

Operations Research

Prepared by Dr. Mohamed Eassa

IT 262
Operations Research

Lecture 7



Transportation Applications

Part Two

Lecture 7

IT 262

Operations Research

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Dr. Mohamed Eassa

Lecture 7 Topics

- ❖ **Transportation Model**
- ❖ **Unbalanced Problem**
- ❖ **The Optimal Solution**

Transportation Applications

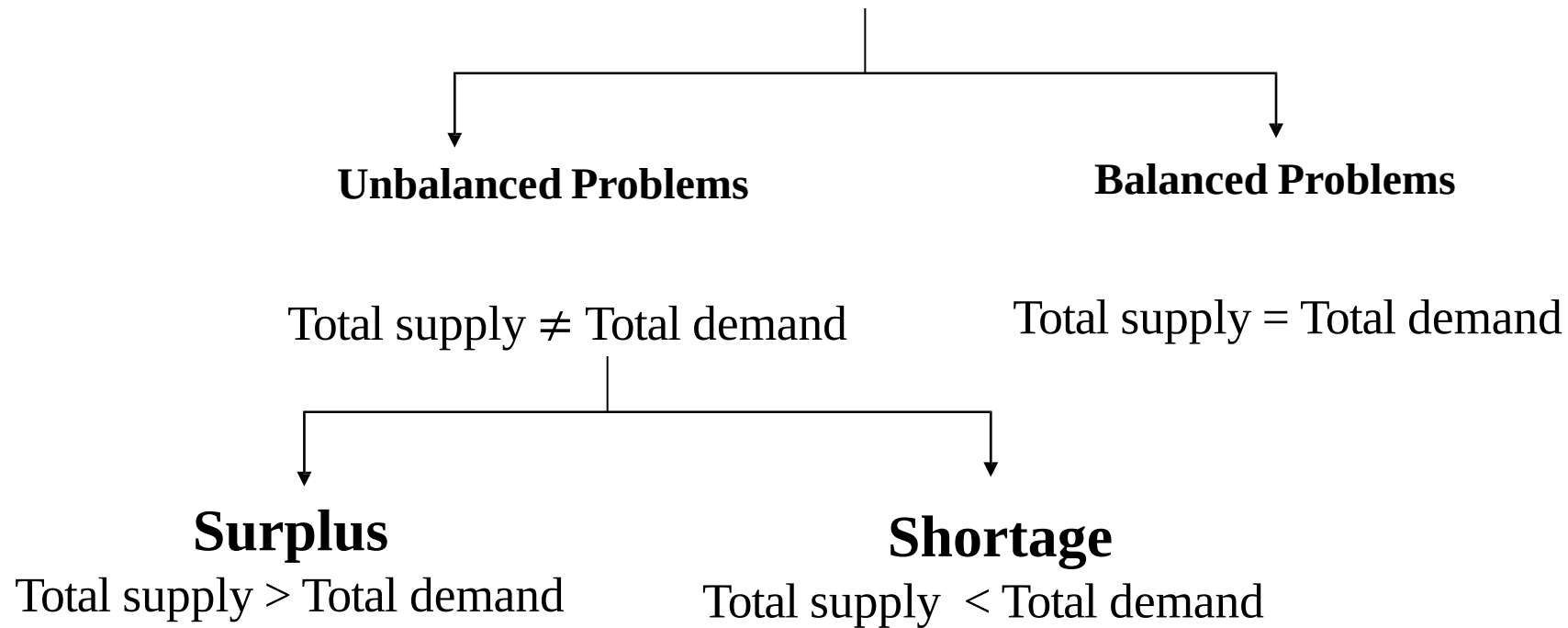
GENERAL FORM OF TP

X_{ij} = NUMBER OF UNITS SHIPPED FROM SUPPLY POINT i TO
DEMAND POINT j . $i = 1, 2, \dots, M, \quad j = 1, 2, \dots, N$

$$\begin{aligned}
 & \text{Minimize } Z = \sum_{i=1}^n \sum_{j=1}^m c_{ij} x_{ij} \\
 & \text{s.t.} \\
 & \sum_{j=1}^n x_{ij} \leq s_i \quad i = 1, 2, \dots, m \\
 & \sum_{i=1}^m x_{ij} \geq d_j \quad j = 1, 2, \dots, n \\
 & x_{ij} \geq 0
 \end{aligned}$$

Transportation Applications

Transportation Models



UNBALANCED TRANSPORTATION PROBLEM

UNBALANCED PROBLEMS

SURPLUS

$$\sum S_i > \sum D_j$$

SURPLUS VALUE:

$$\sum S_i - \sum D_j = D_{\Delta}$$

THERE EXIST DUMMY DEMAND NODE EQUAL SURPLUS VALUE

Unbalanced Transportation

EXAMPLE:

