 1      | $1/2$  | $1/2$  | 0     | 8      | $8 / (1/2) = 16$ |
| $(R_2)$ | 0      | $1/2$  | $-5/2$ | 1     | 5      | $5 / (1/2) = 10$ |
| (obj)   | 0      | -2     | 25     | 0     | 400    |                  |

**Step 3: The Pivot must to be 1**

## Simplex Method: *Problem 1*

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test   |
|-------|--------|--------|-------|-------|--------|--------------|
| $R_1$ | 2      | 1      | 1     | 0     | 16     | $16/2 = 8$   |
| $R_2$ | 5      | 3      | 0     | 1     | 45     | $45 / 5 = 9$ |
| obj   | -50    | -27    | 0     | 0     | 0      |              |

|       | $X_1'$ | $X_2'$ | $S_1$  | $S_2$ | Const. | Ratio Test       |
|-------|--------|--------|--------|-------|--------|------------------|
| $R_1$ | 1      | $1/2$  | $1/2$  | 0     | 8      | $8 / (1/2) = 16$ |
| $R_2$ | 0      | $1/2$  | $-5/2$ | 1     | 5      | $5 / (1/2) = 10$ |
| obj   | 0      | -2     | 25     | 0     | 400    |                  |

|                                           | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test |
|-------------------------------------------|--------|--------|-------|-------|--------|------------|
| $(R_2)_{New} = \frac{(R_2)_{old}}{(1/2)}$ | 0      | 1      | -5    | 2     | 10     |            |

$$P - 50X_1' - 27 X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

**Step 3: The Pivot must to be 1**

## Simplex Method: *Problem 1*

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test   |
|-------|--------|--------|-------|-------|--------|--------------|
| $R_1$ | 2      | 1      | 1     | 0     | 16     | $16/2 = 8$   |
| $R_2$ | 5      | 3      | 0     | 1     | 45     | $45 / 5 = 9$ |
| obj   | -50    | -27    | 0     | 0     | 0      |              |

|       | $X_1'$ | $X_2'$ | $S_1$  | $S_2$ | Const. | Ratio Test       |
|-------|--------|--------|--------|-------|--------|------------------|
| $R_1$ | 1      | $1/2$  | $1/2$  | 0     | 8      | $8 / (1/2) = 16$ |
| $R_2$ | 0      | $1/2$  | $-5/2$ | 1     | 5      | $5 / (1/2) = 10$ |
| obj   | 0      | -2     | 25     | 0     | 400    |                  |

|                                                | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test |
|------------------------------------------------|--------|--------|-------|-------|--------|------------|
| $(R_1)_{New} = -1/2 (R_2)_{New} + (R_1)_{old}$ | 1      | 0      | 3     | -1    | 3      |            |
| $(R_2)_{New} = \frac{(R_2)_{old}}{(1/2)}$      | 0      | 1      | -5    | 2     | 10     |            |
| $(obj)_{New} = 2(R_2)_{New} + (obj)_{old}$     | 0      | 0      | 15    | 4     | 420    |            |

$$P - 50X_1' - 27 X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

**Step 4: use row operation such that in the Pivot column the other element must be 0**

## Simplex Method: *Problem 1*

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test   |
|-------|--------|--------|-------|-------|--------|--------------|
| $R_1$ | 2      | 1      | 1     | 0     | 16     | $16/2 = 8$   |
| $R_2$ | 5      | 3      | 0     | 1     | 45     | $45 / 5 = 9$ |
| obj   | -50    | -27    | 0     | 0     | 0      |              |

$$P - 50X_1' - 27 X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

|       | $X_1'$ | $X_2'$ | $S_1$  | $S_2$ | Const. | Ratio Test       |
|-------|--------|--------|--------|-------|--------|------------------|
| $R_1$ | 1      | $1/2$  | $1/2$  | 0     | 8      | $8 / (1/2) = 16$ |
| $R_2$ | 0      | $1/2$  | $-5/2$ | 1     | 5      | $5 / (1/2) = 10$ |
| obj   | 0      | -2     | 25     | 0     | 400    |                  |