AN OIL REFINERY HAS TWO LOCATIONS TO ENSURE HEATING FUEL NEEDS FOR THREE CITIES. THE PRODUCTION CAPACITY OF HEATING FUEL FOR THE FIRST REFINERY IS 20 MILLION BARRELS PER DAY, WHILE THE PRODUCTION CAPACITY OF HEATING FUEL FOR THE SECOND REFINERY IS 36 MILLION BARRELS PER DAY. ESTIMATED FUEL DEMAND FOR EACH CITY: 18, 13 AND 12 MILLION BARRELS PER DAY FOR THE FIRST, SECOND AND THIRD CITIES, RESPECTIVELY. THE COST OF TRANSPORTING A BARREL FROM ONE OF THE REFINERIES TO ANY CITY IS 5 POUNDS PER 10 KM. THE CITY -1 IS : 30 KM AND 50 KM FROM REFINERY 1 AND 2, RESPECTIVELY, THE CITY -2: 46 KM AND 24 KM FROM REFINERY 1 AND 2 RESPECTIVELY, AND THE CITY - 3: 18 KM AND 32 KM FROM REFINERY 1 AND 2, RESPECTIVELY.

Unbalanced Transportation

$$\text{TOTAL DEMAND} = 18 + 13 + 12 = 43$$

$$\text{TOTAL SUPPLY} = 20 + 36 = 56 > \Sigma D$$

$$\text{DIFFERENCE} = 56 - 43 = 13 \Leftrightarrow \text{SURPLUS}$$

North-West Corner Method:

	15	23	9	<i>Supply</i>
				20
	25	12	16	36
<i>Demand</i>	18	13	12	

Unbalanced Transportation

	15		23		9		0
18							
	25		12		16		0

Supply

~~20~~ 2

36

Demand

~~18~~

13

12

0

	15		23		9		0
18		2					
	25		12		16		0

Supply

~~20~~ ~~2~~ 0

36

Demand

~~18~~

~~13~~

12

0

11

Unbalanced Transportation

		15		23		9		0	<i>Supply</i>	
	18		2						20	2 0
		25		12		16		0	36	
<i>Demand</i>	18		13			12				
	0		11							

		15		23		9		0	<i>Supply</i>	
	18		2						20	2 0
		25		12		16		0	36	25
<i>Demand</i>	18		13			12				
	0		11							
			0							

Unbalanced Transportation

		15		23		9		0	<i>Supply</i>		
	18		2						20	2	0
		25		12		16		0	36	25	
			11								
<i>Demand</i>	18		13			12					
	0		11								
			0								

		15		23		9		0	<i>Supply</i>		
	18		2						20	2	0
		25		12		16		0	36	25	13
			11		12						
<i>Demand</i>	18		13		12						
	0		11		0						
			0								

Unbalanced Transportation

		15		23		9		0	
	18			2					
		25		12		16		0	
			11		12				
<i>Demand</i>	18		13		12				
	0		1		0				

Supply
~~20~~ ~~2~~ 0
~~36~~ ~~25~~ 13

		15		23		9		0	
	18			2					
		25		12		16		0	
			11		12		13		
<i>Demand</i>	18		13		12		13		
	0		1		0				

Supply
~~20~~ ~~2~~ 0
~~36~~ ~~25~~ 13

Unbalanced Transportation

TOTAL DEMAND = $18 + 13 + 12 = 43$

TOTAL SUPPLY = $20 + 36 = 56 > \Sigma D$

DIFFERENCE = $56 - 43 = 13 \Leftrightarrow$ SURPLUS

	15	23	9	0	<i>Supply</i>
18	2				20
	25	12	16	0	36
	11	12	13		
<i>Demand</i>	<i>18</i>	<i>13</i>	<i>12</i>	<i>13</i>	

UNBALANCED TRANSPORTATION PROBLEM

UNBALANCED PROBLEMS

Shortage or Slack $\Sigma S_i < \Sigma D_j$

Slack VALUE:

$$\Sigma D_j - \Sigma S_i = D_{\Delta}$$

THERE EXIST DUMMY DEMAND NODE EQUAL Slack VALUE

Unbalanced Transportation

TOTAL DEMAND = $18 + 13 + 12 = 43$

TOTAL SUPPLY = $20 + 18 = 38 < \Sigma D$

DIFFERENCE = $43 - 38 = 5 \Leftrightarrow \text{SLACK}$

North-West Corner Method:

	15	23	9	<i>Supply</i>
				<i>20</i>
	25	12	16	<i>18</i>
<i>Demand</i>	<i>18</i>	<i>13</i>	<i>12</i>	

Unbalanced Transportation

	18	15		23		9	
		25		12		16	
		0		0		0	
<i>Demand</i>	18 0		13		12		

Supply
~~20~~
18

	18	15	2	23		9	
		25		12		16	
		0		0		0	
<i>Demand</i>	18 0		13 11		12		

Supply
~~20~~
18

Unbalanced Transportation

	18	15	2	23		9	
		25		12		16	
			11				
		0		0		0	
<i>Demand</i>	18 0		13 11 0		12		

Supply
~~20~~ ~~2~~ 0
~~18~~ 7

	18	15	2	23		9	
		25		12		16	
			11		7		
		0		0		0	
<i>Demand</i>	18 0		13 11		12 5		

Supply
~~20~~ ~~2~~ 0
~~18~~ ~~7~~ 0

Unbalanced Transportation

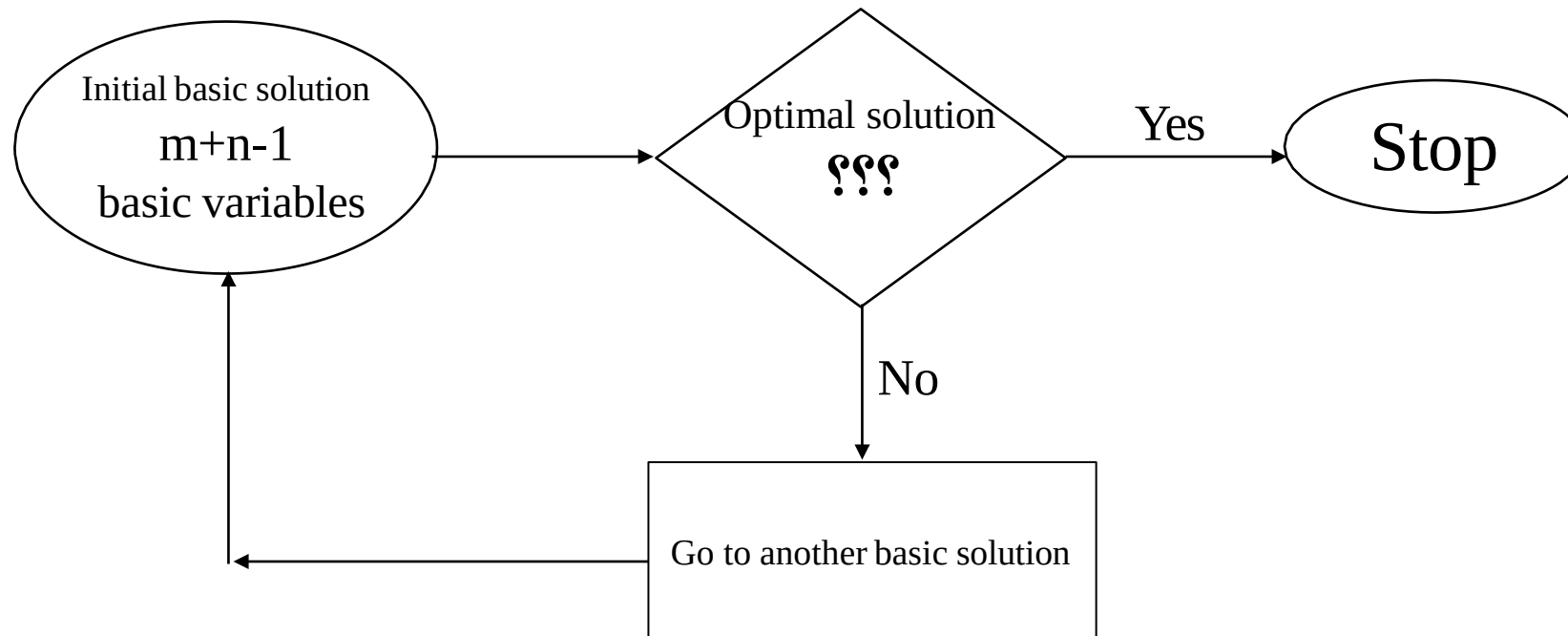
	18	15	2	23		9	<i>Supply</i>
							20 2 0
		25		12		16	18 7 0
			11		7		
		0		0		0	
					5		5
<i>Demand</i>	18		13		12		
	0		11		5		
			0		0		

Unbalanced Transportation

	18	15	2	23		9	<i>Supply</i>
							<i>20</i>
		25		12		16	<i>18</i>
			11		7		
		0		0		0	
					5		5
<i>Demand</i>	<i>18</i>		<i>13</i>		<i>12</i>		

Transportation Simplex

Optimal Solution



Transportation Simplex

TEST OF OPTIMALITY IN TRANSPORTATION TABLE

- FOR EVERY BASIC CELL (FILLED) (i, j) ,
DETERMINE THE WEIGHTS u_i AND v_j SUCH THAT

$$u_i + v_j = c_{ij}$$

LET $u_1 = 0$.

- FOR EVERY ROW (I) HAS THE WEIGHT u_i AND FOR EVERY COLUMN (J) HAS THE WEIGHT v_j .
- FOR EVERY NONBASIC CELL (NOT FILLED) (k, l) CALCULATE THE WEIGHT DIFFERENCES δ_{kl}
SUCH THAT $\delta_{kl} = u_k + v_l - c_{kl}$
IF $\delta_{kl} \leq 0$ FOR ALL NONBASIC CELLS (Negative or zero), THEN THE
SOLUTION IS OPTIMAL.