**Step 4: use row operation such that in the Pivot column the other element must be 0**

|                                                | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test |
|------------------------------------------------|--------|--------|-------|-------|--------|------------|
| $(R_1)_{New} = -1/2 (R_2)_{New} + (R_1)_{old}$ | 1      | 0      | 3     | -1    | 3      |            |
| $(R_2)_{New} = \frac{(R_2)_{old}}{(1/2)}$      | 0      | 1      | -5    | 2     | 10     |            |
| $(obj)_{New} = 2(R_2)_{New} + (obj)_{old}$     | 0      | 0      | 15    | 4     | 420    |            |

*there is no negative numbers in the obj. row*

## Simplex Method: *Problem 1*

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test |
|-------|--------|--------|-------|-------|--------|------------|
| $R_1$ | 2      | 1      | 1     | 0     | 16     | $16/2 = 8$ |
| $R_2$ | 5      | 3      | 0     | 1     | 45     | $45/5 = 9$ |
| obj   | -50    | -27    | 0     | 0     | 0      |            |

|       | $X_1'$ | $X_2'$ | $S_1$  | $S_2$ | Const. | Ratio Test       |
|-------|--------|--------|--------|-------|--------|------------------|
| $R_1$ | 1      | $1/2$  | $1/2$  | 0     | 8      | $8 / (1/2) = 16$ |
| $R_2$ | 0      | $1/2$  | $-5/2$ | 1     | 5      | $5 / (1/2) = 10$ |
| obj   | 0      | -2     | 25     | 0     | 400    |                  |

|                                                | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test |
|------------------------------------------------|--------|--------|-------|-------|--------|------------|
| $(R_1)_{New} = -1/2 (R_2)_{New} + (R_1)_{old}$ | 1      | 0      | 3     | -1    | 3      |            |
| $(R_2)_{New} = \frac{(R_2)_{old}}{(1/2)}$      | 0      | 1      | -5    | 2     | 10     |            |
| $(obj)_{New} = 2(R_2)_{New} + (obj)_{old}$     | 0      | 0      | 15    | 4     | 420    |            |

$$P - 50X_1' - 27X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

From  $S_1$  &  $S_2$  columns

$$X_1 = 15$$

## Simplex Method: *Problem 1*

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test   |
|-------|--------|--------|-------|-------|--------|--------------|
| $R_1$ | 2      | 1      | 1     | 0     | 16     | $16/2 = 8$   |
| $R_2$ | 5      | 3      | 0     | 1     | 45     | $45 / 5 = 9$ |
| obj   | -50    | -27    | 0     | 0     | 0      |              |

|       | $X_1'$ | $X_2'$ | $S_1$  | $S_2$ | Const. | Ratio Test       |
|-------|--------|--------|--------|-------|--------|------------------|
| $R_1$ | 1      | $1/2$  | $1/2$  | 0     | 8      | $8 / (1/2) = 16$ |
| $R_2$ | 0      | $1/2$  | $-5/2$ | 1     | 5      | $5 / (1/2) = 10$ |
| obj   | 0      | -2     | 25     | 0     | 400    |                  |

|                                                | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test |
|------------------------------------------------|--------|--------|-------|-------|--------|------------|
| $(R_1)_{New} = -1/2 (R_2)_{New} + (R_1)_{old}$ | 1      | 0      | 3     | -1    | 3      |            |
| $(R_2)_{New} = \frac{(R_2)_{old}}{(1/2)}$      | 0      | 1      | -5    | 2     | 10     |            |
| $(obj)_{New} = 2(R_2)_{New} + (obj)_{old}$     | 0      | 0      | 15    | 4     | 420    |            |

$$P - 50X_1' - 27 X_2' = 0 \quad (obj.)$$

$$2X_1' + X_2' + S_1 = 16 \quad (R_1)$$

$$5X_1' + 3X_2' + S_2 = 45 \quad (R_2)$$

**Step 4: use row operation such that in the Pivot column the other element must be 0**

*From  $S_1$  &  $S_2$  columns*

$$\rightarrow X_2 = 4$$

## Simplex Method: *Problem 1*

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test |
|-------|--------|--------|-------|-------|--------|------------|
| $R_1$ | 2      | 1      | 1     | 0     | 16     | $16/2 = 8$ |
| $R_2$ | 5      | 3      | 0     | 1     | 45     | $45/5 = 9$ |
| obj   | -50    | -27    | 0     | 0     | 0      |            |