Transportation Simplex

TRANSPORTATION PROBLEMS

	8	6	10	9	<i>Supply</i> 35
35					
10	9	20	20	7	50
	14	9	16	5	40
			10	30	
<i>DEMAND</i> 45		20	30	30	

Total Cost = $(35 \times 8) + (10 \times 9) + (20 \times 12) + (20 \times 13) + (10 \times 16) + (30 \times 5) = 1180$ \$

$$\mathbf{u}_1 = \mathbf{0}$$

$u_1=0$	35 8	6	10	9	Supply
	10 9	20 12	20 13	7	50
	14	9	10 16	30 5	40
DEMAND	45	20	30	30	

Transportation Simplex

TRANSPORTATION PROBLEMS

$$u_1 + v_1 = 8$$

$$0 + v_1 = 8$$

$v_1=8$

$$\mathbf{u}_1 = \mathbf{0}$$

$u_1=0$	35	8	6	10	9	<i>Supply</i>
	10	9	20	12	13	7
	14	9	16	5	30	
<i>DEMAND</i>	45	20	30	30		

Transportation Simplex

TRANSPORTATION PROBLEMS

$$0 + v_1 = 8$$

$$v_1 = 8$$

$$u_1 = 0$$

$$u_2 + 8 = 9$$

$$u_2 = 1$$

	8	6	10	9	
35					
	9	12	13	7	
10		20	20		
	14	9	16	5	
			10	30	
DEMAND	45	20	30	30	

Supply

35

50

40

Transportation Simplex

TRANSPORTATION PROBLEMS

	$0+v_1 = 8$ $v_1=8$	$1+v_2 = 12$ $v_2=11$	$1+v_3 = 13$ $v_3=12$	$4+v_4 = 5$ $v_4=1$																	
$u_1=0$	<table><tr><td></td><td>8</td></tr><tr><td>35</td><td></td></tr></table>		8	35		<table><tr><td></td><td>6</td></tr><tr><td></td><td></td></tr></table>		6			<table><tr><td></td><td>10</td></tr><tr><td></td><td></td></tr></table>		10			<table><tr><td></td><td>9</td></tr><tr><td></td><td></td></tr></table>		9			<i>Supply</i> 35
	8																				
35																					
	6																				
	10																				
	9																				
$u_2+8=9$ $u_2 =1$	<table><tr><td></td><td>9</td></tr><tr><td>10</td><td></td></tr></table>		9	10		<table><tr><td></td><td>12</td></tr><tr><td>20</td><td></td></tr></table>		12	20		<table><tr><td></td><td>13</td></tr><tr><td>20</td><td></td></tr></table>		13	20		<table><tr><td></td><td>7</td></tr><tr><td></td><td></td></tr></table>		7			50
	9																				
10																					
	12																				
20																					
	13																				
20																					
	7																				
$u_3+12=16$ $u_3 =4$	<table><tr><td></td><td>14</td></tr><tr><td></td><td></td></tr></table>		14			<table><tr><td></td><td>9</td></tr><tr><td></td><td></td></tr></table>		9			<table><tr><td></td><td>16</td></tr><tr><td>10</td><td></td></tr></table>		16	10		<table><tr><td></td><td>5</td></tr><tr><td>30</td><td></td></tr></table>		5	30		40
	14																				
	9																				
	16																				
10																					
	5																				
30																					
<i>DEMAND</i>	45	20	30	30																	

Transportation Simplex

TRANSPORTATION PROBLEMS

	$0+v_1 = 8$ $v_1=8$	$1+v_2 = 12$ $v_2=11$	$1+v_3 = 13$ $v_3=12$	$4+v_4 = 5$ $v_4=1$	
$u_1=0$	35	$\delta_{12}=8$ $u_1+v_2 - 6$	$\delta_{13}=6$ $u_1+v_3 - 10$	$\delta_{14}=9$ $u_1+v_4 - 9$	Supply 35
$u_2+8=9$ $u_2 =1$	10	20	20	$\delta_{24}=7$ $u_2+v_4 - 7$	50
$u_3+12=16$ $u_3 =4$	$\delta_{31}=14$ $u_3+v_1 - 14$	$\delta_{32}=9$ $u_3+v_2 - 9$	10	30	40
DEMAND	45	20	30	30	

Transportation Simplex

TRANSPORTATION PROBLEMS

		$0+v_1 = 8$ $v_1=8$	$1+v_2 = 12$ $v_2=11$	$1+v_3 = 13$ $v_3=12$	$4+v_4 = 5$ $v_4=1$	
$u_1=0$						<i>Supply</i>
		8	6	10	9	35
	35		$0+11 - 6$ $\delta_{12}=5$	$0+12 - 10$ $\delta_{13}=2$	$0+1 - 9$ $\delta_{14}=-8$	
$u_2+8=9$ $u_2=1$		9	12	13	7	50
	10		20	20	$1+1 - 7$ $\delta_{24}=-5$	
$u_3+12=16$ $u_3=4$		14	9	16	5	40
	$4+8 - 14$ $\delta_{31}=-2$		$4+11 - 9$ $\delta_{32}^*=6$	10	30	
<i>DEMAND</i>	45	20	30	30		

Transportation Simplex

TRANSPORTATION PROBLEMS

		8	$\delta_{12}=5$	6	$\delta_{13}=2$	$\delta_{14}=-8$	9	<i>Supply</i> 35
	10	9	20	12	20	13	7	50
<i>DEMAND</i>	45	14	$\delta_{31}=-2$	9	$\delta_{32}^*=6$	16	5	40
					10		30	
					30			

Not optimal solution

Transportation Simplex

TRANSFER TO NEW BASIC SOLUTION IN TRANSPORTATION TABLE:

- DETERMINE THE MAXIMUM VALUE FOR δ_{KL} , SAY δ^* .

$$\delta^* = \text{MAX} \{ \delta_{KL} : (K,L) \text{ NONBASIC CELL} \}$$
- CREATE A PIVOT LOOP IN THE TRANSPORTATION TABLE SO THAT IT SATISFIES:
 - 1 SATISFIED THE THREE CONDITION FOR PIVOT LOOP.
 - 2 THE LOOP CONTAINS ONLY ONE NONBASIC CELL.
 - 3 THE NONBASIC CELL IN THE LOOP THAT HAS δ^* .
- DISTRIBUTE THE SIGNS (+) AND (-) ON THE CELLS OF LOOP ALTERNATELY, STARTING FROM THE CELL HAS δ^* .

Transportation Simplex

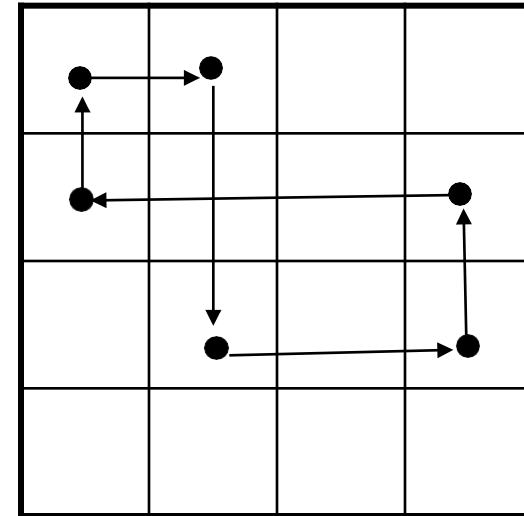
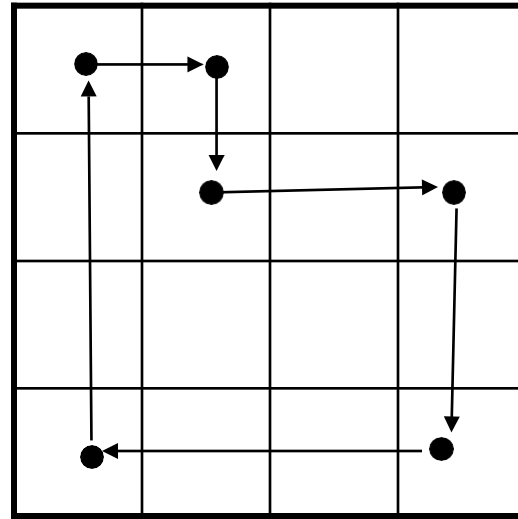
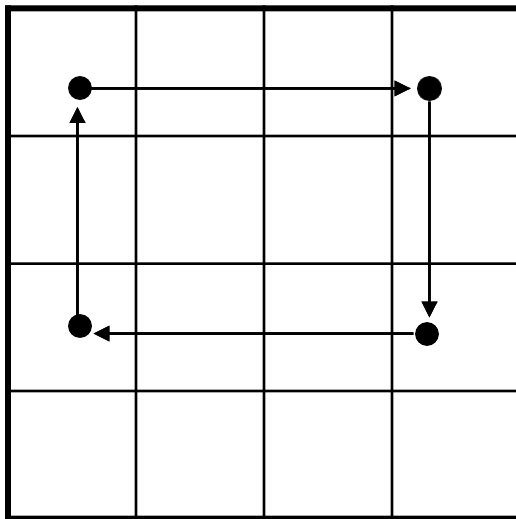
TRANSFER TO NEW BASIC SOLUTION IN TRANSPORTATION TABLE:

DEFINITION : PIVOT LOOP

IT IS A SEQUENCE OF CELLS IN TABLE SATISFIED :

- 1 ANY TWO CONSECUTIVE CELLS SHARE EITHER THE ROW OR THE COLUMN.
- 2 THERE ARE NOT THREE CONSECUTIVE CELLS IN THE SAME ROW OR COLUMN.
- 3 THE LAST CELL IN THE SEQUENCE HAS THE SAME ROW OR COLUMN AS THE FIRST CELL IN THE SEQUENCE.