|       | $X_1'$ | $X_2'$ | $S_1$  | $S_2$ | Const. | Ratio Test       |
|-------|--------|--------|--------|-------|--------|------------------|
| $R_1$ | 1      | $1/2$  | $1/2$  | 0     | 8      | $8 / (1/2) = 16$ |
| $R_2$ | 0      | $1/2$  | $-5/2$ | 1     | 5      | $5 / (1/2) = 10$ |
| obj   | 0      | -2     | 25     | 0     | 400    |                  |

|                                                | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test |
|------------------------------------------------|--------|--------|-------|-------|--------|------------|
| $(R_1)_{New} = -1/2 (R_2)_{New} + (R_1)_{old}$ | 1      | 0      | 3     | -1    | 3      |            |
| $(R_2)_{New} = \frac{(R_2)_{old}}{(1/2)}$      | 0      | 1      | -5    | 2     | 10     |            |
| $(obj)_{New} = 2(R_2)_{New} + (obj)_{old}$     | 0      | 0      | 15    | 4     | 420    |            |

**P** -  $50X_1' - 27X_2' = 0$  (obj.)

$2X_1' + X_2' + S_1 = 16$  ( $R_1$ )

$5X_1' + 3X_2' + S_2 = 45$  ( $R_2$ )

**Step 4:** use row operation such that in the Pivot column the other element must be 0

**Min Z = 420**

$$X_1 = 15, X_2 = 4$$

$$Z = 420$$

*For check you can calculate to make sure the result is true*

$$\text{Min } Z = 16X_1 + 45X_2 = 16(15) + 45(4) = 420$$

**The Result is true**

## Simplex Method

### *The First : Minimization Problem*

#### *Problem 2:*

$$\text{Minimize } Z = 6X_1 + 3X_2$$

*Subject to:*

$$2X_1 + 4X_2 \geq 16$$

$$4X_1 + 3X_2 \geq 24$$

$$X_1, X_2 \geq 0$$

## Simplex Method: *Problem 2*

Convert the problem from minimization problem to maximization problem

Minimize  $Z = 6X_1 + 3X_2$

Subject to:

$$2X_1 + 4X_2 \geq 16$$

$$4X_1 + 3X_2 \geq 24$$

$$X_1, X_2 \geq 0$$

Transformation  
Matrix



$$A = \begin{array}{ccc|c} X_1 & X_2 & \text{RHS} & \\ \hline 2 & 4 & 16 & R_1 \\ 4 & 3 & 24 & R_2 \\ 6 & 3 & Z & \text{Obj.} \end{array}$$

Min  $\rightarrow$  Max



Maximize  $P = 16X_1' + 24X_2'$

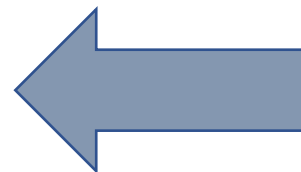
Subject to.

$$2X_1' + 4X_2' \leq 6$$

$$4X_1' + 3X_2' \leq 3$$

$$X_1', X_2' \geq 0$$

Max



$$A^T = \begin{array}{ccc|c} X_1' & X_2' & \text{RHS} & \\ \hline 2 & 4 & 6 & R_1 \\ 4 & 3 & 3 & R_2 \\ 16 & 24 & P & \text{Obj.} \end{array}$$

## Simplex Method: *Problem 2*

$$\text{Max } P = 16X_1' + 24X_2'$$

$$2X_1' + 4X_2' \leq 6$$

$$4X_1' + 3X_2' \leq 3$$

$$X_1', X_2' \geq 0$$

*Steps in the Simplex Method :*

1- **step 1:** Rewrite the problem in standard form.

$$P - 16X_1' - 24X_2' = 0 \quad (\text{obj.})$$

$$2X_1' + 4X_2' + S_1 = 6 \quad (R_1)$$

$$4X_1' + 3X_2' + S_2 = 3 \quad (R_2)$$

$$X_1, X_2 \geq 0$$

*Maximize only*

*Objective function → with R.H.S = 0*

*Constraints:*

*(For  $\leq$ ) → equation by  $(+s_i)$*

*(For  $\geq$ ) → equation by  $(-s_i)$*

## Simplex Method: *Problem 2*

2- **step 2:** *Construct the table and search for the Pivot (Pivot column  $\cap$  Pivot Row)*

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. |  |
|-------|--------|--------|-------|-------|--------|--|
| $R_1$ | 2      | 4      | 1     | 0     | 6      |  |
| $R_2$ | 4      | 3      | 0     | 1     | 3      |  |
| $obj$ | -16    | -24    | 0     | 0     | 0      |  |

$$P - 16X_1' - 24X_2' = 0 \quad (obj.)$$

$$2X_1' + 4X_2' + S_1 = 6 \quad (R_1)$$

$$4X_1' + 3X_2' + S_2 = 3 \quad (R_2)$$

$$X_1, X_2 \geq 0$$

## Simplex Method: *Problem 2*

**2- *step 2*: Construct the table and search for the Pivot (Pivot column  $\cap$  Pivot Row)**

*For select the pivot column get the lower in objective function*

|            | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. |  |
|------------|--------|--------|-------|-------|--------|--|
| $R_1$      | 2      | 4      | 1     | 0     | 6      |  |
| $R_2$      | 4      | 3      | 0     | 1     | 3      |  |
| <i>obj</i> | -16    | -24    | 0     | 0     | 0      |  |

$$P - 16X_1' - 24X_2' = 0 \quad (obj.)$$

$$2X_1' + 4X_2' + S_1 = 6 \quad (R_1)$$

$$4X_1' + 3X_2' + S_2 = 3 \quad (R_2)$$

$$X_1, X_2 \geq 0$$

## Simplex Method: *Problem 2*

**2- *step 2*: Construct the table and search for the Pivot (Pivot column  $\cap$  Pivot Row)**

*For select the pivot column get the lower in objective function*

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. |  |
|-------|--------|--------|-------|-------|--------|--|
| $R_1$ | 2      | 4      | 1     | 0     | 6      |  |
| $R_2$ | 4      | 3      | 0     | 1     | 3      |  |
| obj   | -16    | -24    | 0     | 0     | 0      |  |

$$P - 16X_1' - 24X_2' = 0 \quad (obj.)$$

$$2X_1' + 4X_2' + S_1 = 6 \quad (R_1)$$

$$4X_1' + 3X_2' + S_2 = 3 \quad (R_2)$$

$$X_1, X_2 \geq 0$$

## Simplex Method: *Problem 2*

**2- step 2:** Construct the table and search for the Pivot (Pivot column  $\cap$  Pivot Row)

To select the Pivot row perform the ratio test and get the low value

*Ratio test = constant / value in pivot column for R1&R1*

$$P - 16X_1' - 24 X_2' = 0 \quad (obj.)$$

$$2X_1' + 4X_2' + S_1 = 6 \quad (R_1)$$

$$4X_1' + 3X_2' + S_2 = 3 \quad (R_2)$$

$$X_1, X_2 \geq 0$$

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test        |
|-------|--------|--------|-------|-------|--------|-------------------|
| $R_1$ | 2      | 4      | 1     | 0     | 6      | $6/4 = 3/2 = 1.5$ |
| $R_2$ | 4      | 3      | 0     | 1     | 3      | $3/3 = 1$         |
| obj   | -16    | -24    | 0     | 0     | 0      |                   |

## Simplex Method: *Problem 2*

2- **step 2:** Construct the table and search for the Pivot (Pivot column  $\cap$  Pivot Row)

To select the Pivot row perform the ratio test and get the low value

*Ratio test = constant / value in pivot column for  $R_1$  &  $R_2$*

$$P - 16X_1' - 24X_2' = 0 \quad (obj.)$$

$$2X_1' + 4X_2' + S_1 = 6 \quad (R_1)$$

$$4X_1' + 3X_2' + S_2 = 3 \quad (R_2)$$

$$X_1, X_2 \geq 0$$

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test        |
|-------|--------|--------|-------|-------|--------|-------------------|
| $R_1$ | 2      | 4      | 1     | 0     | 6      | $6/4 = 3/2 = 1.5$ |
| $R_2$ | 4      | 3      | 0     | 1     | 3      | $3/3 = 1$         |
| obj   | -16    | -24    | 0     | 0     | 0      |                   |