PIVOT LOOP (CLOSED LOOP)



Transportation Simplex

TRANSPORTATION PROBLEMS

Figure 1: A 3x4 grid illustrating a supply and demand problem. The grid is divided into four columns by vertical lines and three rows by horizontal lines. The top row is labeled "Supply" on the right, with values 35, 50, and 40. The bottom row is labeled "DEMAND" on the left, with values 45, 20, 30, and 30. The grid contains numbers in various cells: Row 1: (1,1)=35, (1,2)=8, (1,3)=6, (1,4)=10, (1,5)=9. Row 2: (2,1)=10, (2,2)=9, (2,3)=12, (2,4)=13, (2,5)=7. Row 3: (3,1)=14, (3,2)=9, (3,3)=16, (3,4)=5, (3,5)=30. A path of four black dots is shown: (2,2) to (2,3) to (3,3) to (3,2). Arrows connect these dots in sequence. The value 20 is placed next to the dots at (2,2) and (2,3). The value 10 is placed next to the dot at (3,3). The value 30 is placed next to the dot at (3,2). The text $\delta_{32}^*=6$ is written in orange below the dot at (3,2).

Not optimal solution

Transportation Simplex

TRANSPORTATION PROBLEM

[illegible]

Transportation Simplex

- SELECT FROM CELLS WITH A MINUS SIGN (-) THE CELL WITH THE MINIMUM VALUE, LET IT BE θ

$$\theta = \min \{ X_{KL} : (K,L) \text{ NONBASIC CELL WITH } (-) \}$$

- GO TO THE NEW BASIC SOLUTION SUCH THAT THE NEW VALUES IN THE LOOP

AS FOLLOWS:

$$\begin{array}{lll} X_{KL} = X_{KL} + \theta & \text{IF} & (+) \\ X_{KL} = X_{KL} - \theta & \text{IF} & (-) \end{array}$$

THE REST OF THE CELLS REMAIN UNCHANGED

Transportation Simplex

EXAMPLE

$$\theta = \text{MIN} \{ \text{CELLS VALUES WITH } (-) = 20, 10 \} = 10$$

					<i>Supply</i>
	35	8	6	10	9
	10	9	- 20	+ 20	13
		14	+ 0	10	16
				-	30
<i>DEMAND</i>	45	20	30	30	

Transportation Simplex

EXAMPLE

$$\theta = \text{MIN} \{ \text{CELLS VALUES WITH } (-) = 20, 10 \} = 10$$

		8		6		10		9	<i>Supply</i> 35
	35								
	10	9	-	12	+	13		7	50
				$20 - \theta$		$20 + \theta$			
		14	+	9		16		5	40
				$0 + \theta$		$10 - \theta$		30	
					-				
<i>DEMAND</i>	45			20		30		30	

Transportation Simplex

EXAMPLE

	35				35
	10	10	30		50
		10	0	30	40
DEMAND	45	20	30	30	

Transportation Simplex

EXAMPLE

									<i>Supply</i>
	35	8		6		10		9	35
	10	9		10	12	30	13	7	50
		14		10	9		16	30	40
<i>DEMAND</i>	45		20			30		30	

Transportation Simplex

EXAMPLE

$$0 + v_1 = 8$$

$v_1=8$

$$\mathbf{u}_1 = \mathbf{0}$$

$u_1=0$	35	8	6	10	9	Supply
	10	9	10	12	13	7
	14	10	9	16	5	30
DEMAND	45	20	30	30		

Transportation Simplex

EXAMPLE

$0 + v_1 = 8$
 $v_1 = 8$

$u_1 = 0$		8	6	10	9	<i>Supply</i> 35
	35					
$u_2 + 8 = 9$ $u_2 = 1$		9	12	13	7	50
	10	10	30			
		14	9	16	5	40
		10			30	
<i>DEMAND</i>	45	20	30	30		

Transportation Simplex

EXAMPLE

	$0+v_1 = 8$ $v_1=8$	$1+v_2 = 12$ $v_2=11$	$1+v_3 = 13$ $v_3=12$	$-2+v_4 = 5$ $v_4=7$	
$u_1=0$	35				Supply
$u_2+8=9$ $u_2 = 1$	10	10	30		50
$u_3+11=9$ $u_3 = -2$		10		30	40
DEMAND	45	20	30	30	