**Pivot**

## Simplex Method: *Problem 2*

### 3- *step 3*: The Pivot must be 1

|            | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | <i>Const.</i> | <i>Ratio Test</i> |
|------------|--------|--------|-------|-------|---------------|-------------------|
| $R_1$      | 2      | 4      | 1     | 0     | 6             | $6/4 = 3/2 = 1.5$ |
| $R_2$      | 4      | 3      | 0     | 1     | 3             | $3/3 = 1$         |
| <i>obj</i> | -16    | -24    | 0     | 0     | 0             |                   |

$$P - 16X_1' - 24X_2' = 0 \quad (obj.)$$

$$2X_1' + 4X_2' + S_1 = 6 \quad (R_1)$$

$$4X_1' + 3X_2' + S_2 = 3 \quad (R_2)$$

$$X_1, X_2 \geq 0$$

|                                       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | <i>Const.</i> | <i>Ratio Test</i> |
|---------------------------------------|--------|--------|-------|-------|---------------|-------------------|
| $(R_2)_{New} = \frac{(R_2)_{old}}{3}$ | $4/3$  | 1      | 0     | $1/3$ | 1             |                   |

## Simplex Method: *Problem 2*

**4- step 4:** use row operation such that in the Pivot column the other element must be 0

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test        |
|-------|--------|--------|-------|-------|--------|-------------------|
| $R_1$ | 2      | 4      | 1     | 0     | 6      | $6/4 = 3/2 = 1.5$ |
| $R_2$ | 4      | 3      | 0     | 1     | 3      | $3/3 = 1$         |
| obj   | -16    | -24    | 0     | 0     | 0      |                   |

$$P - 16X_1' - 24X_2' = 0 \quad (obj.)$$

$$2X_1' + 4X_2' + S_1 = 6 \quad (R_1)$$

$$4X_1' + 3X_2' + S_2 = 3 \quad (R_2)$$

$$X_1, X_2 \geq 0$$

|                                             | $X_1'$  | $X_2'$ | $S_1$ | $S_2$  | Const. | Ratio Test |
|---------------------------------------------|---------|--------|-------|--------|--------|------------|
| $(R_1)_{New} = -4(R_2)_{New} + (R_1)_{old}$ | $-10/3$ | 0      | 1     | $-4/3$ | 2      |            |
| $(R_2)_{New} = \frac{(R_2)_{old}}{3}$       | $4/3$   | 1      | 0     | $1/3$  | 1      |            |
| $(obj)_{New} = 24(R_2)_{New} + (obj)_{old}$ | 16      | 0      | 0     | 8      | 24     |            |

## Simplex Method: *Problem 2*

**4- step 4:** use row operation such that in the Pivot column the other element must be 0

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test        |
|-------|--------|--------|-------|-------|--------|-------------------|
| $R_1$ | 2      | 4      | 1     | 0     | 6      | $6/4 = 3/2 = 1.5$ |
| $R_2$ | 4      | 3      | 0     | 1     | 3      | $3/3 = 1$         |
| obj   | -16    | -24    | 0     | 0     | 0      |                   |

$$P - 16X_1' - 24X_2' = 0 \quad (obj.)$$

$$2X_1' + 4X_2' + S_1 = 6 \quad (R_1)$$

$$4X_1' + 3X_2' + S_2 = 3 \quad (R_2)$$

$$X_1, X_2 \geq 0$$

|                                             | $X_1'$  | $X_2'$ | $S_1$ | $S_2$  | Const. | Ratio Test |
|---------------------------------------------|---------|--------|-------|--------|--------|------------|
| $(R_1)_{New} = -4(R_2)_{New} + (R_1)_{old}$ | $-10/3$ | 0      | 1     | $-4/3$ | 2      |            |
| $(R_2)_{New} = \frac{(R_2)_{old}}{3}$       | $4/3$   | 1      | 0     | $1/3$  | 1      |            |
| $(obj)_{New} = 24(R_2)_{New} + (obj)_{old}$ | 16      | 0      | 0     | 8      | 24     |            |

there is no negative numbers in the obj. row

## Simplex Method: *Problem 2*

**4- step 4:** use row operation such that in the Pivot column the other element must be 0

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test        |
|-------|--------|--------|-------|-------|--------|-------------------|
| $R_1$ | 2      | 4      | 1     | 0     | 6      | $6/4 = 3/2 = 1.5$ |
| $R_2$ | 4      | 3      | 0     | 1     | 3      | $3/3 = 1$         |
| obj   | -16    | -24    | 0     | 0     | 0      |                   |