Transportation Simplex

EXAMPLE

	$0+v_1 = 8$ $v_1=8$	$1+v_2 = 12$ $v_2=11$	$1+v_3 = 13$ $v_3=12$	$-2+v_4 = 5$ $v_4=7$	
$u_1=0$	8 35	6 $0+11-6$ $\delta_{12}=5$	10 $0+12-10$ $\delta_{13}=2$	9 $0+7-9$ $\delta_{14}=-2$	Supply 35
$u_2+8=9$ $u_2=1$	9 10	12 10	13 30	7 $1+7-7$ $\delta_{24}=1$	50
$u_3+11=9$ $u_3=-2$	14 $-2+8-14$ $\delta_{31}=-8$	9 10	16 $-2+12-16$ $\delta_{31}=-6$	5 30	40
DEMAND	45	20	30	30	

Not optimal solution

Transportation Simplex

EXAMPLE

		8	6	10	9	<i>Supply</i> 35 50 40
	35	$\delta^*=5$	$\delta_{13}=2$	$\delta_{14}=-2$		
	10	10	30	$\delta_{24}=1$		
		14	9	16	5	
	$\delta_{31}=-4$	10	$\delta_{31}=-6$	30		
<i>DEMAND</i>	45	20	30	30		

Not optimal solution

Transportation Simplex

EXAMPLE

		8	6	10	9	<i>Supply</i>
	35		$\delta^*=5$			35
	10	9	10	12	13	7
			30			50
		14	9	16	5	
			10		30	40
<i>DEMAND</i>	45		20	30	30	

Transportation Simplex

$$\delta^* = \delta_{12} = 5 \Rightarrow X_{12} \text{ ENTER}$$

$$\theta = \text{MIN} \{10, 35\} = 10 \Rightarrow X_{22} \text{ LEAVE}$$

	-	35	8	+	6	10	9	Supply
				$\delta^*=5$				35
	+	10	9		10	12	13	50
				-		30	7	
			14		9	16	5	40
				10			30	
DEMAND		45		20		30	30	

Transportation Simplex

$$\delta^* = \delta_{12} = 5 \Rightarrow X_{12} \text{ ENTER}$$

$$\theta = \text{MIN} \{10, 35\} = 10 \Rightarrow X_{22} \text{ LEAVE}$$

	-	35	8	+	0	6	10	9	<i>Supply</i>
									35
	+	10	9		10	12	30	13	50
					-				
			14		10	9	16	5	40
								30	
<i>DEMAND</i>		45		20			30	30	

Transportation Simplex

$$\delta^* = \delta_{12} = 5 \Rightarrow X_{12} \text{ ENTER}$$

$$\theta = \text{MIN} \{10, 35\} = 10 \Rightarrow X_{22} \text{ LEAVE}$$

	-	8	+	6		10		9	<i>Supply</i>
	35-10		0 + 10						35
		9		12		13		7	50
	+10 + 10		10 - 10		30				
			-						
		14		9		16		5	40
			10				30		
<i>DEMAND</i>	45		20		30		30		

Transportation Simplex

$$\delta^* = \delta_{12} = 5 \Rightarrow X_{12} \text{ ENTER}$$

$$\theta = \text{MIN} \{10, 35\} = 10 \Rightarrow X_{22} \text{ LEAVE}$$

				<i>Supply</i>
	8	6	10	35
25		10		
	9	12	13	50
20			30	
	14	9	16	40
		10	30	
<i>DEMAND</i>	45	20	30	30

Transportation Simplex

$0 + v_1 = 8$
 $v_1 = 8$

$u_1 = 0$

	8	6	10	9	<i>Supply</i>
25		10			35
9		12	13	7	50
20			30		
14		9	16	5	40
		10		30	
<i>DEMAND</i>	45	20	30	30	

Transportation Simplex

$0 + v_1 = 8$
 $v_1 = 8$

$u_1 = 0$		8	6	10	9	<i>Supply</i> 35
	25		10			
$u_2 + 8 = 9$ $u_2 = 1$		9	12	13	7	50
	20			30		
		14	9	16	5	40
			10		30	
<i>DEMAND</i>	45		20	30	30	

Transportation Simplex

	$0+v_1 = 8$ $v_1=8$	$0+v_2 = 6$ $v_2=6$	$1+v_3 = 13$ $v_3=12$	$3+v_4 = 5$ $v_4=2$	
$u_1 = 0$	25	10			Supply 35
$u_2+8=9$ $u_2 = 1$	20		30		50
$u_3+6=9$ $u_3 = 3$		10		30	40
DEMAND	45	20	30	30	

Transportation Simplex

	$0+v_1 = 8$ $v_1=8$	$0+v_2 = 6$ $v_2=6$	$1+v_3 = 13$ $v_3=12$	$3+v_4 = 5$ $v_4=2$	
$u_1 = 0$	25 8	10 6	$0+12 - 10$ $\delta^*=\delta_{13}= 2$	$0+2 - 9$ $\delta_{14}=-7$	Supply 35
$u_2+8=9$ $u_2 = 1$	20 9	$1+6 - 12$ $\delta_{22}=-5$	30 13	$1+2 - 7$ $\delta_{24}= - 4$	50
$u_3+6=9$ $u_3 = 3$	$3+8 - 14$ $\delta_{31}= - 3$	10 9	$3+12 - 16$ $\delta_{31}= -1$	30 5	40
DEMAND	45	20	30	30	

Not optimal solution

Transportation Simplex

	-	25	8	10	6	+	10	9	<i>Supply</i> 35
						$\delta_{13}=2$			
	+	20	9	12	13			7	50
						-	30		
			14	10	9	16		5	40
<i>DEMAND</i>		45		20		30		30	

	8	6	10	9	<i>Supply</i>
-	25	10	0		35
	9	12	13	7	50
+	20		30		
	14	9	16	5	40
		10		30	
<i>DEMAND</i>	45	20	30	30	

		8		6	+	10		9	<i>Supply</i>
	-	25-25		10		0+25			35
		●	→	●					
	+			12		13		7	50
		20+25	←	●		30-25			
				●		-			
		14		9		16		5	40
				10				30	
<i>DEMAND</i>		45		20		30		30	

Transportation Simplex

		8		6		10		9	<i>Supply</i>
			10		25				35
	9		12		13		7		50
45				5					
	14		9		16		5		40
		10				30			
<i>DEMAND</i>	45		20		30		30		

Transportation Simplex

$0 + v_2 = 6$
 $v_2 = 6$

$u_1 = 0$

	8	6	10	9	<i>Supply</i> 35
	10	25			
	9	12	13	7	50
45		5			
	14	9	16	5	40
	10		30		
<i>DEMAND</i>	45	20	30	30	

Transportation Simplex

$0+v_2 = 6$ $0+v_3 = 10$
 $v_2=6$ $v_3=10$

$u_1=0$

	8	6	10	9	<i>Supply</i> 35
	10	25			
	9	12	13	7	50
	45	5			
	14	9	16	5	40
	10		30		
<i>DEMAND</i>	45	20	30	30	

Transportation Simplex

		$0+v_2 = 6$ $v_2=6$		$0+v_3 = 10$ $v_3=10$		
$u_1=0$ $u_2+10=13$ $u_2=3$		8	6	10	9	<i>Supply</i>
		10	25			35
		9	12	13	7	50
	45			5		
	14	9	16	5		
		10		30		40
<i>DEMAND</i>	45	20	30	30		

Transportation Simplex

		$3+v_1 = 9$ $v_1=6$	$0+v_2 = 6$ $v_2=6$	$0+v_3 = 10$ $v_3=10$	
$u_1=0$		8	6	10	9
		10	25		
$u_2+10=13$ $u_2=3$		9	12	13	7
		45	5		
		14	9	16	5
		10		30	
<i>DEMAND</i>	45	20	30	30	
					<i>Supply</i>
					35
					50
					40

Transportation Simplex

		$3+v_1 = 9$ $v_1=6$	$0+v_2 = 6$ $v_2=6$	$0+v_3 = 10$ $v_3=10$	$3+v_4 = 5$ $v_4=2$	
$u_1=0$		8	6	10	9	<i>Supply</i> 35
		10	25			
$u_2+10=13$ $u_2 = 3$		9	12	13	7	50
		45	5			
$u_3+6=9$ $u_3 = 3$		14	9	16	5	40
		10	30			
<i>DEMAND</i>	45	20	30	30		

Transportation Simplex

		$3+v_1 = 9$ $v_1=6$	$0+v_2 = 6$ $v_2=6$	$0+v_3 = 10$ $v_3=10$	$3+v_4 = 5$ $v_4=2$	
$u_1=0$		8	6	10	9	Supply
		$0+6 - 8$ $\delta_{12} = -2$	10	25	$0+2 - 9$ $\delta_{14} = -7$	35
$u_2+10=13$ $u_2 = 3$		9	12	13	7	50
		45	$3+6 - 12$ $\delta_{22} = -3$	5	$3+2 - 7$ $\delta_{24} = -2$	
$u_3+6=9$ $u_3 = 3$		14	9	16	5	40
		$3+6 - 14$ $\delta_{31} = -5$	10	$3+10 - 16$ $\delta_{31} = -3$	30	
DEMAND		45	20	30	30	

Optimal Solution

There is no positive number in δ

Transportation Simplex

The Optimal Solution is

	8	6	10	9	<i>Supply</i> 35
	10	25			
45	9	12	13	7	50
		5			
	14	9	16	5	40
		10		30	
<i>DEMAND</i>	45	20	30	30	

Total Cost = $(45 \times 9) + (10 \times 6) + (10 \times 9) + (25 \times 10) + (5 \times 13) + (30 \times 5) = 1020$ \$

TRANSPORTATION PROBLEM

NOTES:

- **THE OPTIMAL SOLUTION IS UNIQUE IF ALL THE VALUES OF $\delta < 0$ or**

$$\delta = 0$$



Thank You