$$P - 16X_1' - 24X_2' = 0 \quad (obj.)$$

$$2X_1' + 4X_2' + S_1 = 6 \quad (R_1)$$

$$4X_1' + 3X_2' + S_2 = 3 \quad (R_2)$$

$$X_1, X_2 \geq 0$$

|                                             | $X_1'$  | $X_2'$ | $S_1$ | $S_2$  | Const. | Ratio Test |
|---------------------------------------------|---------|--------|-------|--------|--------|------------|
| $(R_1)_{New} = -4(R_2)_{New} + (R_1)_{old}$ | $-10/3$ | 0      | 1     | $-4/3$ | 2      |            |
| $(R_2)_{New} = \frac{(R_2)_{old}}{3}$       | $4/3$   | 1      | 0     | $1/3$  | 1      |            |
| $(obj)_{New} = 24(R_2)_{New} + (obj)_{old}$ | 16      | 0      | 0     | 8      | 24     |            |

From  $S_1$  &  $S_2$  columns

$$X_1 = 0$$

## Simplex Method: *Problem 2*

**4- step 4:** use row operation such that in the Pivot column the other element must be 0

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test        |
|-------|--------|--------|-------|-------|--------|-------------------|
| $R_1$ | 2      | 4      | 1     | 0     | 6      | $6/4 = 3/2 = 1.5$ |
| $R_2$ | 4      | 3      | 0     | 1     | 3      | $3/3 = 1$         |
| obj   | -16    | -24    | 0     | 0     | 0      |                   |

$$P - 16X_1' - 24X_2' = 0 \quad (obj.)$$

$$2X_1' + 4X_2' + S_1 = 6 \quad (R_1)$$

$$4X_1' + 3X_2' + S_2 = 3 \quad (R_2)$$

$$X_1, X_2 \geq 0$$

|                                             | $X_1'$  | $X_2'$ | $S_1$ | $S_2$  | Const. | Ratio Test |
|---------------------------------------------|---------|--------|-------|--------|--------|------------|
| $(R_1)_{New} = -4(R_2)_{New} + (R_1)_{old}$ | $-10/3$ | 0      | 1     | $-4/3$ | 2      |            |
| $(R_2)_{New} = \frac{(R_2)_{old}}{3}$       | $4/3$   | 1      | 0     | $1/3$  | 1      |            |
| $(obj)_{New} = 24(R_2)_{New} + (obj)_{old}$ | 16      | 0      | 0     | 8      | 24     |            |

From  $S_1$  &  $S_2$  columns

$$\rightarrow X_2 = 8$$

## Simplex Method: *Problem 2*

**4- step 4:** use row operation such that in the Pivot column the other element must be 0

|       | $X_1'$ | $X_2'$ | $S_1$ | $S_2$ | Const. | Ratio Test        |
|-------|--------|--------|-------|-------|--------|-------------------|
| $R_1$ | 2      | 4      | 1     | 0     | 6      | $6/4 = 3/2 = 1.5$ |
| $R_2$ | 4      | 3      | 0     | 1     | 3      | $3/3 = 1$         |
| obj   | -16    | -24    | 0     | 0     | 0      |                   |

$$P - 16X_1' - 24X_2' = 0 \quad (obj.)$$

$$2X_1' + 4X_2' + S_1 = 6 \quad (R_1)$$

$$4X_1' + 3X_2' + S_2 = 3 \quad (R_2)$$

$$X_1, X_2 \geq 0$$

|                                             | $X_1'$  | $X_2'$ | $S_1$ | $S_2$  | Const. | Ratio Test |
|---------------------------------------------|---------|--------|-------|--------|--------|------------|
| $(R_1)_{New} = -4(R_2)_{New} + (R_1)_{old}$ | $-10/3$ | 0      | 1     | $-4/3$ | 2      |            |
| $(R_2)_{New} = \frac{(R_2)_{old}}{3}$       | $4/3$   | 1      | 0     | $1/3$  | 1      |            |
| $(obj)_{New} = 24(R_2)_{New} + (obj)_{old}$ | 16      | 0      | 0     | 8      | 24     |            |

**Min Z = 24**

$$X_1 = 0, X_2 = 8$$

$$Z = 24$$

*For check you can calculate to make sure the result is true*

$$\text{Minimize } Z = 6X_1 + 3X_2 = 6(0) + 3(8) = 24$$

**The Result is true**



Thank You